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THE PRINCIPLES OF ELECTRIC POWER TRANSMISSION

BY ALTERNATING CURRENTS

BY

H. WADDICOR

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LONDON

CHAPMAN & HALL, LTD.

11 HENRIETTA ST., W.C. 2

1928

PRINTED IN GREAT BRITAIN
BY THE ABERDEEN UNIVERSITY PRESS
ABERDEEN, SCOTLAND

REF
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AUTHOR'S PREFACE.

THERE appears to be a need for a suitable textbook for use by students pursuing a course of study in Electric Power Transmission at Universities and Technical Colleges. This book is intended primarily to meet this need, but the practical side has been kept in view, and it is hoped that the subject-matter will also be of direct utility to designers and those engineers responsible for the operation of power transmission systems. It is not claimed that anything novel is presented herein, but rather that the work is a systematic exposition of the principles underlying the electrical design of transmission lines, and contains in a complete and concise form results hitherto recorded in a very scattered literature.

In the treatment of the subject familiarity with the use of complex quantities is presupposed, as a general course in alternating currents can hardly be considered complete without a discussion of this method of calculation. A knowledge of hyperbolic functions is an advantage when reading Chapter V., but the hyperbolic method of solution can be readily used for the solution of problems without this knowledge in the same way that tables of logarithms can be efficiently employed without understanding their theory or method of derivation. Wherever possible fully-worked numerical examples have been introduced in order to clarify the subject.

It is not suggested in connection with the exact mathematical methods of solution that these should be used for all line calculations. In actual practice charts and nomograms are often used to determine line constants, regulation, etc., and probably furnish results to the degree of accuracy warranted by the state of knowledge of the data of the problem. This, however, does not detract from the importance of the more

rigorous methods, and there are many problems which cannot properly be solved without their assistance. As Miles Walker has said in connection with another branch of electrical engineering, the art of the designer is to know when to calculate and when to use the more approximate methods.

The author wishes to thank the following individuals and firms: British Insulated Cables, Ltd., W. T. Henley's Telegraph Works Co., Ltd., and Metropolitan-Vickers Electrical Co., Ltd., for various data; the editor of *World Power* for permission to reprint the substance of an article on the "Hyperbolic Method of Line Solution," published by the author in the August and September (1924) issues of the journal; also the authors and publishers of the following books: *Electrical Characteristics of Transmission Circuits* (Westinghouse Technical Night School Press), by W. Nesbit; *Electrical Phenomena in Parallel Conductors* (John Wiley & Sons), by F. E. Pernot; *Electric Power Transmission* (McGraw-Hill Book Co.), by A. Still; *Constant-Voltage Transmission* (John Wiley & Sons), by H. B. Dwight; and *Protective Relays* (McGraw-Hill Book Co.), by V. H. Todd, for permission to reproduce various diagrams and material. The first-named book, in particular, is a mine of information, and should be on the bookshelf of every transmission line engineer.

In preparing the book full advantage has been taken of the information contained in original papers widely scattered throughout technical literature. References to the more important of these papers have been given in the hope that they will serve as a basis for the student's further reading.

As regards the illustrations practically all of these have been specially drawn for the book by Mr. A. Nash, Assoc.M.C.T., to whom the author is greatly indebted.

Great care has been taken with the numerical work, but it is too much to hope that no inaccuracies have crept in, and the author would be grateful if readers would bring any such errors to his notice.

H. WADDICOR.

WEMBLEY, MIDDLESEX,
January, 1928.

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LIST OF SYMBOLS.

A . . . = cross-sectional area of conductor.
= resistance per cm. length of cable to flow of heat from outside surface to surrounding air.
= $\cosh \sqrt{ZY} = A_1 + jA_2$, auxiliary constant of line.

$A, A_1, A_2,$. = constants in solution for differential equations of line (Art. 41).

A_1 . . . = real component of auxiliary constant A.

A_2 . . . = imaginary component of auxiliary constant A.

a . . . = distance between centres of conductors and axis of overhead line or cable.
= spacing between conductors B and C.
= co-ordinate of centre of circle in circle diagram.
= percentage to cover annual interest and depreciation on cost of conductors.

B . . . = susceptance of conductor to neutral.
= magnetic flux density.
= $\sqrt{\frac{Z}{Y}} \sinh \sqrt{ZY} = B_1 + jB_2$, auxiliary constant of line.

B_1 . . . = real component of auxiliary constant B.
= susceptance of receiving circuit.

B_2 . . . = imaginary component of auxiliary constant B.

b . . . = spacing between conductors A and C.
= co-ordinate of centre of circle in circle diagram.
= susceptance of conductor per unit length of line.
= barometric pressure of air.

b_0 . . . = primary exciting susceptance of transformer.

C . . . = capacitance of conductor.
= $\sqrt{\frac{Y}{Z}} \sinh \sqrt{ZY} = C_1 + jC_2$, auxiliary constant of line.

C' . . . = capacitance of bunched conductors to sheath in multi-core cable per cm. length.

C_0 . . . = capacitance of conductor per mile.

C_0' . . . = capacitance per mile of single conductor to earth.

C_1 . . . = real component of auxiliary constant C.

C_2 . . . = imaginary component of auxiliary constant C.

C_A . . = capacitance of conductor A per cm.

C'_A . . = capacitance of conductor A per cm., including effect of earth.

C_n . . . = total capacitance of string of suspension insulators.

c . . . = spacing between conductors A and B.
= radius of circle in circle diagram.

c_1 . . . = capacitance to earth of suspension-insulator disc.
= capacitance between conductor and intersheath of graded cable.
c_2 . . . = capacitance between adjacent suspension-insulator discs.
= capacitance between intersheath and outer sheath of graded cable.
c_1, c_2, c_3 . . = see Art. 203.
c_c, c_s, . . = cable capacitances (Art. 130).
c_e . . . = equivalent capacitance of transformer circuit at frequencies greater than f_k.
c_g . . . = capacitance of transformer coils to earth per unit length.
c_s . . . = capacitance between transformer coils per unit length.
D . . . = electrostatic flux density.
D_A . . = electrostatic flux density due to conductor A.
D_B . . = electrostatic flux density due to conductor B.
d . . . = distance between centres of conductors.
= conductor diameter.
= $\sqrt[3]{abc}$, equivalent delta spacing of line.
E . . . = voltage to neutral or star voltage of line.
= voltage of generator.
E' . . . = voltage to neutral at terminals of condenser in middle-condenser method.
E_0 . . . = disruptive critical voltage of line.
E_1, E_2 . . = voltage across 1st and 2nd sections of graded cable.
E_1, E_3, E_5, etc. = harmonic voltages.
E_a . . . = flash-over voltage of insulator string.
E_c . . . = voltage to neutral along line at point C.
E_m . . = crest value of voltage to neutral.
E_r, E_{ro} . . = voltage to neutral at receiving end of line under load and no-load conditions respectively.
E_s, E_{so} . . = voltage to neutral at sending end of line under load and no-load conditions respectively.
E_s' . . . = real component of E_s.
E_s'' . . = imaginary component of E_s.
E_v . . . = visual critical voltage of line.
$E_\rho, E_{\rho o}$ . . = voltage to neutral along line at point P under load and no-load conditions respectively.
e . . . = e.m.f. of self-inductance.
= instantaneous value of voltage wave.
= base of Napierian logarithms.
e_0 . . . = instantaneous value of transmitted voltage wave.
e_1 . . . = instantaneous value of reflected voltage wave.
e_1, e_2, e_m . = voltage across 1st, 2nd, and mth unit of suspension-insulator string.
e_7 . . . = voltage across capacitance of circuit due to 7th harmonic.
e_a . . . = flash-over voltage of suspension-insulator disc.
e_{sh} . . . = voltage induced in open-circuited sheath of single-core cable.
F . . . = electrostatic force.
= mechanical force between conductors.
f . . . = frequency in cycles per second.
f_k . . . = critical frequency of transformer circuit.
G . . . = conductance of conductor to neutral.
= thermal resistance of soil.
= weight per cm. length of conductor.
G' . . . = sum of conductances of conductors.

G_1 . . . = conductance of receiving circuit.
g . . . = conductance to neutral per unit length of line.
= potential gradient.
= thermal resistivity of soil.
= specific gravity of conductor material.
g_0 . . . = primary exciting conductance of transformer.
= disruptive strength of air at 0° C. and 76 cm. barometric pressure.
g_{max} . . = maximum potential gradient (gradient at surface of conductor).
g_{min} . . = minimum potential gradient (potential gradient at inside of cable sheath).
g_v . . . = apparent disruptive strength of air at surface of conductor when corona begins.
H . . . = magnetic field intensity.
= heat generated in cable conductors due to I^2R losses.
h . . . = height of conductor above earth's surface.
= emissivity constant of outside surface of cable.
I . . . = current in each conductor.
= R.M.S. current during twelve months (Art. 144).
= R.M.S. current in conductor during short-circuit (Art. 172).
I' . . . = current inside conductor within radius x (Art. 14).
I_1 . . . = current in high-tension winding of transformer.
I_2 . . . = current in low-tension winding of transformer.
I_7 . . . = seventh harmonic of current wave.
I_A, I_B, I_C . = current in conductors A, B, and C respectively.
I_c . . . = charging or capacitance current of conductor.
= current along line at point C.
I_{cr}, I_{cs} . . = current consumed by receiving- and sending-end condenser respectively.
I_p . . . = active current at receiving end of line.
I_q, I_{qo} . . = reactive current at receiving end of line under load and no-load conditions respectively.
I_r, I_{ro} . . = current at receiving end of line under load and no-load conditions respectively.
I_7' . . . = see Art. 47.
I_s, I_{so} . . = current at sending end of line under load and no-load conditions respectively.
I_s' . . . = real component of I_s.
I_s'' . . . = imaginary component of I_s.
I_{sc} . . . = permanent short-circuit current.
I_{sc}' . . . = initial short-circuit current.
I_{sh} . . . = induced current flowing in short-circuited sheath of single-core cable.
I_α, I_β, I_γ . = current per conductor in single-phase, two-phase, and three-phase systems of transmission respectively (Arts. 9 and 10).
I_ρ, $I_{\rho o}$ . . = current along line at point P under load and no-load conditions respectively.
i . . . = instantaneous value of current wave.
i_0 . . . = instantaneous value of transmitted current wave.
i_1 . . . = instantaneous value of reflected current wave.
i_1, i_2, i_l . . = see Art. 203.
j . . . = $\sqrt{-1}$, an operator.
K . . . = thermal resistivity of cable dielectric.
= a constant (Art. 123).

K_1 . . . = average thermal resistivity of protective covering of cable.
K_d . . . = Luke's dissipation constant (Art. 147).
k . . . = ratio of reduction of I^2R losses due to influence of charging current.
= $\frac{\text{mutual capacitance of suspension-insulator units}}{\text{earth capacitance of suspension-insulator units.}}$
k_0, k_1, k_2 . = constants (Arts. 141 and 142).
kVA_r . . = kilovolt-amperes per phase at receiving end of line.
kW_r . . = kilowatts per phase at receiving end of line.
kVA_s, kVA_{so} . = kilovolt-amperes per phase at sending end of line under load and no-load conditions respectively.
kW_s, kW_{so} . = kilowatts per phase at sending end of line under load and no-load conditions respectively.
kW_{max} . . = maximum load transmitted by a constant-voltage line.
L . . . = inductance of conductor.
= armature leakage inductance of alternator.
= inductance of transformer coils per unit length.
L_0 . . . = inductance of conductor per mile.
L_0' . . . = inductance per mile of single conductor to earth.
L_1 . . . = external inductance of conductor.
L_2 . . . = internal inductance of conductor.
L_e . . . = equivalent inductance of transformer circuit at frequencies less than fk.
l . . . = length of line.
= depth of cable axis below surface of ground.
m . . . = weight of conductor 1·0 sq. in. cross-section in lbs. per mile.
= $\frac{2\pi^2 r^2 f \mu}{\rho} 10^{-9}$ (Art. 59).
= an integer.
m_o, m_v . . = irregularity factors in corona formulæ.
N . . . = total number of wires in stranded conductor.
n . . . = number of phases or conductors.
= number of line sections.
= number of units in string of suspension insulators.
= number of layers in stranded conductor.
= fractional conductivity of copper.
= an integer.
n_1 . . . = number of turns on high-tension winding of transformer.
n_2 . . . = number of turns on low-tension winding of transformer.
P . . . = kW. per phase delivered at receiving end of line.
P_c . . . = corona power loss per phase per mile.
P_1 . . . = annual charge on capital cost of line (or conductors).
P_2 . . . = annual cost of energy losses in line.
p . . . = power loss in line per phase.
p_1 . . . = cost of conductors in pence per lb.
p_2 . . . = cost of one kWH. of energy in pence.
Q . . . = electrostatic charge.
= reactive kVA. per phase delivered at receiving end of line.
Q_1 . . . = reactive kVA. of load.
Q_{pm}, Q_{pmo} . = reactive kVA. taken by synchronous phase modifiers under load and no-load conditions respectively.
q . . . = electrostatic charge per cm. of conductor.
R . . . = resistance of conductor.
= resistance of conductor to flow of steady current (Art. 59).
= internal radius of lead sheath.

R' . . . = effective resistance of conductor to flow of alternating current (Art. 59).
= insulation resistance of cable (bunched conductors to sheath).
R_0 . . . = resistance of conductor at 0° C.
R_1 . . . = external radius of lead sheath.
= resistance of receiving circuit.
R_2 . . . = overall radius of cable.
R_{eff} . . = effective resistance of single-core cable
R_m . . . = resistance of conductor 1·0 sq. in. cross-section in ohms per mile.
R_s . . . = resistance of lead sheath (Art. 124).
R_t . . . = resistance of conductor at t° C.
= equivalent resistance of transformer.
R_α, R_β, R_γ, . = resistance of conductors in single-phase, two-phase, and three-phase systems of transmission respectively.
R_θ . . . = resistance of conductor at temperature corresponding to temperature rise of θ° C.
r . . . = radius of conductor.
= resistance of conductor per unit length of line.
= resistance of conductor per cm. (Art. 172).
r_1 . . . = resistance of high-tension winding of transformer.
= external radius of 1st layer of insulation in graded cables.
r_2 . . . = resistance of low-tension winding of transformer.
= external radius of intersheath in graded cable.
S . . . = $E_s^2 - 2\,000\,(PR + QX)$ . . . (Art. 26).
= specific heat of conductor material.
= thermal resistance per cm. between conductors and outer surface of cable.
S' . . . = thermal resistance per cm. between conductors and sheath.
S'' . . . = thermal resistance per cm. between sheath and outside of cable.
T . . . = $\left(R^2 + X^2\right)\left(\frac{1\,000\,P}{\cos\,\phi_r}\right)^2$ . . . (Art. 26).
= conductor-insulation thickness.
t . . . = time in seconds.
= belt-insulation thickness.
= temperature.
t_1 . . . = temperature in degrees C.
V . . . = voltage or potential difference between conductors.
= $\pm\sqrt{zy}$ . . . (Art. 41).
V_m . . . = crest value of voltage between conductors.
v . . . = potential.
= velocity of propagation of electric field.
= modulus or size of line angle (Art. 49).
v_m . . . = potential at cross-arm end of mth unit of suspension-insulator string.
W_d . . . = total dielectric losses of cable per phase.
W_h . . . = hysteresis losses of cable per phase.
W_l . . . = leakage losses of cable per phase.
X . . . = reactance of conductor.
X_1 . . . = reactance of receiving circuit.
X_2 . . . = internal reactance of conductor with a steady current.
X_2' . . . = effective internal reactance of conductor with alternating currents. (Art. 59).
X_c, X_m . . = see Art. 124.
X_{eff} . . = effective reactance of single-core cable.

X_t . . . = equivalent reactance of transformer.
x . . . = a distance.
= reactance of conductor per unit length of line.
= iron loss of transformer.
= argument of sinh x, etc.
= interaxial spacing of grouped cables.
x_1 . . . = reactance of high-tension winding of transformer.
x_2 . . . = reactance of low-tension winding of transformer.
x_l . . . = inherent or leakage reactance of alternator.
x_r . . . = component of synchronous reactance of alternator due to armature reaction.
x_s . . . = synchronous reactance of alternator.
Y . . . = admittance of conductor to neutral.
Y_0 . . . = surge or natural admittance of line.
Y_1 . . . = admittance of receiving circuit.
Y_m . . = see Art. 124.
y . . . = admittance of conductor per unit length of line.
= copper loss of transformer at full load.
y_0 . . . = primary exciting admittance of transformer.
Z . . . = impedance of conductor.
Z_0 . . . = surge or natural impedance of line.
Z_0' . . = surge or natural impedance of terminal circuit.
Z_1 . . . = impedance of receiving circuit.
Z_c, Z_2 . . = see Art. 124.
Z_{eff} . . = effective impedance of single-core cable.
Z_s . . . = impedance of lead sheath (Art. 124).
z . . . = impedance of conductor per unit length of line.
α . . . = real component of complex angle.
= temperature coefficient of resistance of copper.
$= \left(\frac{a+r}{R}\right)^3$ . . . (Art. 114).
α, α_1, α_2 . . = impulse ratio of spark gaps.
α_0, α_t, α_{t_1} . = temperature coefficient of resistance of copper at 0°, t°, and t_1° C. respectively.
β . . . = imaginary component of complex angle.
$= \left(\frac{a-3r}{a+3r}\right)\alpha$ . . . (Art. 114).
β, β_1, β_2 . . = setting factor of spark gaps.
δ . . . = air density factor in corona formulæ.
δ_c . . . = position angle along line at point C.
δ_r, δ_{ro} . . = position angle at receiving end of line under load and no-load conditions respectively.
δ_s . . . = position angle at sending end of line.
δ_ρ, $\delta_{\rho o}$ . . = position angle along line at point P under load and no-load conditions respectively.
ϵ . . . = permittivity.
ϵ_1, ϵ_2, ϵ_3 . . = permittivity of 1st, 2nd, and 3rd layers of dielectric in graded cable.
θ . . . = $\sqrt{ZY}$ = complex angle of line.
= $2\pi ft$ (Art. 56).
= temperature rise in deg. C.
$= \tan^{-1}\frac{R}{X}$ . . . (Art. 24).
θ_1 . . . = temperature rise of cable conductors when running alone.

θ_r, θ_{ro} . . = complex angle of receiving circuit under load and no-load conditions respectively.

θ_v . . . = argument or slope of line angle (Art. 49).

θ_x . . . = temperature rise of cable conductors due to effect of neighbouring cable.

μ . . . = permeability.

ν_1, ν_2, ν_3 . . = dielectric strength of 1st, 2nd, and 3rd layers of dielectric in graded cable.

π . . . = 3·14159 . . . .

ρ . . . = resistivity of conductor material.
= resistivity of cable dielectric.

ρ_0, ρ_t . . = resistivity of cable dielectric at 0° and t° C. respectively.

σ . . . = impedance load to earth at receiving end of line (Art. 54).

Φ . . . = magnetic flux.

Φ_1 . . . = magnetic flux surrounding a conductor (external flux).

Φ_2 . . . = magnetic flux inside cross-section of conductor (internal flux).

Φ_1, Φ_2, Φ_3 . = magnetic flux surrounding each cm. of conductor A due to A, B, and C in initial condition and after 1st and 2nd transposition respectively.

Φ_A, Φ_B, Φ_C . = magnetic flux between conductor A and neutral per cm. length of line due to A, B, and C respectively.

ϕ . . . = phase-angle.

ϕ_d . . . = angle of advance of charging current of cable.

ϕ_r . . . = receiving-end phase-angle.

ϕ_s, ϕ_{so} . . = sending-end phase-angle under load and no-load conditions respectively.

ϕ_ρ, $\phi_{\rho o}$ . . = phase-angle along line at point P under load and no-load conditions respectively.

ψ . . . = dielectric flux.

ω . . . = $2\pi f$ = electrical angular velocity.

THE PRINCIPLES OF ELECTRIC POWER TRANSMISSION

BY ALTERNATING CURRENTS

CHAPTER I.

ELEMENTARY ECONOMIC AND ELECTRICAL PRINCIPLES.

1. Electrical Transmission of Power.—Compared with water, steam, compressed air, or mechanical power, electrical power possesses enormous advantages as it may be carried in far greater quantities, and to far greater distances, at a very high efficiency. Atmospheric temperatures and pressures, and differences of land elevation, have little or no effect on its transmission, and methods of control are extremely elastic and convenient. Electric power, in fact, is the highest form of power known since it can be changed to other forms, heat, light, motive power, or chemical action, with unparalleled directness and simplicity.

Electric power transmission has its *raison d'être* and attains successful results owing to the following facts:—

1. By its aid distant and otherwise impracticable natural sources of power such as waterfalls may supersede steam generating stations.

2. Power can be produced more cheaply by steam in one large generating station than in numerous small stations.

Thus there exist two main types of transmission: those starting from water-power sites and converging towards the industrial centres; those from large steam-driven stations, originating at the large centres of population and radiating into the country, supplying current for railways, light and power in outlying towns and villages.

Hydro-electric power schemes usually involve a large initial

capital outlay but the running expenses are extremely small, and thus they have a long economic radius of transmission. Steam plants cannot transmit power economically over such great distances as are employed by hydro-electric plants, but nevertheless quite a considerable area may be served. This is due to savings effected in point of: (1) the reduced capital cost per kilowatt of large, compared with small, generating units; (2) the reduced fuel costs, and lower attendance and maintenance costs, per kilowatt-hour made possible by the large prime movers and higher load factor; (3) the lower percentage of reserve plant required in a large central station compared with several small stations. Other factors being equal, wholesale production of electric power in large generating stations operated by experts in power production enables energy to be sold to a distant consumer more cheaply than he could produce it himself.

2. Power Supply to Consumers.—Electric power being proportional to the product of two factors, *viz.* current and voltage, can be supplied to consumers either at constant current or constant voltage.

There are, however, serious limitations to the former method of supply, *e.g.* the design of a satisfactory constant-speed constant-current motor presents difficulties, and this method of supply is of no importance commercially.

At the present day the supply of electric power is effected by the use of an approximately constant voltage; the current being the variable quantity. The various consuming devices, such as lamps, motors, or heating appliances in which energy is applied to some useful purpose, are designed to operate at a definite voltage, and the pressure supplied to the consumers' terminals is maintained as close as possible to this figure.

An upper limit is fixed to the supply voltage by the following considerations:—

1. The maximum voltage for which the consuming devices may be constructed so as to have a reasonably high efficiency and long life.

2. The danger to human life which arises when the voltage exceeds a certain figure.

3. Systems of Transmission.—Although energy can be transmitted electrically either by means of alternating currents or continuous currents, practically all existing transmission schemes make

use of three-phase alternating currents. It will be shown later that for economy in the transmission of power, high line voltages must be employed, and the longer the distance of transmission the higher the economic voltage of the system. With alternating currents, no matter what the generating-station voltage, it can be stepped up by transformers with ease and efficiency to a high line voltage. At the receiving end of the line pressure transformations can be again effected to the voltage suitable for distribution. In fact the system is extremely flexible, and the distribution of power in scattered districts at any voltage desired by the consumer is a very simple matter.

The popularity of the three-phase system of transmission rather than the somewhat simpler single-phase system must without doubt be largely ascribed to the superior operating characteristics of poly-phase plant, and also to the fact that three-phase generators and motors are considerably cheaper than single-phase ones of the same output. When applied to electric traction work, however, the three-phase system possesses a serious drawback in that two insulated conductors have to be provided—the rails being employed as the third phase conductor. The line construction at junctions and crossings is thus exceedingly complicated. A further disadvantage of the three-phase system is that induction motors are not suitable for economical speed variation. Single-phase power transmission has accordingly found a limited sphere of application in connection with electric traction work. By its adoption the construction of the line is simplified, and it is possible to take full advantage of the variable-speed characteristic of the single-phase commutator motor.

As regards continuous currents the chief obstacle to their use for transmission work has been the difficulty of generating sufficiently high voltages and of utilising the high-voltage energy at the receiving end of the line. In many respects, however, continuous currents have an advantage, particularly for very long-distance transmissions or for underground systems, and there are several European installations operating on the Thury system. This method of transmission contains unique features, but it can only be considered a partial solution of the problem. More recently the invention of the 'transverter' by Calverley and Highfield has again directed attention to the merits of the continuous-current system, and there is no doubt that the successful development of this

apparatus would lead to the adoption of continuous currents for many power transmission schemes.[1]

4. Typical Alternating-current Power Scheme.—Fig. 1 gives a single-line diagram of a typical alternating-current power

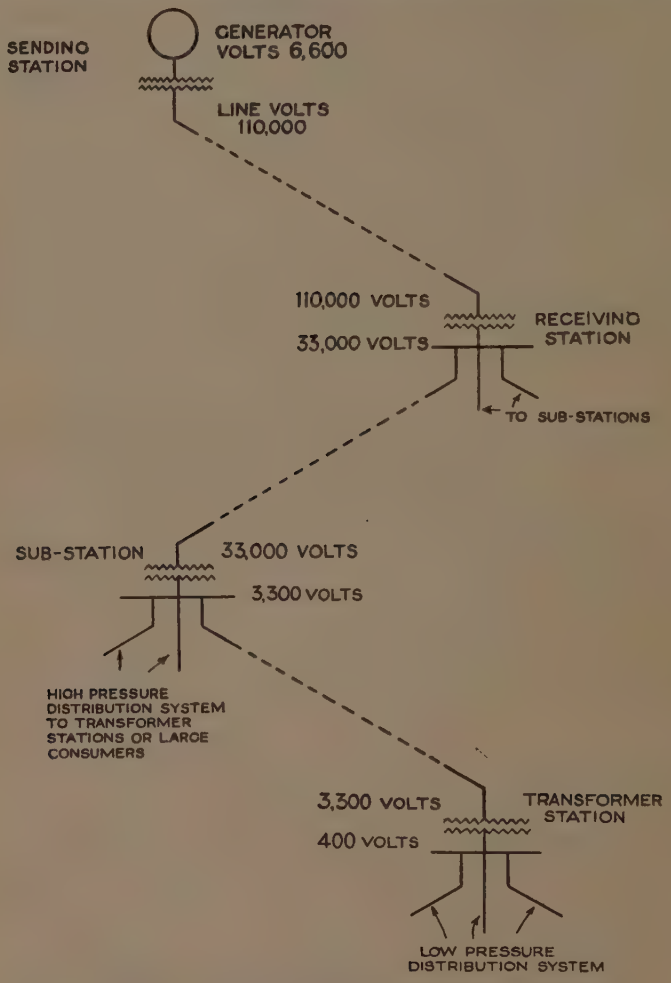


FIG. 1.—Typical alternating-current power transmission scheme.

supply scheme. The generating station, which is situated near a cheap source of power, such as a waterfall or easily worked coal deposit, operates at a moderately high voltage of 6 600 or 11 000 volts. This pressure is then stepped up by transformers to the

very much higher pressure required for the transmission line itself —this may be anything up to a couple of hundred kilovolts, depending to a great extent on the length of the line and the amount of power to be transmitted. The other end of the line terminates at the receiving station, which is located at a convenient point near to the industrial district to be supplied with power. At this station the voltage is reduced by step-down transformers to a value suitable for supplying the various substations dotted up and down the district. From these substations radiate the distributing mains to which the actual consumers are connected. In the majority of cases there are two distinct sets of distributing mains in use :—

1. A high-pressure distribution system operating at about 2 200 or 3 300 volts for power supply to large consumers.

2. A low-pressure distribution system from transformer stations installed near groups of consumers. This supply may be at about 400 / 230 volts, using the line voltage for power and the star voltage for lighting loads.

5. Overhead *versus* Underground Lines.—An underground system is usually more expensive than an overhead system designed for the same operating voltage, consequently lines for power transmission are placed overhead when the conditions are suitable.

Thus overhead lines are used in the following cases :—

1. For the extremely high pressures which must be used for transmitting power over long distances, where this method of construction must be used in order to provide adequate insulation for the conductors.

2. For the lower pressures (where underground cables would also be technically suitable), when the line route runs through open country.

In the neighbourhood of cities, however, underground cables are generally employed, partly to eliminate the danger to human life which would be present with overhead conductors, and partly to avoid the unsightly appearance and inconvenience of pole lines running down the main thoroughfares.

Hitherto the principal use of cables has been in connection with city distribution systems at comparatively low pressures, and only to a very limited extent have they been used for the straight transmission of power. During the last few years, however, great improvements have been made in the art of cable making, and multi-core cables for working at 55 000 and 66 000 volts are now

a commercial proposition. Underground transmission at 132 000 volts, using single-core cables, is also being tried out, and there is every probability that in the near future more use will be made of high-tension underground cables for moderate-distance power transmission in densely populated areas.

6. Influence of Voltage on Amount of Conductor Material Required.—In most schemes of power transmission the cost of the conductor material is a large item in the capital expenditure and has to be considered when fixing the line pressure.

In general, in transmitting a given amount of power at a given loss over a given distance, the amount of copper required in the conductors is inversely proportional to the square of the pressure employed. For if the pressure is raised m times the current in the line required to transmit the same amount of power is reduced m times, and the line losses which are proportional to the product of the square of the current and the line resistance are reduced m^2 times. So to transmit with the same losses as before the resistance of the line can be increased m^2 times, that is the amount of copper in the conductors can be decreased m^2 times.

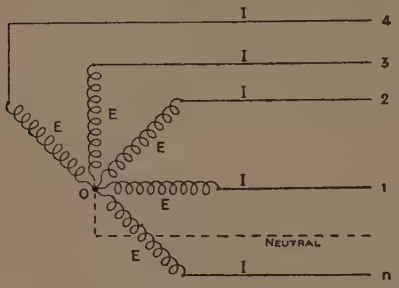


FIG. 2.—Polyphase system of transmission.

This fact can also be shown very clearly as follows:—

Let E be the voltage to neutral of an n-phase system of transmission represented diagrammatically in Fig. 2. I is the current, lagging behind the voltage by ϕ degrees on each phase of the system.

Then the total power transmitted is

$$P = \frac{EI \cos \phi}{1\,000} \text{ kW. per phase.} \quad . \quad . \quad (1)$$

If R is the resistance of each line conductor, the power loss will be

$$p = \frac{I^2R}{1\,000} \text{ kW. per phase.} \quad . \quad . \quad (2)$$

Substituting for I^2 from equation (1) gives

$$p = \frac{1\,000P^2R}{E^2 \cos^2 \phi} \quad . \quad . \quad . \quad . \quad (3)$$

The volume of copper per phase equals lA ; where l is the length of the line, and A is the cross-sectional area of the conductors. If ρ is the resistivity of the copper,

$$R = \frac{l\rho}{A}.$$

Substituting for R in (3) gives

$$p = \frac{1\,000P^2l\rho}{AE^2\cos^2\phi},$$

or

$$A = \frac{1\,000P^2l\rho}{pE^2\cos^2\phi}.$$

Thus the volume of copper in each line conductor is

$$lA = \left(\frac{1\,000P^2l^2\rho}{p}\right)\frac{1}{E^2\cos^2\phi} \quad . \quad . \quad . \quad (4)$$

the quantity in brackets being a constant for any given line and transmission efficiency and given amount of power to be delivered.

The above formula, besides showing the great saving of copper possible by employing a high line voltage, indicates also the importance of working as near unity power factor as possible. The effect of operating at, say, 0·8 power factor instead of unity, is to increase the amount of copper used in the ratio of $1 : \frac{1}{0{\cdot}8^2}$, or an increase of 56 %. A very considerable economy can be effected in the conductor material if a high load power factor can only be guaranteed.

7. Practical Limits of Line Voltage.—From formula (4) it might appear advisable to use the highest possible obtainable line pressures for the transmission of energy as by so doing the expenditure on conductors would be reduced to a minimum. On each particular transmission, however, there is a superior limit fixed for the pressure beyond which there is nothing to be gained in the matter of economy. This limit is reached when the saving in cost of conductor material due to the higher voltage is more than counterbalanced by (1) the increased cost of insulating the conductors, and (2) the increased cost of transformers, switchgear and other terminal apparatus, which is greater for the higher voltages.

The determination of the best line voltage is thus essentially an economic problem and will be discussed more fully in a later chapter.

8. Basis of Comparison of Systems as regards Copper Efficiencies.—From a technical point of view it is evident that in long-distance transmission lines the pressure is only limited by the problem of insulating the conductors against disruptive discharge, and hence when comparing the respective amounts of copper required in various systems the proper comparison is on the basis of equality of the maximum potential difference; that is equal maximum stress on the dielectric. The comparison voltage may be either the potential difference between any two conductors of the system, or the potential difference between any conductor and earth, depending on the character of the circuit.

In an overhead system it is easy to avoid disruptive discharge between the conductors at points intermediate to the line supports by suitably spacing the conductors. At the supporting towers themselves, the insulation between conductors is indirectly provided by insulating each conductor from earth (represented by the tower cross-arm). Thus the disruptive stress is from conductor to earth, and the comparison of systems in this case has to be made on the basis of maximum potential difference between conductor and earth. It is usually assumed that the neutral point of the system is earthed, but whether or not if the system is of any considerable extent the neutral will be maintained at earth potential by the capacitance currents flowing.

With underground transmission by means of multi-core cables the chief stress on the insulation is from core to core. Hence in this case, equality of the maximum potential difference between conductors must be taken as the basis of comparison of systems.

In an underground transmission using single-core cables provided with lead sheaths, the sheaths being bonded together and earthed, the disruptive stress is from conductor to earth. Systems employing single-core cables must then be compared on the basis of maximum potential difference between conductor and earth.

9. Copper Efficiencies of Systems on the Basis of Equal Maximum Potential Difference between any Conductor and Earth.—In comparing the relative amounts of conductor material necessary for different systems of transmission, similar conditions will be assumed in each case, *viz.* (1) equal amounts of power transmitted; (2) equal lengths of line; and (3) equal power losses in the line.

Case 1. *Continuous-current.* (Fig. 3 (*a*).)—Let E_m be the

maximum potential difference between any conductor and earth, and I the current. Assuming that the middle point of the system is earthed the line voltage is $2E_m$, the power transmitted is $2E_mI$, and if R is the resistance of each conductor the power loss is $2RI^2$.

Case 2. *Single-phase.* (Fig. 3 (*b*).)—E_m again being the maximum potential difference to earth the R.M.S. voltage to earth is

$$\frac{E_m}{\sqrt{2}},$$

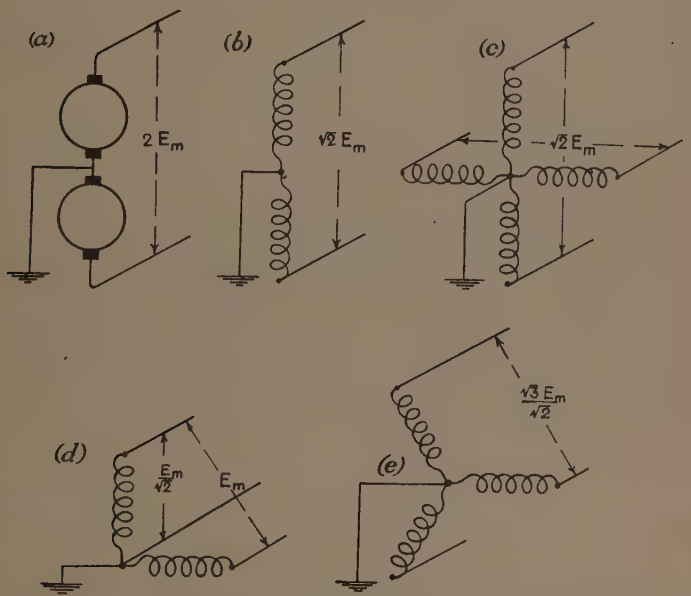


Fig. 3.—Transmission systems with same maximum potential above earth.

and between conductors its value is

$$\frac{2E_m}{\sqrt{2}} \text{ or } \sqrt{2}E_m.$$

The amount of power to be transmitted is the same as before, *viz.* $2E_mI$, so the line current at a power factor $\cos\phi$ is

$$I_a = \frac{2E_mI}{\sqrt{2}E_m \cos\phi} = \frac{\sqrt{2}I}{\cos\phi}.$$

If R_a stands for the resistance of each conductor, the power loss in transmission is

$$2R_aI_a^2 = \frac{4R_aI^2}{\cos^2\phi},$$

and for equality of losses

$$\frac{4R_aI^2}{\cos^2\phi} = 2RI^2 \quad \text{or} \quad \frac{R_a}{R} = \frac{\cos^2\phi}{2}.$$

The cross-section of each conductor and the total amount of copper required must therefore be

$$\frac{2}{\cos^2\phi}$$

of the corresponding values in the continuous-current case.

Case 3. *Two-phase, Four-wire.* (Fig. 3 (*c*).)—This system can be considered as two independent single-phase systems each transmitting one-half the total power. The line conductors can thus be reduced to one-half the size required in the single-phase case, but as there are now four conductors instead of two the total amount of copper required will still be the same, or $\frac{2}{\cos^2\phi}$ times the quantity used in the continuous-current transmission.

Case 4. *Two-phase, Three-wire.* (Fig. 3 (*d*).)—The R.M.S. voltage between each outgoing conductor and the common return is

$$\frac{E_m}{\sqrt{2}},$$

and if I_β is the current in each of the outgoing wires the total power transmitted is $\sqrt{2}E_mI_\beta \cos\phi$. Comparing this with the continuous-current case $\sqrt{2}E_mI_\beta \cos\phi$ must equal $2E_mI$, or

$$I_\beta = \frac{\sqrt{2}I}{\cos\phi},$$

and if R_β is the resistance of the outgoing wires, the power loss in each of these is $R_\beta I_\beta^2$. Since the common return carries $\sqrt{2}$ times the current in each of the others the cross-section must be increased in the same ratio to maintain the same current density. The resistance of this common wire is therefore

$$\frac{R_\beta}{\sqrt{2}},$$

and the power loss in it is

$$\frac{R_\beta}{\sqrt{2}}(\sqrt{2}I_\beta)^2 = \sqrt{2}R_\beta I_\beta^2.$$

The total power loss in the system is thus

$$2R_\beta I_\beta^2 + \sqrt{2}R_\beta I_\beta^2 = (2 + \sqrt{2})R_\beta I_\beta^2.$$

Substituting in this for I_β gives the total loss as

$$\frac{2(2 + \sqrt{2})R_\beta I^2}{\cos^2 \phi}.$$

Equating this to the power loss in the continuous-current case gives

$$\frac{2(2 + \sqrt{2})R_\beta I^2}{\cos^2 \phi} = 2RI^2, \text{ or } \frac{R_\beta}{R} = \frac{\cos^2 \phi}{2 + \sqrt{2}}.$$

The ratio of the amount of copper required in this case to that required in the continuous-current system is therefore

$$\left(\frac{2 + \sqrt{2}}{2}\right)\frac{2 + \sqrt{2}}{\cos^2 \phi} = \frac{5{\cdot}82}{\cos^2 \phi}.$$

Case 5. *Three-phase.* (Fig. 3 (*e*).)—The voltage to neutral (R.M.S.) is

$$\frac{E_m}{\sqrt{2}},$$

and if I_γ denotes the current in each conductor, the power transmitted is

$$\frac{3E_m}{\sqrt{2}}I_\gamma \cos \phi,$$

and for equality with the continuous-current system

$$\frac{3E_m}{\sqrt{2}}I_\gamma \cos \phi = 2E_m I, \text{ or } I_\gamma = \frac{2\sqrt{2}I}{3 \cos \phi}.$$

If R_γ is the resistance of each conductor, the total power loss is $3R_\gamma I_\gamma^2$, and substituting for I_γ this gives

$$\frac{8R_\gamma I^2}{3 \cos^2 \phi}.$$

For the same transmission loss as before

$$\frac{8R_\gamma I^2}{3 \cos^2 \phi} = 2RI^2, \text{ or } \frac{R_\gamma}{R} = \frac{3 \cos^2 \phi}{4}.$$

Since, however, there are three conductors in the three-phase system as against two in the continuous-current system, the amount of copper necessary in the former case will be

$$\frac{4}{3\cos^2\phi} \times \frac{3}{2} = \frac{2}{\cos^2\phi}$$

times the amount required by the latter.

10. Copper Efficiencies of Systems on the Basis of Equal Maximum Potential Difference between any Two Conductors.—Equal distances and loads transmitted, and equal line losses will again be assumed.

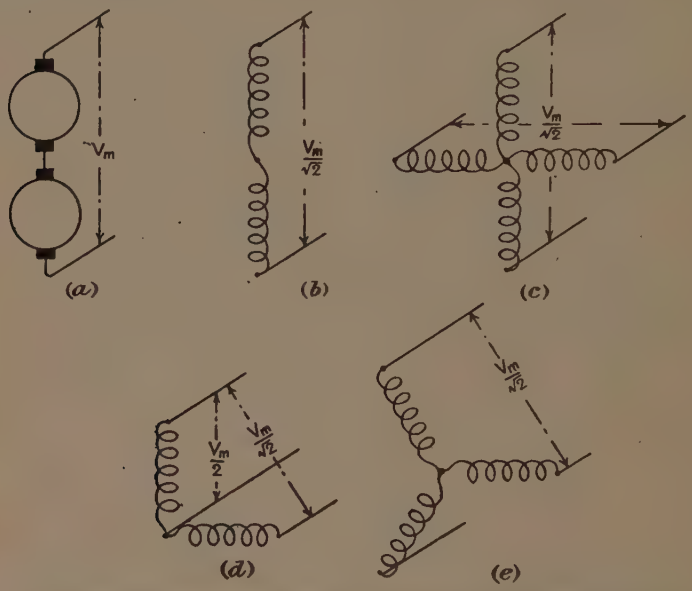


FIG. 4.—Transmission systems with same maximum potential between conductors.

Case 1. *Continuous-current.* (Fig. 4 (*a*).)—If V_m denotes the maximum potential difference between the conductors it will also be the working voltage in this case. R and I being respectively the resistance of each conductor and current flowing, the power transmitted is $V_m I$, and the loss in transmission $2RI^2$.

Case 2. *Single-phase.* (Fig. 4 (*b*).)—In the case of the single-phase system the R.M.S. line voltage is

$$\frac{V_m}{\sqrt{2}},$$

the corresponding current

$$\frac{\sqrt{2}I}{\cos\phi},$$

and if R_a denotes the resistance of each conductor the transmission loss will be

$$\frac{4R_aI^2}{\cos^2\phi}.$$

For equality of loss

$$\frac{4R_aI^2}{\cos^2\phi} \text{ must equal } 2RI^2, \text{ or } \frac{R_a}{R} = \frac{\cos^2\phi}{2}.$$

Thus, the single-phase system requires $\frac{2}{\cos^2\phi}$ times as much copper as the continuous-current system.

Case 3. *Two-phase, Four-wire.* (Fig. 4 (*c*).)—A two-phase system with its phases entirely independent is clearly on the same footing as the single-phase system and will also require $\frac{2}{\cos^2\phi}$ times as much copper as would be necessary on a continuous-current system.

Case 4. *Two-phase, Three-wire.* (Fig. 4 (*d*).)—The maximum voltage, V_m, here occurs between the two outgoing conductors; the maximum voltage per phase is therefore

$$\frac{V_m}{\sqrt{2}},$$

and the R.M.S. voltage per phase

$$\frac{V_m}{2}.$$

If I_β denotes the current in each of the two outgoing wires the total power transmitted is $V_mI_\beta \cos\phi$. Hence $V_mI_\beta \cos\phi$ must equal V_mI, or

$$I_\beta = \frac{I}{\cos\phi}.$$

Supposing the common return to be worked at the same current density as the other two conductors (each of which has a resistance R_β), its resistance is

$$\frac{R_\beta}{\sqrt{2}}$$

and the current flowing in it is

$$\frac{\sqrt{2}I}{\cos\phi}.$$

Hence the total loss in transmission is

$$\frac{2R_\beta I^2}{\cos^2\phi} + \frac{\sqrt{2}R_\beta I^2}{\cos^2\phi} = \frac{(2+\sqrt{2})R_\beta I^2}{\cos^2\phi},$$

and for equality of loss

$$\frac{(2+\sqrt{2})R_\beta I^2}{\cos^2\phi} = 2RI^2, \text{ or } \frac{R_\beta}{R} = \frac{2\cos^2\phi}{2+\sqrt{2}}.$$

Thus the ratio of the amount of copper required in this case to that required in the continuous-current system is

$$\frac{(2+\sqrt{2})\left(\frac{2+\sqrt{2}}{2\cos^2\phi}\right)}{2} = \frac{2{\cdot}91}{\cos^2\phi}.$$

Case 5. *Three-phase.* (Fig. 4(*e*)).—Finally considering the case of the three-phase system, since the R.M.S. voltage between conductors is

$$\frac{V_m}{\sqrt{2}}$$

the total power transmitted is

$$\frac{\sqrt{3}}{\sqrt{2}}V_m I_\gamma \cos\phi$$

where I_γ is the line current. Hence

$$\frac{\sqrt{3}}{\sqrt{2}}V_m I_\gamma \cos\phi$$

must equal $V_m I$, or

$$I_\gamma = \frac{\sqrt{2}I}{\sqrt{3}\cos\phi}.$$

If R_γ is the resistance of each conductor the loss in transmission is

$$3R_\gamma I_\gamma^2 = \frac{2R_\gamma I^2}{\cos^2\phi},$$

so that for equality of power loss

$$\frac{2R_\gamma I^2}{\cos^2\phi} = 2RI^2, \text{ or } \frac{R_\gamma}{R} = \cos^2\phi.$$

As three conductors are required in the three-phase system, as against two in the continuous-current case, the ratio of the amounts of copper required is

$$\frac{1{\cdot}5}{\cos^2\phi}.$$

11. Results of Comparison of Systems.—The results obtained above are summarised in Table 1:—

TABLE 1.—*Copper Efficiencies of Various Systems of Transmission.*

System.	Amount of Conductor Material required.	
	On Basis of Maximum P.D. between Conductor and Earth.	On Basis of Maximum P.D. between any Two Conductors.
Continuous-current . .	Unity	Unity
Single-phase . . .	$\frac{2}{\cos^2\phi}$ units	$\frac{2}{\cos^2\phi}$ units
Two-phase, Four-wire .	$\frac{2}{\cos^2\phi}$ "	$\frac{2}{\cos^2\phi}$ "
Two-phase, Three-wire .	$\frac{5{\cdot}82}{\cos^2\phi}$ "	$\frac{2{\cdot}91}{\cos^2\phi}$ "
Three-phase . . .	$\frac{2}{\cos^2\phi}$ "	$\frac{1{\cdot}5}{\cos^2\phi}$ "

The great economy of conductor material possible by adopting the continuous-current system is clearly shown, particularly when it is remembered that the power factor of an alternating-current system is usually considerably less than unity.

The two-phase three-wire system is obviously quite unsuitable for long-distance transmission and needs no further consideration.

Considering the other alternating-current systems among themselves, it will be noted that there is a decided saving in copper with the three-phase system in the cases compared on the basis of equal maximum potential difference between conductors. In the case where the maximum potential difference between conductor

and earth is the criterion, the three-phase system has no superiority over the single-phase or the two-phase four-wire system.

The advantage of the three-phase system, which causes its practically universal use over the single-phase system, is the greater usefulness of polyphase power, and the advantage over the two-phase four-wire system is the use of only three conductors against four in the latter system.

12. Transmission Voltages in Use.—In Great Britain there are no water-powers of any considerable magnitude except tidal energy, and the latter is still of problematical utility. Energy is generated in coal-burning stations, and areas of supply are small as most of the industrial areas are situated on the coalfields. This fact to a very large extent obviates the necessity for long-distance transmission of power. There is also no doubt that the growth of overhead transmission lines has been hindered by legislative restrictions, and the difficulty and delay experienced in securing suitable rights of way. Hitherto voltages of 33 kV. have been considered sufficiently high, but two or three 66-kV. lines are now in operation, or being constructed. Future developments in connection with the Government superpower scheme, which will bring into being a high-tension transmission network covering the whole country, will undoubtedly call for a very much higher line voltage.

In the British Dominions overseas there are many high-voltage transmission systems in operation. Canada, Australia, New Zealand, South Africa, and India furnish instances of lines employing pressures of 80-132 kV., and the Irish Free State is now constructing a scheme to utilise the water-power of the Shannon which involves transmission at 100 kV.

On the continent of Europe and in many other parts of the world, transmission schemes—chiefly associated with water-power—are too numerous to mention, and line voltages range chiefly from 60 to 150 kV. In several countries, Government schemes are being actively developed for the co-ordination of electricity supply stations by means of interconnected high-tension lines. Examples of these superpower schemes are to be found in Germany, France, Italy, and Sweden where many new lines have been constructed for ultimate operation at 220 kV.; although in most cases a lower voltage will be used till the higher voltage is required.

Undoubtedly the most striking examples of power transmission

are to be found in the United States. This vast country, with its enormous industrial demands situated at long distances from equally large water-powers, can be considered as the home of power transmission schemes. Line pressures up to 220 kV. are in actual use, and a large portion of the country is one immense network of high-voltage overhead lines.

Speaking generally, the trend is towards higher line pressures owing to the larger amounts of power being transmitted. As previously pointed out the choice of voltage is primarily an economic consideration, but recent improvements in the design and construction of insulators and high-tension transformers have undoubtedly materially assisted this development. In this connection it is interesting to note that a pressure of 1 000 kV. has been produced in the laboratory and a short experimental transmission effected at this voltage. It has been established that the empirical laws of corona and spark-over discharge found satisfactory at the ordinary transmission voltages apply with very little modification to pressures of this magnitude. The problems to be solved are mainly practical, concerned with the method of mounting conductors of the diameter, tubular shape and distance apart indicated, but sufficient has been done to prove that line pressures very much higher than any of those employed at the present day can be used when the conditions demand them.[2]

REFERENCES.

[1] 'The English Electric Transverter,' *Electrician*, Vol. 92, p. 567 (1924).

[2] J. Liston, 'Some Developments in the Electrical Industry during 1921,' *Gen. El. Rev.*, Vol. 25, p. 25 (1922).

CHAPTER II.

INDUCTANCE AND CAPACITANCE OF CONDUCTORS.

13. Electric Field between Two Round Parallel Conductors.—The lines of magnetic force due to a single round conductor (that is, with the return conductor assumed at an infinite distance), consist of a series of concentric circles, and the lines of dielectric force are straight lines radiating from the conductor.

In an ordinary two-wire circuit, however, the influence of the return wire is not negligible, and the lines of magnetic and dielec-

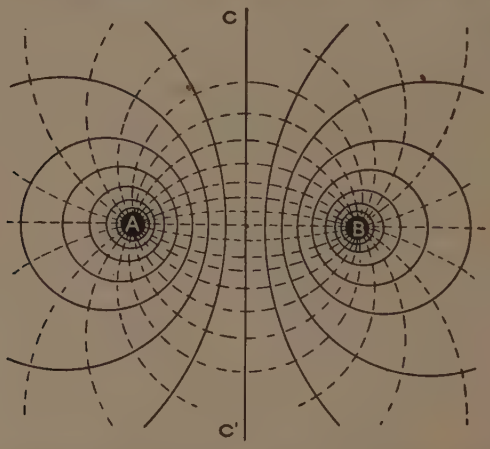


FIG. 5.—Electric field between two round parallel conductors.

tric force are crowded together between the conductors. The former become circles with centres on the line passing through the centres of the conductors, and the latter are also circles intersecting in two points in the conductors as shown in Fig. 5.

14. Inductance of Round Parallel Conductors.—Fig. 6 represents two parallel conductors A and B, carrying equal currents I c.g.s. units but in opposite directions, and forming part of an ordinary two-wire circuit.

If Φ is the magnetic flux associated with each wire the inductance of the wire by definition is

$$L = \frac{\Phi}{I}. \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Let r = radius of each wire in centimetres,
and d = distance between centres in centimetres.
The magnetic field intensity at the point P distant x centimetres from A and $d - x$ centimetres from B is

$$H = \frac{2I}{x} + \frac{2I}{d - x} \quad \text{dynes,}$$

and the flux density is

$$B = H = \frac{2I}{x} + \frac{2I}{d - x} \quad \text{lines per sq. cm.;}$$

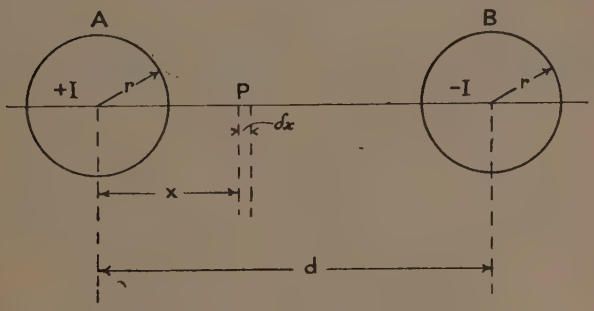


Fig. 6.—Inductance of parallel conductors.

the flux in a section of width dx and length 1 centimetre is

$$d\Phi = Bdx \text{ lines.}$$

The flux between the wires per centimetre length produced by the current flowing is

$$\int_r^d Bdx = \int_r^d \left(\frac{2I}{x} + \frac{2I}{d - x}\right)dx$$

$$= 4I \log_e \frac{d}{r};$$

the flux surrounding each wire being one-half this quantity is given by

$$\Phi_1 = 2I \log_e \frac{d}{r},$$

and therefore the inductance of each wire per centimetre length is

$$L_1 = \frac{\Phi_1}{I} = 2 \log_e \frac{d}{r}. \quad . \quad . \quad . \quad (6)$$

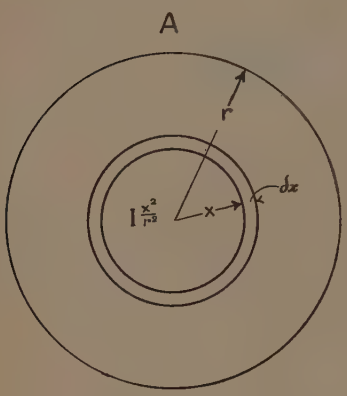


Fig. 7.—Flux inside a conductor.

This quantity is the external inductance of the conductor.

The flux inside the conductor has been neglected so far, but its inductive effect can be calculated very easily if it is assumed that the current is distributed uniformly over the section of the wire. In Fig. 7 the section of wire A is shown enlarged.

If μ is the permeability of the conductor material the flux density at a distance x centimetres from the centre of A is

$$B = \mu H = \frac{2\mu I'}{x},$$

where I' is the current inside the radius x centimetres and its value is

$$I' = I\frac{x^2}{r^2} \quad \text{c.g.s. units.}$$

The flux in the ring of radius x, width dx, and length 1 centimetre is

$$d\Phi' = \frac{2\mu I'}{x}dx = \frac{2\mu I x}{r^2}dx \quad \text{lines;}$$

this flux surrounds only the current I' and is equivalent to a smaller flux surrounding the current I, of value

$$d\Phi = d\Phi'\frac{x^2}{r^2} = \frac{2\mu I x^3}{r^4}dx;$$

the flux equivalent to the flux inside the whole section is

$$\Phi_2 = \int_0^r d\Phi = \int_0^r \frac{2\mu I x^3}{r^4}dx$$
$$= \frac{\mu I}{2} \quad \text{lines,}$$

and the inductance per centimetre length due to the flux inside the conductor is

$$L_2 = \frac{\Phi_2}{I} = \frac{\mu}{2} \quad \text{c.g.s. units,} \quad . \quad . \quad . \quad (7)$$

which is the internal inductance of the conductor.

The total inductance of each wire per centimetre is therefore

$$L_1 + L_2 = 2 \log_e \frac{d}{r} + \frac{\mu}{2} \quad \text{c.g.s. units.} \quad . \quad . \quad (8)$$

For the usual non-magnetic conductors used in practice $\mu = 1$, and changing into practical units, the inductance of each wire is

$$L_0 = 2{\cdot}54 \times 12 \times 5\ 280\left(2 \times 2{\cdot}303 \log_{10} \frac{d}{r} + \frac{1}{2}\right)10^{-9} \text{ henries per mile}$$

$$= 0{\cdot}080 + 0{\cdot}741 \log_{10} \frac{d}{r} \text{ millihenries per mile.} \quad . \quad . \quad (9)$$

15. Capacitance of Round Parallel Conductors.—As before A and B (Fig. 8) represent two parallel conductors each of radius

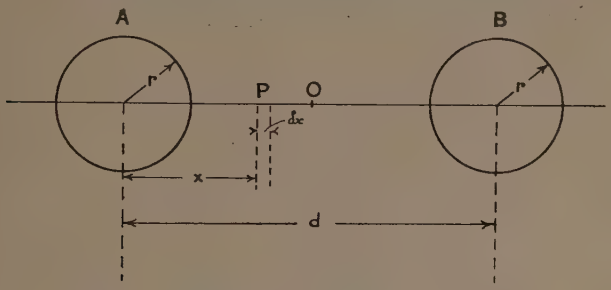


FIG. 8.—Capacitance of parallel conductors.

r centimetres, suspended in air at a distance of d centimetres between centres.

When a potential difference V is applied between them a dielectric flux ψ passes across from A to B and a positive charge

$$Q = \frac{\psi}{4\pi}$$

appears on A and an equal negative charge

$$-Q = -\frac{\psi}{4\pi}$$

on B.

The electrostatic force at any point P on the line joining the centres of conductors, distant x centimetres from A and $d - x$

centimetres from B, is then the resultant of the forces exerted by the charges on A and B acting independently.

Assuming that the electrostatic charges are distributed uniformly over the surfaces of the conductors their effect at any external point can be calculated as though they were concentrated on the lines forming the axes of the conductors.

Let q and $-q$ be the charges per centimetre length on A and B respectively.

Then from each centimetre length of A, $4\pi q$ lines of dielectric flux extend out normally and produce at the point P a flux density

$$D_A = \frac{4\pi q}{2\pi x} = \frac{2q}{x} \quad \text{lines per square centimetre.}$$

Similarly the charge $-q$ on B produces at P a flux density

$$D_B = \frac{2q}{d - x} \quad \text{lines per square centimetre,}$$

and the resultant flux density at P is

$$D = D_A + D_B = \frac{2q}{x} + \frac{2q}{d - x} \quad \text{lines per square centimetre.}$$

The electrostatic force at the point P is

$$F = D = \frac{2q}{x} + \frac{2q}{d - x} \text{ dynes} \quad . \quad . \quad (10)$$

and acts from A to B.

The work done in moving a unit positive charge from B to A is equal to the potential difference between the two conductors and is

$$V = \int_r^{d-r} \left(\frac{2q}{x} + \frac{2q}{d - x}\right) dx$$

$$= 4q \log_e \frac{d - r}{r}, \quad . \quad . \quad . \quad (11)$$

but since r is always small compared with d in overhead lines

$$V = 4q \log_e \frac{d}{r}. \quad . \quad . \quad . \quad (12)$$

The capacitance of the two conductors per centimetre length is therefore

$$\frac{q}{V} = \frac{q}{4q \log_e \frac{d}{r}}$$

$$= \frac{1}{4 \log_e \frac{d}{r}} \quad \text{electrostatic units.}$$

It is more convenient, however, to separate the circuit capacitance into the capacitance of the two conductors forming it. The potential of the point O, midway between the two wires, is zero, and the potential of the wire A is

$$E = \frac{V}{2} = 2q \log_e \frac{d}{r};$$

therefore the capacitance of A per centimetre length between the wire and the neutral or point of zero potential is

$$C_A = \frac{q}{E}$$

$$= \frac{1}{2 \log_e \frac{d}{r}} \quad \text{electrostatic units,} \quad . \quad . \quad (13)$$

and the capacitance of each wire is

$$C_0 = 2{\cdot}54 \times 12 \times 5\,280 \left(\frac{1}{2 \times 2{\cdot}303 \log_{10} \frac{d}{r}} \right) \frac{1}{9 \times 10^{11}} \text{ farads per mile}$$

$$= \frac{0{\cdot}0388}{\log_{10} \frac{d}{r}} \quad \text{microfarads per mile.} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

If the two conductors had been suspended in a medium of permittivity ϵ the capacitance would have been increased in the ratio ϵ. The capacitance of each conductor would thus be

$$\frac{0{\cdot}0388\,\epsilon}{\log_{10} \frac{d}{r}} \text{ microfarads per mile.} \quad . \quad . \quad (15)$$

16. Potential and Potential Gradient between Parallel Conductors.—The potential at the point P in Fig. 8 is the work done in moving unit positive charge from the neutral point O to the point P; it is

$$v = \int_x^{\frac{d}{2}} \left(\frac{2q}{x} + \frac{2q}{d-x}\right) dx$$

$$= 2q \log_e \frac{d-x}{x}, \quad . \quad . \quad . \quad . \quad (16)$$

but $$q = \frac{V}{4 \log_e \frac{d-r}{r}}$$ from equation (11)

and substituting for q in (16) gives

$$v = V \frac{\log_e \frac{d-x}{x}}{2 \log_e \frac{d-r}{r}}. \quad . \quad . \quad . \quad . \quad (17)$$

The potential of conductor A is

$$\frac{V}{2} = E$$

and that of B is

$$-\frac{V}{2} = -E.$$

The potential gradient at any point P is equal to the intensity of the electrostatic field at that point and is given by

$$g = \frac{dv}{dx} = \frac{2q}{x} + \frac{2q}{d-x}$$

$$= 2q\left\{\frac{d}{x(d-x)}\right\}$$

$$= \frac{V}{2 \log_e \frac{d-r}{r}}\left\{\frac{d}{x(d-x)}\right\}, \quad . \quad . \quad (18)$$

or expressed in terms of the potential to neutral it is

$$g = \frac{E}{\log_e \frac{d-r}{r}}\left\{\frac{d}{x(d-x)}\right\}. \quad . \quad . \quad . \quad (19)$$

For small values of x and r compared with d this can be written

$$g = \frac{E}{x \log_e \frac{d}{r}}. \quad . \quad . \quad . \quad . \quad (20)$$

The maximum gradient occurs at the surface of the conductor and is

$$g_{max} = \frac{E}{r \log_e \frac{d}{r}} \quad . \quad . \quad . \quad . \quad (21)$$

All these values of potential gradient are in volts per centimetre if V and E are expressed in volts.

Assuming the same potential difference applied between the

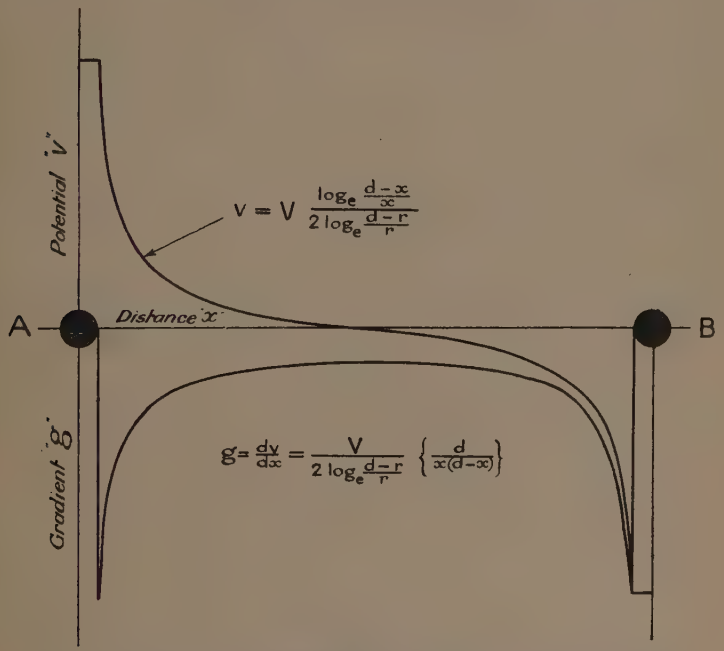


FIG. 9.—Potential and potential gradient between parallel conductors.

two conductors, the distribution of potential around them is unaffected by the value of the permittivity ϵ of the medium in which the conductors are placed.

Fig. 9 shows the values of potential and potential gradient at points along the line joining the centres of conductors as plotted from equations (17) and (18).

17. Inductance and Capacitance of a Single Conductor to Earth.—Referring to Fig. 5 it will be seen from considerations of symmetry that the centre line CC' is a line of magnetic force, and

since the lines of electrostatic force intersect it at right angles, it is also a dielectric equipotential line. Hence if it were replaced by a conducting plane of perfect conductivity this would exert no effect on the magnetic or dielectric fields between the conductors A and B.

If, then, the earth's surface is assumed to be a perfectly conducting plane, the electric field between the earth and a single overhead conductor will be exactly similar to that between A and CC' in Fig. 5. In other words, if the conductor is at a potential E and carrying a current I the magnetic and dielectric flux distribution is exactly similar to that which would be produced on the assumption of an image conductor having a potential $-E$ and carrying a current $-I$ placed at the same distance below the ground as the conductor is above ground. That is, the formulæ obtained for the inductance and capacitance of a wire with return wire at distance d can be immediately applied to the inductance and capacitance of a wire to earth by using, as distance d, twice the distance of the wire from ground.

Thus if h is the height of the wire above the earth's surface, the inductance is

$$L'_0 = 0{\cdot}080 + 0{\cdot}741 \log_{10} \frac{2h}{r} \quad \text{millihenries per mile,} \qquad (22)$$

and the capacitance is

$$C'_0 = \frac{0{\cdot}0388}{\log_{10} \frac{2h}{r}} \quad \text{microfarads per mile.} \qquad (23)$$

The above case is a simple application of the Theory of Electric Images due to Lord Kelvin, a theory which is found of great utility in the solution of many electrical problems.

18. Effect of Earth on Capacitance of Line.—Although a considerable capacitance exists between earth and a single overhead wire, the effect of the earth's presence on a transmission line consisting of outgoing and return wires is very small.

Considering the case of a single-phase line, image wires A' and B' must be assumed for both the wires A and B, as represented in Fig. 10, and the capacitance of each wire is then calculated as follows:—

Let h be the height of the wires from the ground, d their dis-

tance apart from centre to centre, and r the radius of the wires, all in centimetres.

If A carries a charge of $+ q$ units per centimetre length, A' will carry $- q$ units, B, $- q$ units, and B', $+ q$ units.

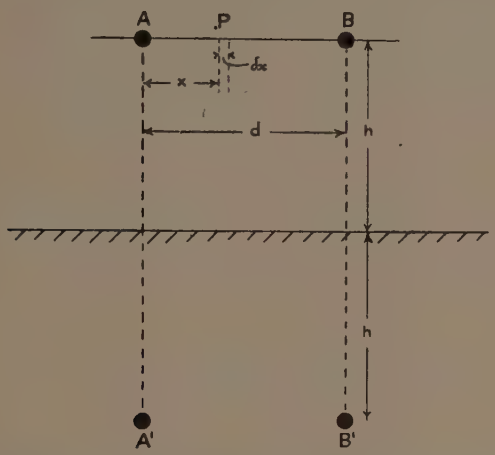


FIG. 10.—Effect of earth on capacitance of single-phase transmission line.

Then at a point P lying between A and B and distant x centimetres from A the electrostatic force is

$$\frac{2q}{x} \text{ dynes due to } A,$$

$$\frac{2q}{d - x} \text{ dynes due to } B,$$

$$- \frac{2qx}{4h^2 + x^2} \text{ dynes due to } A',$$

$$- \frac{2q(d - x)}{4h^2 + (d - x)^2} \text{ dynes due to } B',$$

The resultant force at P is

$$\frac{2q}{x} + \frac{2q}{d - x} - \frac{2qx}{4h^2 + x^2} - \frac{2q(d - x)}{4h^2 + (d - x)^2} \text{ dynes.}$$

The work done in moving unit positive charge from B to A is equal to the potential difference between the two wires; it is

$$V = \int_r^d \frac{2q}{x}dx + \int_r^d \frac{2q}{d-x}dx - \int_r^d \frac{2qx}{4h^2+x^2}dx - \int_r^d \frac{2q(d-x)}{4h^2+(d-x)^2}dx$$

$$= 2q \log_e \frac{d}{r} + 2q \log_e \frac{d}{r} - q \log_e \frac{4h^2+d^2}{4h^2} + q \log_e \frac{4h^2}{4h^2+d^2}$$

$$= 4q \log_e \frac{d}{r} + 2q \log_e \frac{4h^2}{4h^2+d^2}.$$

Midway between the two wires the potential is zero, and the potential of A is

$$E = \frac{V}{2} = 2q \log_e \frac{d}{r} + q \log_e \frac{4h^2}{4h^2+d^2}$$

$$= 2q \log_e \frac{d}{r} + 2q \log_e \frac{2h}{\sqrt{4h^2+d^2}}, \quad . \quad (24)$$

and therefore the capacitance of A between the wire and neutral is

$$C'_A = \frac{q}{E} = \frac{1}{2 \log_e \frac{d}{r} + 2 \log_e \frac{2h}{\sqrt{4h^2+d^2}}}$$

$$= \frac{1}{2 \log_e \frac{d}{r}\left(\frac{2h}{\sqrt{4h^2+d^2}}\right)} \text{ electrostatic units per cm.} \quad (25)$$

The formula given in Art. 15 for the capacitance of a wire as measured between wire and neutral was

$$C_A = \frac{1}{2 \log_e \frac{d}{r}} \text{ electrostatic units per cm.} \quad . \quad (13)$$

Hence the effect of the earth's proximity is to increase slightly the capacitance of an overhead line.

Taking as an average case

$$h = 30' = 360'',$$
$$d = 8' = 96'',$$

and

$$r = 0{\cdot}2'',$$

we find the capacitance is changed by less than 0·2 °/₀, so that the effect of the earth in increasing the capacitance of an overhead system is negligible.

19. Inductance and Capacitance of Three-phase Overhead Line.—In order to understand the derivation of the formulæ for the inductance and capacitance of a three-phase system consider

first of all that the three phases are kept entirely separate along the line, six conductors being used. In Fig. 11 (*a*) *A*, *B*, and *C* are the three outgoing wires occupying the vertices of an equilateral triangle and *A′*, *B′*, and *C′* respectively, the return wires running along near the neutral axis of the line.

The system is thus divided into three circuits or loops symmetrically placed, and assuming the same load on each phase it is obvious that all calculations may be based on the phenomena occurring in one of these loops, say *AA′*.

It is therefore the inductance and capacitance of *AA′* that is required for the purpose of line calculations, but in obtaining these values the effect of the loops *BB′* and *CC′* has also to be taken into consideration.

Now the three return wires *A′*, *B′*, and *C′* being at the same potential may be amalgamated into one common return wire with

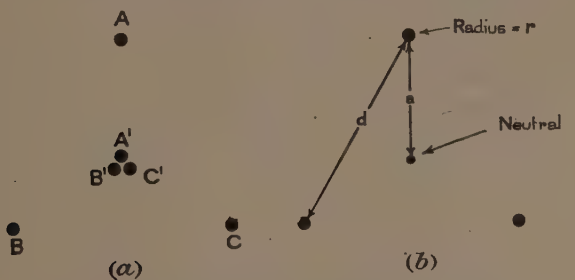


FIG. 11.—Inductance of three-phase transmission line.

its centre on the neutral axis of the line without any difference being made to the flux distribution of the system.

Also, since in a balanced three-phase system the sum of the currents in the three outgoing wires is zero, the current in the common return wire will also be zero, and the wire may be (and in practice always is) omitted without affecting the magnetic and dielectric fields between the other three wires.

The inductance and capacitance of a three-phase line is thus the inductance and capacitance between any one wire and the neutral axis of the line.

Referring to Fig. 11 (*b*) let

d = distance between centres of wires in centimetres,

a = distance between centres of wires and neutral in centimetres,

and r = radius of wires in centimetres.

The currents flowing in the wires have phase differences of 120 degrees so if the current in wire A is I c.g.s. units, the current in B will be

$$\left(-0{\cdot}5 + \frac{\sqrt{3}}{2}j\right)I$$

$$= (-0{\cdot}5 + 0{\cdot}866j)I \quad \text{c.g.s. units,}$$

and in wire C

$$(-0{\cdot}5 - 0{\cdot}866j)I \quad \text{c.g.s. units.}$$

Assuming the positive direction of the current along the line as away from the reader it is evident that the fluxes due to B and C threading the loop formed by wire A and the neutral are in opposition to the flux produced by A itself.

Therefore the magnetic flux between A and neutral per centimetre length of line is

$$\Phi_A = \frac{I}{2} + 2I \log_e \frac{a}{r} \quad \text{due to } A,$$

$$\Phi_B = (-0{\cdot}5 + 0{\cdot}866j)2I \log_e \frac{d}{a} \quad \text{due to } B,$$

and

$$\Phi_C = (-0{\cdot}5 - 0{\cdot}866j)2I \log_e \frac{d}{a} \quad \text{due to } C.$$

The resultant flux is

$$\Phi = \Phi_A - \Phi_B - \Phi_C$$

$$= \frac{I}{2} + 2I \log_e \frac{a}{r} + I \log_e \frac{d}{a} - 1{\cdot}732jI \log_e \frac{d}{a} + I \log_e \frac{d}{a} + 1{\cdot}732jI \log_e \frac{d}{a}$$

$$= \frac{I}{2} + 2I \log_e \frac{a}{r} + 2I \log_e \frac{d}{a}$$

$$= \frac{I}{2} + 2I \log_e \frac{d}{r}.$$

The inductance between wire and neutral is

$$\tfrac{1}{2} + 2 \log_e \frac{d}{r} \quad \text{c.g.s. units per centimetre,}$$

and changing into practical units the inductance of each wire is

$$L_0 = 0{\cdot}080 + 0{\cdot}741 \log_{10} \frac{d}{r} \quad \text{millihenries per mile.}$$

Similarly the line capacitance may be represented by condensers connected between each wire and the neutral as shown in Fig. 12,

and after obtaining the dielectric flux distribution in an analogous manner to the above calculation the capacitance of each wire is found to be

$$C_0 = \frac{0{\cdot}0388}{\log_{10} \frac{d}{r}} \text{ microfarads per mile.}$$

On comparing these results with those derived in Arts. 14 and 15, it is seen that the same formulæ apply for the inductance and capacitance of an overhead circuit whether single-phase or three-phase, providing these quantities are measured between conductor and neutral.

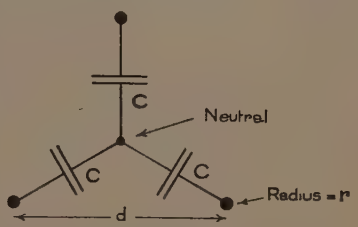


FIG. 12.—Capacitance of three-phase transmission line.

20. Inductance and Capacitance with Irregular Spacing of Conductors. — It has been assumed in the above discussion that the three line conductors were symmetrically arranged, each occupying the corner of an equilateral triangle. In many overhead transmission systems, however, the conductors are irregularly spaced, and the above formulæ are not directly applicable.

When the wires of a three-phase system are spaced so that they are not equidistant, the inductances and capacitances are not the same in the different phases. It is the practice with such lines to interchange or transpose the wires at intervals along the line so that each of the three wires occupies a particular position relatively to the other wires for one-third the total length of transmission. This has the effect of completely eliminating any out-of-balance effect.

The average inductance of an irregularly-spaced three-phase line in which the wires are transposed at regular intervals may be calculated as follows.

Let Fig. 13 represent the actual spacing of the wires A, B, and C of an overhead line, and the currents in the wires be represented by the vectors OP, OQ, and OR respectively.

Then if the current in wire A is

$$I_A = OP = I \quad \text{c.g.s. units,}$$

the current in B is

$$I_B = OQ = (-0{\cdot}5 + 0{\cdot}866j)I \quad \text{c.g.s. units,}$$

and the current in C is

$$I_C = OR = (-0{\cdot}5 - 0{\cdot}866j)I \quad \text{c.g.s. units.}$$

The magnetic flux surrounding each centimetre length of A is

$$\Phi_1 = \left(\tfrac{1}{2} + 2\log_e \frac{\infty}{r}\right)I \quad \text{due to } A;$$

$$+ (-0{\cdot}5 + 0{\cdot}866j)\left(2\log_e \frac{\infty}{c}\right)I \quad \text{due to } B;$$

$$+ (-0{\cdot}5 - 0{\cdot}866j)\left(2\log_e \frac{\infty}{b}\right)I \quad \text{due to } C,$$

where r is the radius of the wires.

Suppose at the end of one-third the total length of transmission the line is transposed so that the wires A, B, and C now occupy the positions B, C, and A respectively.

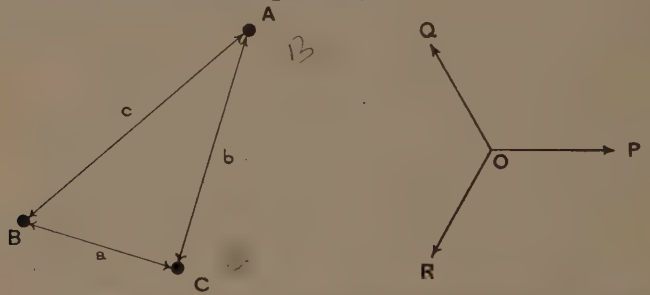


Fig. 13.—Inductance of three-phase line with irregular spacing of conductors.

The flux now surrounding each centimetre length of A is

$$\Phi_2 = \left(\tfrac{1}{2} + 2\log_e \frac{\infty}{r}\right)I \quad \text{due to } A;$$

$$+ (-0{\cdot}5 + 0{\cdot}866j)\left(2\log_e \frac{\infty}{a}\right)I \quad \text{due to } B;$$

$$+ (-0{\cdot}5 - 0{\cdot}866j)\left(2\log_e \frac{\infty}{c}\right)I \quad \text{due to } C.$$

Let the line be transposed again so that the wires A, B, and C occupy the positions C, A, and B for the last one-third of the total distance of transmission.

Then the flux surrounding each centimetre length of A is

$$\Phi_3 = \left(\tfrac{1}{2} + 2\log_e \frac{\infty}{r}\right)I \quad \text{due to } A;$$

$$+ (-0{\cdot}5 + 0{\cdot}866j)\left(2\log_e \frac{\infty}{b}\right)I \quad \text{due to } B;$$

$$+ (-0{\cdot}5 - 0{\cdot}866j\left(2\log_e \frac{\infty}{a}\right)I \quad \text{due to } C.$$

The average value of the magnetic flux surrounding each centimetre length of A (or either of the other wires) is

$$\Phi = \frac{\Phi_1 + \Phi_2 + \Phi_3}{3}$$

$$= \tfrac{1}{3}\left(1{\cdot}5I + 6I \log_e \frac{\infty}{r} - 2I \log_e \frac{\infty}{a} - 2I \log_e \frac{\infty}{b} - 2I \log_e \frac{\infty}{c}\right)$$

$$= \tfrac{1}{3}\left(1{\cdot}5I + 6I \log_e \frac{\infty}{r} - 6I \log_e \frac{\infty}{\sqrt[3]{abc}}\right)$$

$$= \tfrac{1}{3}\left(1{\cdot}5I + 6I \log_e \frac{\sqrt[3]{abc}}{r}\right).$$

Therefore the average inductance of each wire is

$$\tfrac{1}{2} + 2 \log_e \frac{\sqrt[3]{abc}}{r} \quad \text{c.g.s. units per centimetre,}$$

or in practical units the average inductance of each wire is

$$L_0 = 0{\cdot}080 + 0{\cdot}741 \log_{10} \frac{\sqrt[3]{abc}}{r} \quad \text{millihenries per mile.} \quad (26)$$

In the same way it may be shown that the average value of the capacitance of each wire is

$$C_0 = \frac{0{\cdot}0388}{\log_{10} \dfrac{\sqrt[3]{abc}}{r}} \quad \text{microfarads per mile.} \quad (27)$$

Hence for purposes of calculation the line may be treated as though the conductors were spaced symmetrically at an equivalent delta spacing of $d = \sqrt[3]{abc}$.

CHAPTER III.

PERFORMANCE OF SHORT TRANSMISSION LINES.

21. Effect of Line Inductance and Capacitance on Transmission of Alternating Currents.—In consequence of the necessary spacing between the conductors of a transmission circuit, each conductor is surrounded by a considerable magnetic flux which generates in it an e.m.f. of self-inductance. This e.m.f. is in quadrature with the current flowing in the conductor and has the value

$$e = 2\pi fLI$$
$$= IX \text{ volts}; \quad . \quad . \quad . \quad (28)$$

where f = frequency of supply in cycles per second,
L = inductance per conductor in henries,
I = current in amperes,
and X = reactance per conductor in ohms.

The voltage impressed on the circuit at the sending end has thus to provide components to overcome the resistance pressure drop IR, which is in phase with the current, and the reactance pressure drop IX, which is in quadrature with the current.

The precise effect on the receiving-end voltage depends on the difference of phase between the current and the voltage at this end of the line. When the load has a lagging or unity power factor the receiving-end voltage is less than the sending-end voltage, but with a load of leading power factor there may be a rise of pressure at the receiving end of the line.

The effect of the line capacitance is to produce a current—usually called the charging current—which is in quadrature with the voltage. This component of current in the conductor has its maximum value at the sending end of the circuit and decreases continuously as the receiving end of the circuit is approached at which point its value is zero.

The value of the charging current at the sending end is given by

$$I_c = 2\pi fCE$$
$$= EB \text{ amperes}; \quad . \quad . \quad . \quad (29)$$

where C = capacitance per conductor in farads,
E = voltage to neutral,
and B = susceptance per conductor in mhos.

It should be noted that both the reactance drop and the charging current of the circuit are proportional to the frequency, and thus have a far greater influence on the performance of a 50-cycle than on a 25-cycle line. Also that with overhead lines reactance effects are relatively important due to the wide spacing of conductors which must be adopted, whilst in the case of transmission by underground cables reactance effects are small and the effect of capacitance predominates.

In the case of 50-cycle overhead lines not exceeding, say, 50 miles in length, the effect of capacitance on the line performance is so small that it may generally be neglected.

22. Treatment of Polyphase Systems.—A three-phase transmission circuit carrying an equal load on each phase is a particular example of a balanced symmetrical polyphase system, and in making calculations and drawing diagrams of such a system it is only necessary to take into account one of the star circuits of the system.

For instance, the polyphase system represented in Fig. 2 at balanced load can be considered as consisting of the equal single-phase systems: 0 – 1; 0 – 2; 0 – 3; . . . 0 – n lying between the conductors and the neutral axis.

In investigating a polyphase system it is only necessary to deal with the phenomena occurring in one of these constituent circuits, which carries $\frac{1}{n}$th the total load on the system, and has impressed on it the star voltage or voltage to neutral of the system. In a three-phase system each of the star circuits carries one-third of the total load and has a voltage equal to $\frac{1}{\sqrt{3}}$ times the voltage between conductors.

23. Vector Diagram for Short Line.—Fig. 14 is the vector diagram showing the relation between the various elements in a

short transmission circuit, the effect of capacitance being neglected. Both the diagram itself, and the upper part of the figure which illustrates the circuit schematically, are drawn for one phase only, in accordance with the rules laid down in the preceding article.

The various quantities are represented as follows:

OD is the current.

OR is the voltage to neutral at the receiving end of the line drawn ϕ_r degrees in advance of *OD*: cos ϕ_r being the load power factor (assumed lagging in this case).

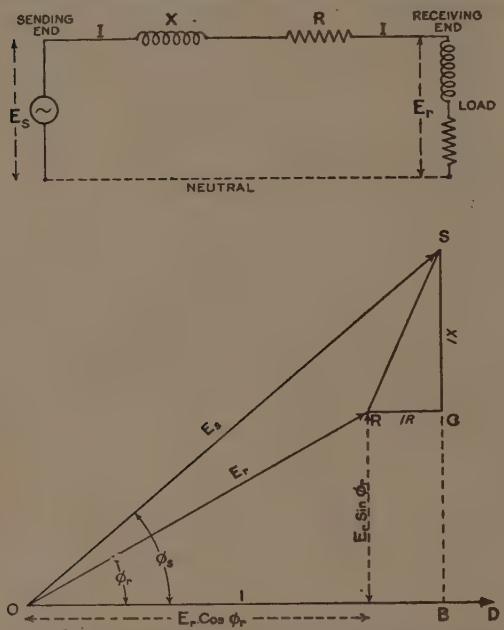


FIG. 14.—Vector diagram for short transmission line.

RC drawn parallel to *OD* is the voltage consumed in the resistance of the conductor.

CS drawn 90 degrees ahead of *OD* is the voltage consumed in the reactance of the conductor.

OS is the voltage to neutral at the sending end.

ϕ_s is the phase-angle at the sending end: cos ϕ_s being the power factor at this end.

The component of the sending-end voltage in phase with the current is

$$OB = E_r \cos \phi_r + IR,$$

and the component in quadrature ahead of the current is

$$BS = E_r \sin \phi_r + IX,$$

and therefore the sending-end voltage is

$$E_s = \sqrt{(E_r \cos \phi_r + IR)^2 + (E_r \sin \phi_r + IX)^2}. \quad . \quad (30)$$

Expressed in another form

$$E_s = E_r \sqrt{\left(\cos \phi_r + \frac{IR}{E_r}\right)^2 + \left(\sin \phi_r + \frac{IX}{E_r}\right)^2}, \quad . \quad (31)$$

which is rather more convenient for purposes of calculation as the quantity under the radical sign is then of the order of unity.

The sending-end phase-angle is

$$\phi_s = \tan^{-1} \frac{\sin \phi_r + \frac{IX}{E_r}}{\cos \phi_r + \frac{IR}{E_r}}. \quad . \quad . \quad . \quad (32)$$

Hence, knowing the conditions existing at one end of the line, those at the other end can easily be obtained by means of the above formulæ. Problems may also be solved graphically by drawing the vector diagram to scale, but owing to the small size of the impedance triangle compared with the rest of the diagram no great accuracy can be obtained. It is advisable, therefore, to calculate all quantities trigonometrically, using the graphical method as a rough check.

24. Regulation.—The regulation of a transmission line can be defined as the change of voltage at the receiving end when full load is thrown off, the sending-end voltage being kept constant. The percentage regulation is the regulation expressed as a percentage of the full-load receiving-end voltage.

When the load is thrown off, the only current in the conductors is the charging current of the system, and if this is of negligible magnitude the receiving-end voltage will equal the constant impressed voltage at the sending end of the line. Hence in the case of short lines, the regulation is equal to $E_s - E_r$, the arithmetical difference between the sending-end and receiving-end voltages.

The regulation for any given load power factor can be graphically determined by means of Kapp's regulation diagram, which is a slight modification of the fundamental impedance diagram of the line. In Fig. 15 if OD is kept fixed and ϕ_r varied, S lies on a

circle with O as centre. RC and CS remain constant in length and move parallel to themselves, consequently R lies on an equal circle with centre at O' where OO' is equal and parallel to SR.

These circles are drawn in the figure. If OR is produced to meet the other circle in E, then $OE = OS$, and therefore

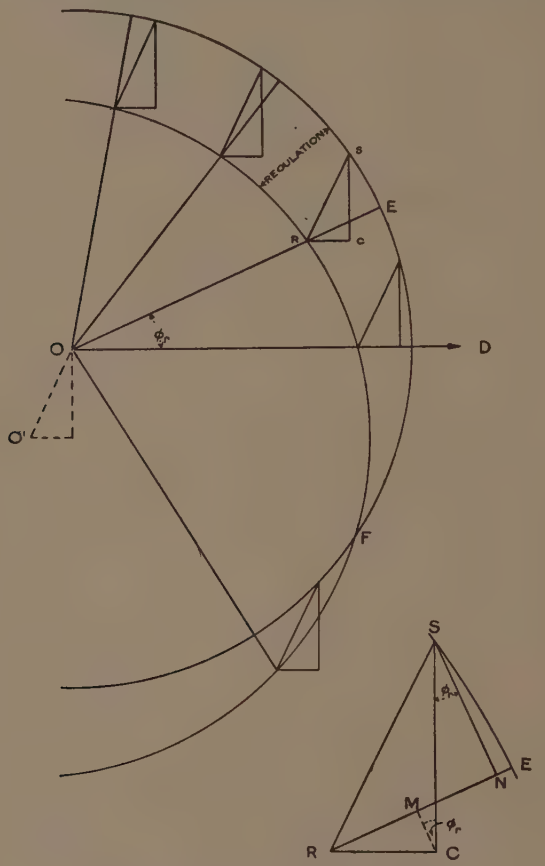


FIG. 15.—Kapp's regulation diagram.

$RE = OS - OR$, the pressure drop along the line and also the regulation.

All that is necessary then, in order to determine the regulation at any power factor $\cos \phi_r$ is to draw a line making an angle ϕ_r with OD, and measure the length of the intercept between the two circles.

An inspection of the diagram shows that as the angle of lag increases, the regulation increases, reaching a maximum value when OR is in line with OO', *i.e.*, when

$$\phi_r = \tan^{-1}\frac{X}{R}.$$

For still greater angles of lag the regulation diminishes again.

With leading power factor the regulation diminishes as the angle of lead increases, becomes zero when $\phi_r = \angle DOF$, and beyond this has a negative value, *i.e.*, the receiving-end pressure falls when the load is thrown off.

A formula giving the regulation directly may be found by referring to the part of the diagram shown enlarged in Fig. 15. RS is usually small in comparison with OS, and so the arc SE may be replaced with very little error by the perpendicular SN from S on to OR produced. If CM is drawn at right angles to RE we have

$$\begin{aligned}\text{Regulation} &= \overline{RM} + \overline{MN} \\ &= IR\cos\phi_r + IX\sin\phi_r, \quad . \quad . \quad (33)\end{aligned}$$

or expressed in another form

$$\begin{aligned}\text{Regulation} &= IZ\left(\frac{IR}{IZ}\cos\phi_r + \frac{IX}{IZ}\sin\phi_r\right) \\ &= IZ(\sin\theta\cos\phi_r + \cos\theta\sin\phi_r) \\ &= IZ\sin(\phi_r + \theta), \quad . \quad . \quad . \quad . \quad (34)\end{aligned}$$

where the angle $\theta = \tan^{-1}\frac{R}{X}$.

The regulation is a maximum when $\sin(\theta + \phi_r) = 1$, or $\phi_r = 90 - \theta$, that is, at a load power factor

$$\cos\phi_r = \sin\theta = \frac{R}{Z},$$

and is equal to the impedance drop IZ.

25. Numerical Example of Short Line Solution.—When the voltage, current, and power factor either at the sending or receiving end are given, the conditions at the other end of the line can easily be found by applying formulæ (31) and (32). To illustrate the method of solution the following example will be taken :—

Problem 1.—Find the characteristics of the load at the sending

end of a three-phase line 30 miles long, consisting of three hard-drawn, copper conductors, spaced in a four-foot delta. Temperature taken as 20° C. Load conditions at receiving end assumed as 8 000 kVA. (6 400 kW. at 80 % power factor lagging), 30 000 volts, 50 cycles; transmission loss to be approximately 10 %.

At the receiving end of the line

$$\text{kVA}_r = \frac{8\,000}{3} = 2\,667 \quad \text{kVA. per phase.}$$

$$\text{kW}_r = \frac{6\,400}{3} = 2\,133 \quad \text{kW. per phase.}$$

$$E_r = \frac{30\,000}{\sqrt{3}} = 17\,320 \quad \text{volts to neutral.}$$

$$\cos \phi_r = 80\ \%_0 \quad \text{lagging.}$$

$$I = \frac{2\,667}{17\,320} \times 10^3 = 154 \quad \text{amperes.}$$

For a transmission loss of 10 % the resistance of each conductor will be

$$R = \frac{2\,133 \times 10^3}{10 \times (154)^2}$$
$$= 8{\cdot}99 \quad \text{ohms,}$$

or 0·300 ohms per mile.

From wire tables[1] the nearest standard size of wire is found to be No. 00000 S.W.G., which has a resistance of 0·302 ohms per mile (assuming a 97 % conductivity), and a diameter of 0·432 inches.

The resistance of each conductor is therefore

$$R = 0{\cdot}302 \times 30 = 9{\cdot}06 \quad \text{ohms.}$$

The inductance of each conductor by formula (9) is

$$L = \left(0{\cdot}080 + 0{\cdot}741 \log_{10} \frac{48}{0{\cdot}216}\right) 10^{-3} \times 30$$
$$= 0{\cdot}0546 \text{ henries,}$$

and the reactance is

$$X = 2\pi \times 50 \times 0{\cdot}0546$$
$$= 17{\cdot}1 \quad \text{ohms.}$$

We then have

$$IR = 154 \times 9{\cdot}06 = 1\,395 \quad \text{volts (resistance drop).}$$
$$IX = 154 \times 17{\cdot}1 = 2\,633 \quad \text{volts (reactance drop).}$$
$$E_s = 17\,320\sqrt{\left(0{\cdot}8 + \frac{1\,395}{17\,320}\right)^2 + \left(0{\cdot}6 + \frac{2\,633}{17\,320}\right)^2}$$
$$= 17\,320\sqrt{(0{\cdot}8805)^2 + (0{\cdot}7520)^2}$$
$$= 20\,060 \quad \text{volts.}$$
$$\phi_s = \tan^{-1}\frac{0{\cdot}\,7520}{0{\cdot}\,8805}$$
$$= 40{\cdot}5°$$
$$\cos\phi_s = 76{\cdot}04\ \%\ \text{lagging.}$$

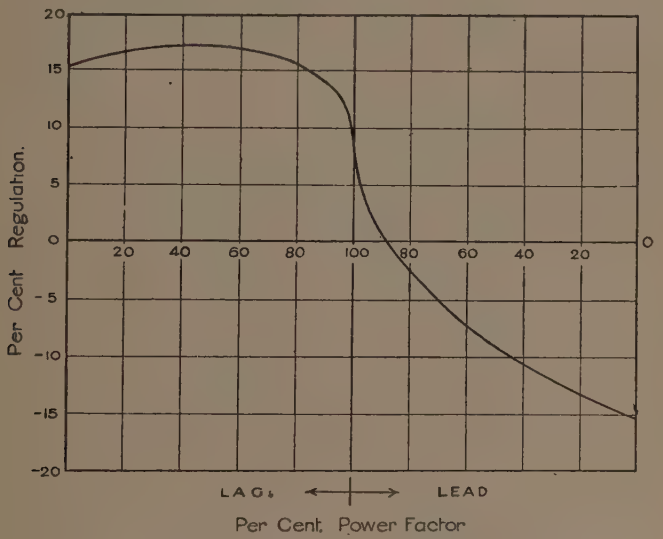


Fig. 16.—Regulation curve for short transmission line.

$$\text{Loss} = \frac{(154)^2 \times 9{\cdot}06}{1\,000}$$
$$= 214{\cdot}9\ \text{kW.}$$
$$\text{Efficiency} = \frac{2\,133 - 214{\cdot}9}{2\,133} \times 100\ \%$$
$$= 89{\cdot}92\ \%.$$
$$\text{Regulation} = 20\,060 - 17\,320$$
$$= 2\,740 \quad \text{volts.}$$
$$\text{Percentage Regulation} = \frac{2\,740}{17\,320} \times 100\ \%$$
$$= 15{\cdot}82\ \%.$$

The percentage regulation derived from formula (33) is

$$\frac{IR\cos\phi_r + IX\sin\phi_r}{E_r} \times 100\ \%$$

$$= \frac{154 \times 9{\cdot}06 \times 0{\cdot}8 + 154 \times 17{\cdot}1 \times 0{\cdot}6}{17\,320} \times 100\ \%$$

$$= 15{\cdot}6\ \%,$$

which agrees fairly well with the accurate value.

The complete regulation curve for the line has also been calculated by the above formula and is shown in Fig. 16. The curve gives the percentage regulation of the line at all power factors, the current being the same in each case, *viz.* 154 amperes.

26. Mixed Sending- and Receiving-end Conditions.—Frequently in practice the case occurs where the voltage at the sending end is fixed, and also the power and power factor at the receiving end. In such a case the solution becomes more complicated. The following method of treatment is due to P. de Bancarel:—[2]

Referring to Fig. 14 the following equations can be written down:—

$$E_r\cos\phi_r + RI = E_s\cos\phi_s, \quad . \quad . \quad (35)$$

$$E_r\sin\phi_r + XI = E_s\sin\phi_s, \quad . \quad . \quad (36)$$

and also

$$E_r\cos\phi_r \times RI = 1\,000PR, \quad . \quad . \quad (37)$$

$$E_r\sin\phi_r \times XI = 1\,000QX, \quad . \quad . \quad (38)$$

where P = the power in kW. per phase delivered at the receiving end of the line,

and Q = the reactive kVA. per phase delivered at the receiving end of the line.

Squaring equations (35) and (36) and adding gives

$$E_r^2 + 2RIE_r\cos\phi_r + 2XIE_r\sin\phi_r + (R^2 + X^2)I^2 = E_s^2,$$

whence by substituting for the second and third terms from equations (37) and (38)

$$E_r^2 + 2\,000(PR + QX) + (R^2 + X^2)I^2 = E_s^2. \quad . \quad (39)$$

Substituting

$$\frac{1\,000P}{E_r\cos\phi_r} = I,$$

and multiplying throughout by E_r^2 gives

$$E_r^4 - [E_s^2 - 2\,000(PR + QX)]E_r^2 + (R^2 + X^2)\left(\frac{1\,000P}{\cos\phi_r}\right)^2 = 0. \quad (40)$$

Let

$$S = E_s^2 - 2\,000(PR + QX),$$

and

$$T = (R^2 + X^2)\left(\frac{1\,000P}{\cos \phi_r}\right)^2,$$

then equation (40) can be written

$$E_r^4 - SE_r^2 + T = 0, \quad . \quad . \quad . \quad (41)$$

whence

$$E_r = \sqrt{\frac{S \pm \sqrt{(S^2 - 4T)}}{2}}. \quad . \quad . \quad . \quad (42)$$

In general, two values of the receiving-end voltage can be found to satisfy the conditions laid down, depending on whether the + or − sign is used in the formula. The solution giving the higher value of E_r and consequently higher transmission efficiency is the one required in practice, and thus the + sign should be taken.

The following example will illustrate the use of the formula given above :—

Problem 2.—Find the characteristics of the load at the receiving end of a three-phase line ten miles long, consisting of three hard-drawn, No. 0 copper conductors, spaced in a thirty-inch delta. Temperature taken as 20° C. Load at receiving end assumed as 1 500 kVA. (1 200 kW. at 80 °/₀ power factor lagging). Voltage at sending end given as 11 000 volts, 50 cycles.

From wire tables the resistance per mile of conductor is found to be 0·531 ohms, and the diameter is 0·324 inches.

Hence the resistance of each conductor is

$$R = 0{\cdot}531 \times 10 = 5{\cdot}31 \quad \text{ohms.}$$

The inductance of each conductor is

$$L = \left(0{\cdot}080 + 0{\cdot}741 \log_{10} \frac{30}{0{\cdot}162}\right)10^{-3} \times 10 \quad . \quad (9)$$

$$= 0{\cdot}0176 \quad \text{henries,}$$

and the reactance is

$$X = 2\pi \times 50 \times 0{\cdot}0176$$
$$= 5{\cdot}54 \quad \text{ohms.}$$

There is also given

$$P = \frac{1\,200}{3} = 400 \text{ kW. per phase,}$$

$$Q = P \tan \phi_r$$
$$= 400 \times 0{\cdot}75$$
$$= 300 \text{ kVA. per phase,}$$

$$\cos \phi_r = 80\ \%_{\circ} \text{ lagging,}$$

$$E_s = \frac{11\,000}{\sqrt{3}} = 6\,351 \text{ volts to neutral.}$$

Hence

$$S = E_s^2 - 2\,000(PR + QX)$$
$$= 6\,351^2 - 2\,000(400 \times 5{\cdot}31 + 300 \times 5{\cdot}54)$$
$$= 3{\cdot}2763 \times 10^7,$$

and

$$T = (R^2 + X^2)\left(\frac{1\,000P}{\cos \phi_r}\right)^2$$
$$= (5{\cdot}31^2 + 5{\cdot}54^2)\left(\frac{4 \times 10^5}{0{\cdot}8}\right)^2$$
$$= 1{\cdot}4722 \times 10^{13}.$$

The receiving-end voltage

$$E_r = \sqrt{\frac{S + \sqrt{(S^2 - 4T)}}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

$$= \sqrt{\frac{3{\cdot}2763 \times 10^7 + \sqrt{(1{\cdot}0734 \times 10^{15} - 5{\cdot}8888 \times 10^{13})}}{2}}$$

$$= 5\,684 \text{ volts to neutral,}$$

and the current

$$I = \frac{500 \times 1\,000}{5\,684}$$
$$= 88{\cdot}0 \text{ amperes,}$$

also the sending-end power factor

$$\cos \phi_s = \frac{E_r \cos \phi_r + RI}{E_s}$$
$$= \frac{5\,684 \times 0{\cdot}8 + 5{\cdot}31 \times 88{\cdot}0}{6\,351}$$
$$= 79{\cdot}0\ \%_{\circ} \text{ lagging.}$$

27. **Graphical Solution for Mixed Sending- and Receiving-end Conditions.**—If preferred, a semi-graphical solution of this problem may be employed. Referring to Fig. 17,

OS = voltage to neutral at sending end of line,
OR = unknown voltage at receiving end,
and OD = unknown current.

Then the point R lies on an arc of a circle passing through O and S, the centre of which is at G, and the

$$\text{angle } GSO = \frac{\pi}{2} + \phi_r - \tan^{-1}\frac{X}{R}.$$

R also lies on a circle of radius

$$\overline{FR} = \sqrt{\frac{E_s^2}{4} - 1\,000(PR + QX)}, \quad . \quad . \quad (43)$$

the centre of which is at F the mid-point of OS.

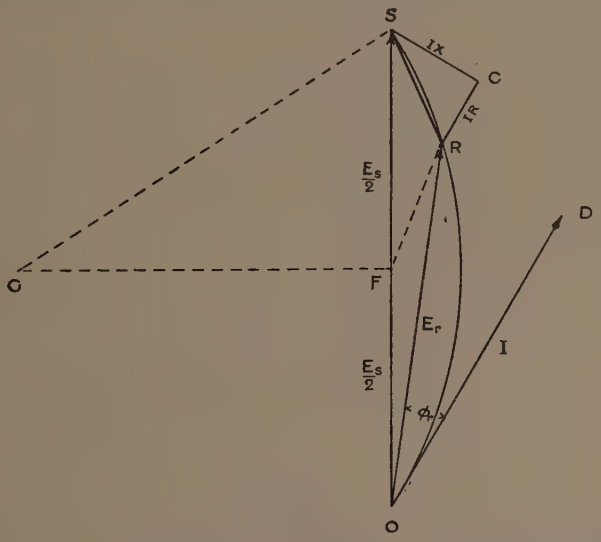


FIG. 17.—Vector diagram for mixed sending- and receiving-end conditions.

For a proof of this solution reference should be made to Fig. 18 in which a circle is drawn on OS as diameter, OR is produced to cut the circle at L, and FR produced both ways to cut the circle at H and K. RC drawn parallel to OD, and CS drawn at 90 degrees to OD, are respectively the ohmic and reactive pressure drops in the conductors.

Consider first of all the circle of which ORS is an arc. The angle ORS at the circumference is half the re-entrant angle OGS at the centre standing on the same chord OS.

The angle $ORS = 2\pi$ – angle ORC – angle CRS

$$= 2\pi - (\pi - \phi_r) - \tan^{-1}\frac{X}{R}$$

$$= \pi + \phi_r - \tan^{-1}\frac{X}{R},$$

hence the re-entrant angle $OGS = 2\pi + 2\phi_r - 2\tan^{-1}\frac{X}{R}$,

$$\text{angle } FGS = -\phi_r + \tan^{-1}\frac{X}{R},$$

and

$$\text{angle } GSO = \frac{\pi}{2} + \phi_r - \tan^{-1}\frac{X}{R}.$$

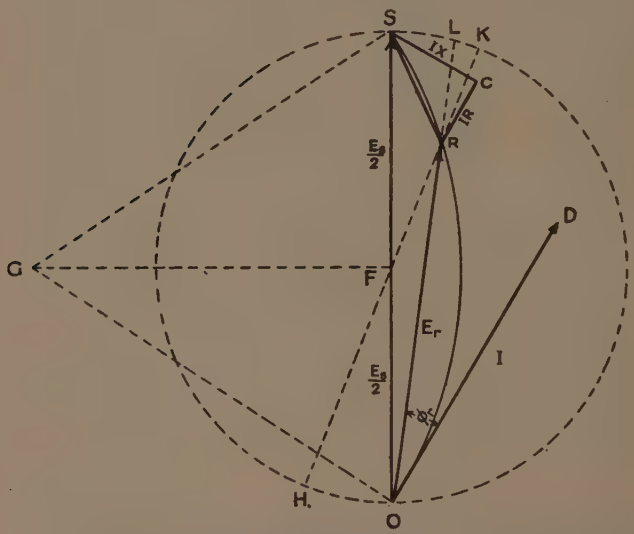


FIG. 18.—Derivation of formula for mixed sending- and receiving-end conditions.

Also in the triangle ORS

$$\overline{OS}^2 = \overline{OR}^2 + \overline{RS}^2 + 2\overline{OR} \cdot \overline{RL},$$

or

$$E_s^2 = E_r^2 + (R^2 + X^2)I^2 + 2\overline{OR} \cdot \overline{RL},$$

and comparing with equation (39) it is seen that

$$\overline{OR} \cdot \overline{RL} = 1\,000(PR + QX).$$

Therefore $\overline{OR} \cdot \overline{RL}$ expresses the power of point R in relation to the circle of diameter $OS = E_s$.

This quantity is constant so

$$\overline{OR} \cdot \overline{RL} = 1\,000(PR + QX) = \overline{HR} \cdot \overline{RK},$$

whence $(\overline{HF} + \overline{FR})(\overline{FK} - \overline{FR}) = 1\,000(PR + QX)$,

or $$\frac{E_s^2}{4} - (\overline{FR})^2 = 1\,000(PR + QX),$$

and $$\overline{FR} = \sqrt{\frac{E_s^2}{4} - 1\,000(PR + QX)}.$$

28. Effect of Transformers.—In the foregoing discussion no account has been taken of the effect of transformers included in the circuit. Transmission lines usually have raising transformers installed at the sending end to step up from the generating to the line pressure, and lowering transformers at the receiving end to step down to the distribution pressure, and it is often required to

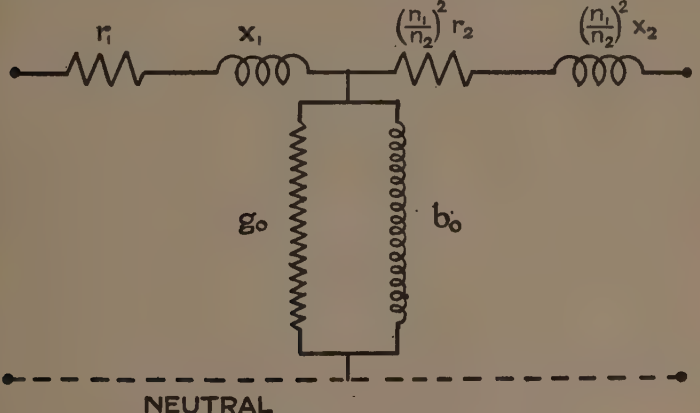


FIG. 19.—Equivalent circuit of transformer.

make calculations on the complete circuit from the generator terminals to the low pressure busbars of the receiving station. Such calculations are conveniently made in terms of the high-voltage circuit and if results are desired in terms of the low-voltage side of the transformers they may be obtained from the ratio of transformation.

Fig. 19 represents the equivalent circuit of a transformer in which r_1 and x_1 are the resistance and reactance of the high-voltage winding, r_2 and x_2 the resistance and reactance of the low-voltage winding, and n_1 and n_2 the number of turns on the high-voltage and low-voltage windings respectively. $y_0 = g_0 - jb_0$ is the primary exciting admittance of the transformer.

In power transformers the exciting current is of the order of 4 to 8 % of the full-load current, but it is nearly in quadrature with the voltage so that when the transformer is on load, the current I_1 in the high-voltage winding can be regarded as equal to $\frac{n_2}{n_1}$ times the current I_2 in the low-voltage winding. Hence, neglecting the effect of y_0, the composite circuit of a transmission line and connected transformers can be represented as in Fig. 20. The equivalent resistance of the transformer in terms of the high-voltage circuit is

$$R_t = r_1 + \left(\frac{n_1}{n_2}\right)^2 r_2, \quad . \quad . \quad . \quad (44)$$

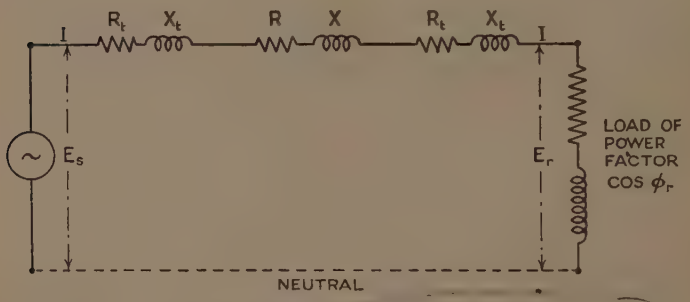


FIG. 20.—Short transmission line including effect of transformers.

and the equivalent reactance is

$$X_t = x_1 + \left(\frac{n_1}{n_2}\right)^2 x_2. \quad . \quad . \quad . \quad (45)$$

In many cases these values are not known but can be obtained from other known data.

For instance, take the case of an 8 000-kVA. transformer to be used in conjunction with the 30-mile transmission, particulars of which were given in Art. 25. The assumed performance is as follows :—

Efficiency at full load = 98 %.
Efficiency at one-fourth load = 96 %.
Regulation at full load 80 % power factor = 4 %.

The equivalent resistance and reactance of the transformer (to neutral) can then be found by the method outlined below. At full load the input is

$$\frac{8\,000}{3 \times 0{\cdot}98} = 2\,721{\cdot}09 \quad \text{kW. per phase,}$$

so that the total loss per phase at full load is 54 420 watts.

At one-fourth load the input is

$$\frac{2\,000}{3 \times 0{\cdot}96} = 694{\cdot}44 \quad \text{kW. per phase,}$$

so that the total loss per phase is 27 770 watts.

The iron loss is constant at all loads, but the copper loss varies approximately as the square of the load, so that if x = the iron loss, and y = the copper loss at full load

$$x + y = 54\,420 \quad \text{watts,}$$

and

$$x + \frac{y}{16} = 27\,770 \quad \text{watts,}$$

from which

$$x = 26\,000 \quad \text{watts,}$$

and

$$y = 28\,420 \quad \text{watts.}$$

The current at 17 320 volts to neutral is 154 amperes, so that the equivalent resistance of the transformer referred to the high-voltage circuit is

$$R_t = \frac{28\,420}{154 \times 154}$$
$$= 1{\cdot}198 \quad \text{ohms (to neutral).}$$

The equivalent reactance can be obtained from formula (33) as applied to the regulation of a transformer.

$$\text{Regulation} = IR_t \cos\phi + IX_t \sin\phi,$$

or

$$X_t = \frac{\text{Regulation} - IR_t \cos\phi}{I \sin\phi}$$
$$= \frac{\frac{4}{100} \times 17\,320 - 154 \times 1{\cdot}198 \times 0{\cdot}8}{154 \times 0{\cdot}6}$$
$$= 5{\cdot}900 \quad \text{ohms (to neutral).}$$

Supposing, now, this transformer to be installed at the receiving end of the line, the resistance and reactance can be added to the corresponding values for the line conductors, and the characteristics of the load at the high-voltage side of the sending-end transformer calculated as though there were no transformers in circuit. In a similar way the effect of the sending-end transformer can be taken

into account, and the electrical conditions determined at the terminals of the generator.

REFERENCES.

[1] Overhead conductors have now been standardised in British practice and reference should be made to the following publications:—

(*a*) 'British Standard Specification for Hard-drawn Copper Solid and Stranded Circular Conductors for Overhead Power Transmission Purposes,' *B.E.S.A. Report*, No. 125.—1924.

(*b*) 'British Standard Specification for Hard-drawn Aluminium and Steel-cored Aluminium Conductors for Overhead Power Transmission Purposes,' *B.E.S.A. Report*, No. 215.—1925.

[2] P. de Bancarel, 'Note sur le calcul des lignes de transmission d'énergie,' *Rev. Gen. d'Él.*, Vol. 2, p. 43 (1917).

CHAPTER IV.

PERFORMANCE OF LONG TRANSMISSION LINES (LOCALISED-CAPACITANCE METHODS OF SOLUTION).

29. Representation of Line Capacitance by Condensers.—In the transmission of power by overhead lines having a length of over 50 miles (and in nearly all cases where high-pressure underground cables are employed) the capacitance current of the system is appreciable and must be reckoned with in all calculations. This capacitance current is always flowing in the line while the generating-station switches are closed even though the receiving end of the line may be open-circuited. The actual value of the capacitance current flowing at any point along the line is that required to charge the section of line between the given point and the receiving end, hence it has a maximum value at the sending end and diminishes at a practically uniform rate down to zero at the receiving end.

The effect of the capacitance current can be approximately taken into account by assuming that the capacitance of the system is divided up and 'lumped' in the form of condensers shunted across the line at one or more points.

The commonest localised-capacitance methods of representation are:—

1. The middle-condenser, or nominal-T method in which the capacitance of each conductor is represented by a condenser of equal capacitance shunted between conductor and neutral midway along the line.

2. The split-condenser, or nominal-π method in which one-half the total capacitance of each conductor is assumed to be shunted between conductor and neutral at each end of the circuit.

30. Nominal-T Method.—This method assumes that the total capacitance of the line may be concentrated at its middle point and so the full charging current flows over half the line. The arrangement of the circuit is then as shown schematically in

the upper part of Fig. 21. At the terminals of the condenser the voltage has a value E' intermediate between the sending-end voltage E_s and the receiving-end voltage E_r. A capacitance current I_c, leading E' by 90 degrees, accordingly flows along the sending-end half of the line to charge the condenser. The current in the receiving-end half of the line is the load current I_r, and in the sending-end half of the line it is I_s the vector sum of the capacitance current I_c and the load current I_r.

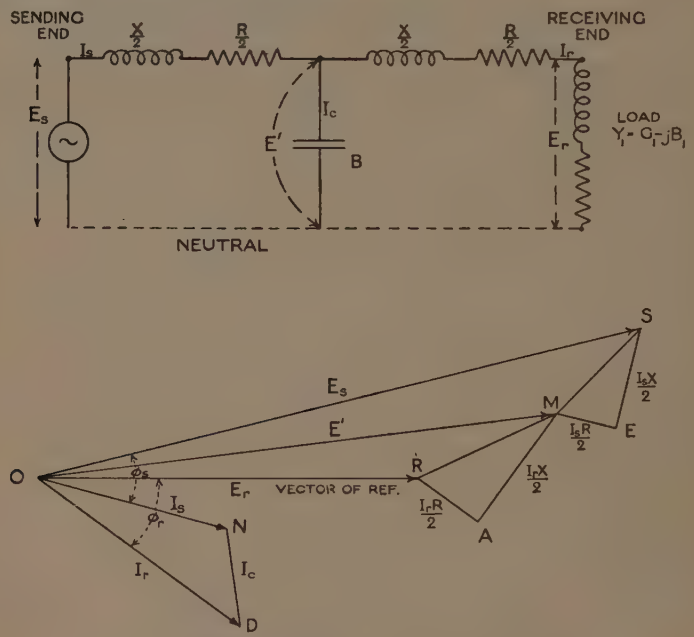


FIG. 21.—Middle-condenser or nominal-T method.

In the vector diagram the various quantities are represented as follows:—

OD is the current in the receiving circuit.

OR is the voltage to neutral at the receiving end drawn ϕ_r degrees in advance of *OD*: $\cos \phi_r$ being the load power factor.

RA drawn parallel to *OD* is the ohmic pressure drop in the receiving-end half of the line.

AM drawn 90 degrees ahead of *OD* is the reactive pressure drop in the receiving-end half of the line.

OM is the voltage at the centre of the line.

DN drawn 90 degrees ahead of OM is the charging current of the line.

ON which is the vector sum of OD and DN is the current at the sending end.

ME drawn parallel to ON is the ohmic pressure drop in the sending-end half of the line.

ES drawn 90 degrees ahead of ON is the reactive pressure drop in the sending-end half of the line.

OS is the voltage at the sending end.

ϕ_s is the phase-angle at the sending end: $\cos \phi_s$ being the power factor at this end.

In order to determine the performance of the line by means of complex quantities let

$Y_1 = G_1 - jB_1$ = admittance of receiving circuit,

$Z = R + jX$ = impedance of line,

and B = condensive susceptance of line.

Then referring to the schematic diagram in the upper part of Fig. 21 we have

$$\dot{I}_r = \dot{E}_r(G_1 - jB_1).$$

$$\dot{E}' = \dot{E}_r + \frac{R + jX}{2} I_r$$

$$= \dot{E}_r\left\{1 + \frac{(R + jX)(G_1 - jB_1)}{2}\right\}.$$

$$\dot{I}_c = \dot{E}' jB$$

$$= jB\dot{E}_r\left\{1 + \frac{(R + jX)(G_1 - jB_1)}{2}\right\}.$$

$$\dot{I}_s = \dot{I}_r + \dot{I}_c$$

$$= \dot{E}_r\left[G_1 - jB_1 + jB\left\{1 + \frac{(R + jX)(G_1 - jB_1)}{2}\right\}\right] \quad . \quad (46)$$

$$\dot{E}_s = \dot{E}' + \frac{R + jX}{2} I_s$$

$$= \dot{E}_r\left\{1 + \frac{(R + jX)(G_1 - jB_1)}{2} + \frac{(R + jX)(G_1 - jB_1)}{2} + jB\left(\frac{R + jX}{2}\right) + jB\frac{(R + jX)^2(G_1 - jB_1)}{4}\right\}$$

$$= \dot{E}_r\left\{1 + (R + jX)\left(G_1 - jB_1 + \frac{jB}{2}\right) + \frac{jB}{4}(R + jX)^2(G_1 - jB_1)\right\}. \quad (47)$$

31. Nominal-π Method.—This method assumes that the total capacitance of the line is split up, one-half being placed across the line at each end. In this case one-half of the total charging current flows over the entire line.

The schematic diagram for this arrangement of the circuit is given in the upper part of Fig. 22. The current flowing in the line is I_r the vector sum of the load current I and the current I_{cr} taken by the receiving-end condenser. Knowing this resultant current, the voltage at the sending end can easily be found by the

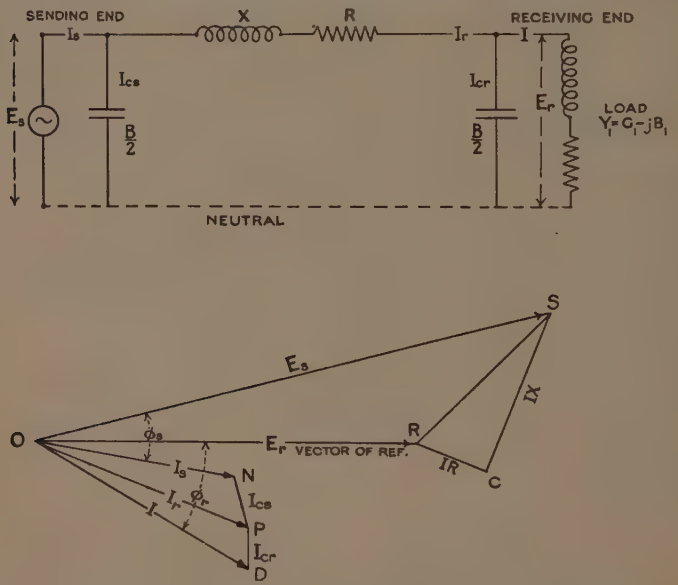


FIG. 22.—Split-condenser or nominal-π method.

fundamental impedance method used for short lines. The sending-end voltage is therefore more readily calculated by this method than by the middle-condenser method which requires the calculation of the two separate halves of the circuit. If, however, the sending-end current and power factor are also required, a second calculation must be made to determine them. In such a case the current I_{cs} consumed by the condenser at the sending end must be added vectorially to that in the line.

In the vector diagram the various quantities are represented as follows:—

OR is the voltage to neutral at the receiving end.

OD is the load current drawn ϕ_r degrees behind OR: $\cos \phi_r$ being the load power factor.

DP drawn 90 degrees ahead of OR is the charging current consumed by the receiving-end condenser.

OP which is the vector sum of OD and DP is the resultant current in the line.

RC drawn parallel to OP is the ohmic pressure drop.

CS drawn 90 degrees ahead of OP is the reactive pressure drop.

OS is the voltage to neutral at the sending end.

PN drawn 90 degrees ahead of OS is the charging current consumed by the sending-end condenser.

ON which is the vector sum of OP and PN is the resultant sending-end current.

ϕ_s is the phase angle at the sending end: $\cos \phi_s$ being the power factor at this end.

Applying complex quantities to the solution of the same circuit we have the following relations:—

$$\begin{aligned}
\dot{I}_r &= \dot{I} + \dot{I}_{cr} \\
&= \dot{E}_r(G_1 - jB_1) + \frac{jB}{2}\dot{E}_r \\
&= \dot{E}_r\left(G_1 - jB_1 + \frac{jB}{2}\right).
\end{aligned}$$

$$\begin{aligned}
\dot{E}_s &= \dot{E}_r + \dot{I}_r(R + jX) \\
&= \dot{E}_r + \dot{E}_r\left(G_1 - jB_1 + \frac{jB}{2}\right)(R + jX) \\
&= \dot{E}_r\left\{1 + \left(G_1 - jB_1 + \frac{jB}{2}\right)(R + jX)\right\}. \qquad . \quad . \quad . \quad (48)
\end{aligned}$$

$$\begin{aligned}
\dot{I}_{cs} &= \frac{jB}{2}\dot{E}_s \\
&= \frac{jB}{2}\dot{E}_r\left\{1 + \left(G_1 - jB_1 + \frac{jB}{2}\right)(R + jX)\right\}.
\end{aligned}$$

$$\begin{aligned}
\dot{I}_s &= \dot{I}_r + \dot{I}_{cs} \\
&= \dot{E}_r\left(G_1 - jB_1 + \frac{jB}{2}\right) + \frac{jB}{2}\dot{E}\left\{1 + \left(G_1 - jB_1 + \frac{jB}{2}\right)(R + jX)\right\} \\
&= \dot{E}_r\left\{G_1 - jB_1 + jB + \frac{jB}{2}\left(G_1 - jB_1 + \frac{jB}{2}\right)(R + jX)\right\}. \quad (49)
\end{aligned}$$

32. Numerical Example.—Transmission line problems may be solved graphically by drawing the vector diagram to scale, but it is difficult to obtain accurate results by this method owing to the varying magnitude of the quantities represented, and it is advisable to calculate all quantities. These calculations may be made trigonometrically, using the appropriate vector diagram as a guide, or the method of solution by means of complex quantities may be employed.

In order to show the nature of the agreement between the results obtained by the two localised-capacitance methods, the same problem will be solved by both methods.

Problem 3.—Find the characteristics of the load at the sending end of a three-phase line 120 miles long, consisting of three No. 000 S.W.G., hard-drawn, copper conductors, spaced in a nine-foot delta arrangement. Temperature taken as 20° C. Load conditions at receiving end assumed as 15,000 kVA. (13 500 kW. at 90 °/₀ power factor lagging), 88 000 volts, 50 cycles.

From wire tables the resistance per mile of wire (97 °/₀ conductivity) is found to be 0·406 ohms, and the diameter of the wire is 0·372 inches.

The resistance of each conductor is therefore

$$R = 0{\cdot}406 \times 120 = 48{\cdot}7 \quad \text{ohms.}$$

The inductance of each conductor is

$$L = \left(0{\cdot}080 + 0{\cdot}741 \log_{10} \frac{108}{0{\cdot}186}\right)10^{-3} \times 120 \quad . \quad (9)$$
$$= 0{\cdot}255 \quad \text{henries,}$$

and the reactance is

$$X = 2\pi \times 50 \times 0{\cdot}255$$
$$= 80{\cdot}2 \quad \text{ohms.}$$

The capacitance of each conductor from formula (14) is

$$C = \frac{0{\cdot}0388}{\log_{10} \frac{108}{0{\cdot}186}} 10^{-6} \times 120$$
$$= 1{\cdot}68 \times 10^{-6} \quad \text{farads,}$$

and the condensive susceptance is

$$B = 2\pi \times 50 \times 1{\cdot}68 \times 10^{-6}$$
$$= 0{\cdot}000529 \quad \text{mhos.}$$

At the receiving end of the line

$$kVA_r = \frac{15\,000}{3} = 5\,000 \quad \text{kVA. per phase.}$$

$$kW_r = \frac{13\,500}{3} = 4\,500 \quad \text{kW. per phase.}$$

$$E_r = \frac{88\,000}{\sqrt{3}} = 50\,810 \quad \text{volts to neutral.}$$

$$\cos \phi_r = 90\ \% \quad \text{lagging.}$$

The load current is

$$I = \frac{5\,000 \times 1\,000}{50\,810} = 98{\cdot}5 \quad \text{amperes.}$$

The load admittance is

$$\gamma_1 = \frac{I}{E_r}$$

$$= \frac{98{\cdot}5}{50\,810} = 0{\cdot}\,001939 \quad \text{mhos per phase.}$$

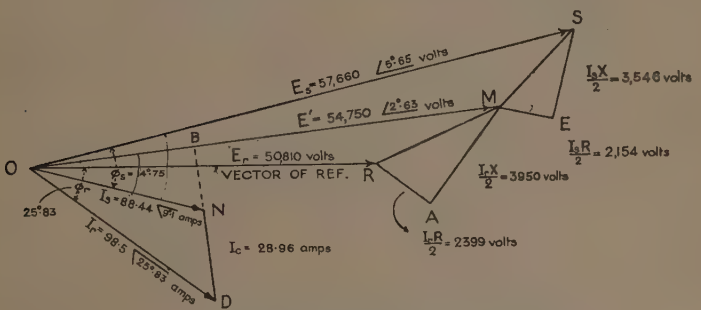


FIG. 23.—Vector solution by nominal-T method.

The load conductance is

$$G_1 = \gamma_1 \cos \phi_r$$

$$= 0{\cdot}\,001939 \times 0{\cdot}9 = 0{\cdot}\,001745 \quad \text{mhos per phase.}$$

The load susceptance is

$$B_1 = \gamma_1 \sin \phi_r$$

$$= 0{\cdot}\,001939 \times 0{\cdot}\,4359 = 0{\cdot}\,0008450 \quad \text{mhos per phase.}$$

33. Nominal-T Solution.—The graphical vector solution for the above problem by the nominal-T method is indicated in Fig. 23, although, for the sake of clearness, this is not drawn strictly to scale.

The electrical conditions at the middle of the line are first of all determined as follows :—

$$\frac{I_r R}{2} = 98{\cdot}5 \times 24{\cdot}35 = 2\,399 \quad \text{volts (resistance drop).}$$

$$\frac{I_r X}{2} = 98{\cdot}5 \times 40{\cdot}1 = 3\,950 \quad \text{volts (reactance drop).}$$

$$E' = 50\,810\sqrt{\left(0{\cdot}9 + \frac{2\,399}{50\,810}\right)^2 + \left(0{\cdot}4359 + \frac{3\,950}{50\,810}\right)^2}$$

from formula (31)

$$= 50\,810\sqrt{(0{\cdot}9472)^2 + (0{\cdot}5135)^2}$$

$$= 54\,750 \quad \text{volts to neutral.}$$

E' leads the current vector OD by $\tan^{-1}\frac{0{\cdot}5135}{0{\cdot}9472} = 28{\cdot}47°$.

So

$$E' = 54\,750 \angle 28{\cdot}47° \quad \text{volts to neutral with reference to vector } OD.$$

$$= 54\,750 \angle 2{\cdot}63° \quad \text{volts to neutral with reference to vector } OR.$$

The current consumed by the condenser leads the voltage E' by 90 degrees and is

$$I_c = 0{\cdot}000529 \times 54\,750 = 28{\cdot}96 \quad \text{amperes.}$$

The current in the sending-end half of the line is determined as follows:—

$$OB = 98{\cdot}5 \cos 28{\cdot}47° = 86{\cdot}59 \quad \text{amperes.}$$

$$BD = 98{\cdot}5 \sin 28{\cdot}47° = 46{\cdot}95 \quad \text{amperes.}$$

$$BN = 46{\cdot}95 - 28{\cdot}96 = 17{\cdot}99 \quad \text{amperes.}$$

$$I_s = ON = \sqrt{86{\cdot}59^2 + 17{\cdot}99^2}$$

$$= 88{\cdot}44 \angle 11{\cdot}73° \quad \text{amperes with reference to vector } OB.$$

$$= 88{\cdot}44 \angle 9{\cdot}10° \quad \text{amperes with reference to vector } OR.$$

The sending-end voltage is then found as follows:—

$$\frac{I_s R}{2} = 88{\cdot}44 \times 24{\cdot}35 = 2\,154 \quad \text{volts (resistance drop).}$$

$$\frac{I_s X}{2} = 88{\cdot}44 \times 40{\cdot}1 = 3\,546 \quad \text{volts (reactance drop).}$$

$$E_s = 54\,750\sqrt{\left(\cos 11{\cdot}73° + \frac{2\,154}{54\,750}\right)^2 + \left(\sin 11{\cdot}73° + \frac{3\,546}{54\,750}\right)^2} \quad (31)$$

$$= 54\,750\sqrt{\left(0{\cdot}9791 + \frac{2\,154}{54\,750}\right)^2 + \left(0{\cdot}2034 + \frac{3\,546}{54\,750}\right)^2}$$

$$= 54\,750\sqrt{(1{\cdot}0184)^2 + (0{\cdot}2682)^2}.$$

$$= 57\,660 \angle 14{\cdot}75° \quad \text{volts to neutral with reference to vector } ON$$

$$= 57\,660 \angle 5{\cdot}65° \quad \text{volts to neutral with reference to vector } OR.$$

The other electrical conditions at the sending end are

$$\cos \phi_s = \cos 14{\cdot}75^\circ = 96{\cdot}70\ \%\quad \text{lagging.}$$
$$\text{kVA}_s = 57\,660 \times 88{\cdot}44 = 5\,099 \quad \text{kVA. per phase.}$$
$$\text{kW}_s = 5\,099 \times 0{\cdot}9670 = 4\,931 \quad \text{kW. per phase.}$$
$$\text{Loss} = 4\,931 - 4\,500 = 431 \quad \text{kW. per phase.}$$
$$\text{Efficiency} = \frac{4\,500 \times 100}{4\,931} = 91{\cdot}26\ \%.$$

In order to solve the same problem by complex quantities, it is only necessary to substitute numerical values in the analytical expressions for voltage and current derived in Art. 30.

The sending-end voltage is

$$\dot{E}_s = \dot{E}_r\left\{1 + (R + jX)\left(G_1 - jB_1 + \frac{jB}{2}\right) + \frac{jB}{4}(R + jX)^2(G_1 - jB_1)\right\} \tag{47}$$

$$= 50\,810\{1 + (48{\cdot}7 + 80{\cdot}2j)(0{\cdot}001745 - 0{\cdot}0008450j + 0{\cdot}0002645j) + 0{\cdot}0001322j(48{\cdot}7 + 80{\cdot}2j)^2(0{\cdot}001745 - 0{\cdot}0008450j)\}$$
$$= 50\,810(1{\cdot}1293 + 0{\cdot}1116j),$$

and its absolute value is

$$E_s = 50\,810\sqrt{(1{\cdot}1293)^2 + (0{\cdot}1116)^2}$$
$$= 57\,660 \angle 5{\cdot}65^\circ \quad \text{volts to neutral.}$$

The sending-end current is

$$\dot{I}_s = \dot{E}_r\left[G_1 - jB_1 + jB\left\{1 + \frac{(R + jX)(G_1 - jB_1)}{2}\right\}\right]. \tag{46}$$

$$= 50\,810[0{\cdot}001745 - 0{\cdot}0008450j + 0{\cdot}000529j\{1 + (24{\cdot}35 + 40{\cdot}1j)(0{\cdot}001745 - 0{\cdot}0008450j)\}]$$
$$= 50\,810(0{\cdot}001719 - 0{\cdot}0002753j),$$

and its absolute value is

$$I_s = 50\,810\sqrt{(0{\cdot}001719)^2 + (0{\cdot}0002753)^2}$$
$$= 88{\cdot}44 \angle 9{\cdot}10^\circ \quad \text{amperes.}$$

These results check exactly with those previously obtained by trigonometrical calculation.

34. Nominal-π Solution.—The graphical vector solution for Problem 3 by the nominal-π method is indicated in Fig. 24, although, like Fig. 23, this diagram is not drawn strictly to scale.

The charging current consumed by the receiving-end condenser leads the voltage at this end by 90 degrees and is,

$$I_{cr} = 0{\cdot}0002645 \times 50\,810 = 13{\cdot}44 \quad \text{amperes.}$$

The resultant current in each conductor is the vector sum of the load and condenser currents, and is determined as follows:—

$$OB = 98{\cdot}5 \times 0{\cdot}9 = 88{\cdot}65 \quad \text{amperes.}$$
$$BD = 98{\cdot}5 \times 0{\cdot}4359 = 42{\cdot}92 \quad \text{amperes.}$$
$$BP = 42{\cdot}92 - 13{\cdot}44 = 29{\cdot}48 \quad \text{amperes.}$$
$$I_r = OP = \sqrt{88{\cdot}65^2 + 29{\cdot}48^2}$$
$$= 93{\cdot}42 \angle 18{\cdot}40^\circ \quad \text{amperes with reference to vector } OR.$$

The voltage consumed by the resistance and reactance of each conductor is

$$I_r R = 93{\cdot}42 \times 48{\cdot}7 = 4\,550 \quad \text{volts (resistance drop).}$$
$$I_r X = 93{\cdot}42 \times 80{\cdot}2 = 7\,492 \quad \text{volts (reactance drop).}$$

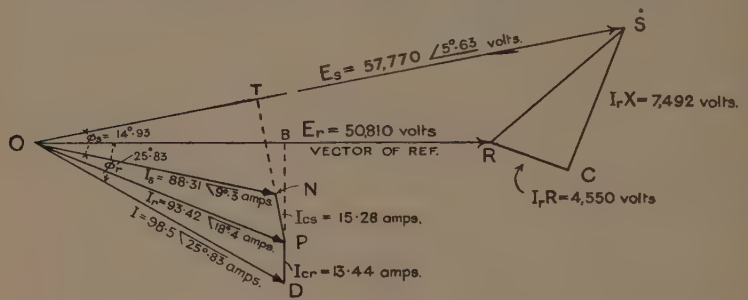


FIG. 24.—Vector solution by nominal-π method.

The sending-end voltage is

$$E_s = 50\,810\sqrt{\left(\cos 18{\cdot}40^\circ + \frac{4\,550}{50\,810}\right)^2 + \left(\sin 18{\cdot}40^\circ + \frac{7\,492}{50\,810}\right)^2}$$
$$= 50\,810\sqrt{(1{\cdot}0384)^2 + (0{\cdot}4631)^2}$$
$$= 57\,770 \angle 24{\cdot}03^\circ \quad \text{volts to neutral with reference to vector } OP.$$
$$= 57\,770 \angle 5{\cdot}63^\circ \quad \text{volts to neutral with reference to vector } OR.$$

The charging current consumed by the condenser at the sending end leads the voltage at this end by 90 degrees and is

$$I_{cs} = 0{\cdot}0002645 \times 57\,770 = 15{\cdot}28 \quad \text{amperes.}$$

The sending-end current is the vector sum of the current flowing in the conductor and the current consumed by the condenser at the sending end. It is found as follows:—

$$OT = 93{\cdot}42 \cos 24{\cdot}03^\circ = 85{\cdot}33 \quad \text{amperes.}$$
$$TP = 93{\cdot}42 \sin 24{\cdot}03^\circ = 38{\cdot}05 \quad \text{amperes.}$$
$$TN = 38{\cdot}05 - 15{\cdot}28 = 22{\cdot}77 \quad \text{amperes.}$$

$$I_s = ON = \sqrt{85{\cdot}33^2 + 22{\cdot}77^2}$$
$$= 88{\cdot}31\angle 14{\cdot}93^\circ \text{ amperes with reference to vector } OS$$
$$= 88{\cdot}31\angle 9{\cdot}30^\circ \text{ amperes with reference to vector } OR.$$

The electrical conditions at the sending end are therefore

$\cos \phi_s = \cos 14{\cdot}93^\circ = 96{\cdot}62$ °/₀ lagging.
$\text{kVA}_s = 57\,770 \times 88{\cdot}31 = 5\,102$ kVA. per phase.
$\text{kW}_s = 5\,102 \times 0{\cdot}9662 = 4\,929$ kW. per phase.
Loss $= 4\,929 - 4\,500 = 429$ kW. per phase.

$$\text{Efficiency} = \frac{4\,500 \times 100}{4\,929} = 91{\cdot}29 \text{ °/₀}.$$

Applying complex quantities to the solution of the problem the sending-end voltage is

$$\dot{E}_s = \dot{E}_r\left\{1 + \left(G_1 - jB_1 + \frac{jB}{2}\right)(R + jX)\right\} \quad . \quad . \quad . \quad (48)$$
$$= 50\,810\{1 + (0{\cdot}001745 - 0{\cdot}0008450j + 0{\cdot}0002645j)$$
$$(48{\cdot}7 + 80{\cdot}2j)\}$$
$$= 50\,810(1{\cdot}1315 + 0{\cdot}1117j)$$
$$= 57\,770\angle 5{\cdot}63^\circ \text{ volts to neutral.}$$

The sending-end current is

$$\dot{I}_s = \dot{E}_r\left\{G_1 - jB_1 + jB + \frac{jB}{2}\left(G_1 - jB_1 + \frac{jB}{2}\right)(R + jX)\right\} \quad (49)$$
$$= 50\,810\{0{\cdot}001745 - 0{\cdot}0008450j + 0{\cdot}000529j$$
$$+ 0{\cdot}0002645j(0{\cdot}001745 - 0{\cdot}0008450j + 0{\cdot}0002645j)$$
$$(48{\cdot}7 + 80{\cdot}2j)\}$$
$$= 50\,810(0{\cdot}001715 - 0{\cdot}0002809j)$$
$$= 88{\cdot}31\angle 9{\cdot}30^\circ \text{ amperes.}$$

The above results correspond exactly with those previously obtained by the trigonometrical method.

35. Ferranti Effect.—On a long transmission line, open-circuited, or only very lightly loaded, a rise of pressure occurs at the receiving end. This is generally known as the 'Ferranti effect' owing to its first being observed on the Deptford mains laid down by S. Z. de Ferranti in 1890.[1]

This pressure rise is due to the e.m.f. of self-inductance of the charging current being in phase with the impressed voltage at the sending end of the line, and thus both capacitance and inductance are necessary to produce the phenomenon.

In order to determine the magnitude of the pressure rise one-half of the total line capacitance will be assumed to be concentrated at the receiving end. In Fig. 25 the receiving-end voltage to neutral is represented by *OR*, while *OD* drawn 90 degrees ahead of *OR* is the charging current consumed by the receiving-end condenser. When the load is entirely disconnected *OD* is the total current in the conductor. The voltage consumed by the resistance of each conductor is *RC* drawn in phase with *OD*, and the voltage consumed by the reactance of each conductor is *CS* drawn 90 degrees ahead of *OD*. Thus the voltage at the sending end of the line is *OS*, which is smaller than that at the receiving end.

In the case of short lines the magnitude of the effect is negligible,

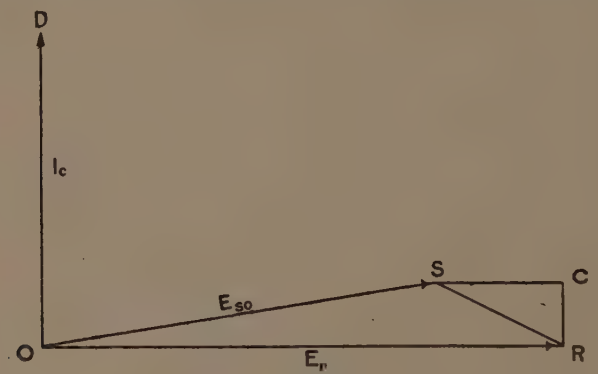


FIG. 25.—Ferranti effect.

but it increases rapidly as the length of the line increases, and on a 300-mile 50-cycle overhead transmission the receiving-end voltage on open-circuit would be 10 to 15 % higher than the sending-end voltage.

Overhead lines having a sufficient length to bring the effect into prominence usually have a fairly large reactance compared with resistance, and by taking advantage of this fact a simple approximate formula to calculate the magnitude of the effect can be found. Thus, referring again to Fig. 25, if *RC* can be neglected in comparison with *CS*, the pressure rise *OR* − *OS* can be taken as equal to the induced volts without introducing any serious error.

The charging current flowing in the conductors is

$$I_c = 2\pi f \frac{C_0}{2} El, \quad . \quad . \quad . \quad . \quad (50)$$

where C_0 = the capacitance per mile of conductor in farads
and l = length of line in miles.

The induced volts are

$$I_c X = 2\pi f L_0 I_c l, \quad . \quad . \quad . \quad . \quad (51)$$

where L_0 = the inductance per mile of conductor in henries.

Substituting for I_c in the above,

$$I_c X = (2\pi f)^2 \frac{L_0 C_0}{2} l^2 E \text{ volts}, \quad . \quad . \quad . \quad (52)$$

but from formulæ (9) and (14) it can be seen that the product $L_0 C_0$ for all overhead lines has a practically constant value of 3×10^{-11}, hence the pressure rise at the end of the line is

$$I_c X = (2\pi f)^2 \frac{3 \times 10^{-11}}{2} l^2 E$$

$$= 5{\cdot}9 E f^2 l^2 10^{-10} \text{ volts}. \quad . \quad . \quad (53)$$

For 50-cycle lines this gives an approximate value of

$$1{\cdot}5E\left(\frac{l}{1\,000}\right)^2 \text{ volts}. \quad . \quad . \quad . \quad (54)$$

In the case of the particular line having the constants given in Problem 3, the pressure rise at zero load is

$$1{\cdot}5 \times 50\,810\left(\frac{120}{1\,000}\right)^2$$

$$= 1\,100 \text{ volts to neutral},$$

or assuming that the receiving-end voltage was held constant at 50 810 volts to neutral, the corresponding sending-end voltage would be

$$E_{so} = 50\,810 - 1\,100$$

$$= 49\,710 \text{ volts to neutral}.$$

36. Charging Current and Losses in Open-circuited Line.—In transmission line operation the voltage at the receiving end is usually maintained at a constant value although the actual load on the system may vary considerably at different times.

The charging current of an open-circuited line then refers to the amount of current flowing into the line at the sending end with normal voltage and zero load at the receiving end.

In many cases the total charging current of the system is determined by multiplying the total susceptance of the line by the receiving-end voltage. This would be correct if the voltage

throughout the entire length of the line had the same value as at the receiving end. As shown above, this condition is only approximately complied with, but this method of finding the total charging current is sufficiently accurate for most commercial lines.

Making then the assumption of voltage equality along the line, the actual value of the charging current will decrease uniformly from its maximum value at the sending end to zero at the receiving end.

The value of the current at a point x miles from the sending end of a line l miles in length is

$$\frac{I_c}{l}(l - x), \quad . \quad . \quad . \quad . \quad (55)$$

where I_c is the total charging current as found by the above method.

The average value of the square of this quantity is

$$\frac{1}{l}\int_0^l \frac{I_c^2}{l^2}(l - x)^2 dx$$

$$= \frac{I_c^2}{l^3}\int_0^l (l - x)^2 dx$$

$$= \frac{I_c^2}{3}. \quad . \quad . \quad . \quad . \quad . \quad (56)$$

Hence at open-circuit, the power losses in each conductor are

$$\frac{I_c^2}{3}R. \quad . \quad . \quad . \quad . \quad . \quad (57)$$

Applying these results to the line described in Problem 3, the charging current is

$$I_c = 50\ 810 \times 0{\cdot}000529$$
$$= 26{\cdot}87 \quad \text{amperes,}$$

the kilovolt-amperes put into the line at the sending end are

$$\text{kVA}_{so} = 50\ 810 \times 26{\cdot}87 \times 10^{-3}$$
$$= 1\ 365 \quad \text{kVA. per phase,}$$

and the open-circuit power losses are

$$\frac{26{\cdot}87^2}{3} \times 48{\cdot}7 \times 10^{-3}$$
$$= 11{\cdot}72 \quad \text{kW. per phase.}$$

37. Effect of Capacitance on Performance of Loaded Line. —When, as usually obtains in practice, the transmission line sup-

plies an inductive load, the effect of capacitance is to reduce the pressure drop along the line and also to decrease the copper losses of the system. The magnitude of this effect depends on the phase-angle at the receiving end.

In any line, the current flowing along the conductors can be split up into two components, the load current and the charging current. The effect of the load current with lagging or unity power factor in the receiving circuit is to produce a voltage at the receiving end lower than that impressed on the line at the sending end. The charging current, however, tends to raise the receiving-end voltage, consequently the pressure drop along the line is diminished by the amount indicated by formula (53).

In this connection the distinction between the following terms should be noted: 1. Impedance drop, 2. Pressure drop, and 3. Regulation.

1. The impedance drop in the line is the voltage consumed in the line impedance. It is the vector difference between the sending-end voltage and the receiving-end voltage.

2. The pressure drop is the arithmetical difference between the sending- and receiving-end voltages.

3. The regulation is the change of pressure at the receiving end when full load is thrown off, the sending-end voltage being held constant. In the case of short lines the regulation and pressure drop are numerically equal. In lines where capacitance effects have to be reckoned with, the regulation is greater than the pressure drop by the pressure rise due to the Ferranti effect.

Thus, referring again to Problem 3, and using the results of the split-condenser method of solution,

$$\begin{aligned} \text{Regulation} &= \text{Pressure drop} + 1\ 100 \\ &= 57\ 770 - 50\ 810 + 1\ 100 \\ &= 8\ 060 \quad \text{volts to neutral,} \end{aligned}$$

and expressed in terms of the receiving-end voltage

$$\text{Percentage regulation} = \frac{8\ 060 \times 100}{50\ 810} = 15{\cdot}86\ \%.$$

The presence of capacitance also influences the power losses of the system by an amount depending on the relative values of the charging current and the quadrature component of the load current.

For instance, referring to Fig. 22, the load current I can be

resolved into the two components $I \cos \phi_r$ in phase with E_r, and $I \sin \phi_r$ in quadrature behind E_r. The charging current I_{cr} is 90 degrees ahead of E_r and hence reduces the value of the quadrature component of the resultant line current from $I \sin \phi_r$ to $I \sin \phi_r - I_{cr}$. The resultant line current is thus

$$I_r = \sqrt{(I \cos \phi_r)^2 + (I \sin \phi_r - I_{cr})^2},$$

and the copper losses are reduced in the ratio

$$k = \frac{(I \cos \phi_r)^2 + (I \sin \phi_r - I_{cr})^2}{I^2}. \quad . \quad . \quad (58)$$

It is easily seen from this formula that a reduction in the losses takes place so long as I_{cr} is less than $2I \sin \phi_r$.

Assuming now that the power factor of the load is unity there will be no quadrature component of the load current. Thus the effect of the charging current in a line supplying a non-inductive load is always to increase the copper losses.

Also if the power factor is lagging, but the actual load on the system is small, I_{cr} may be greater than $2I \sin \phi_r$, giving again an increased line loss.

In general, however, with the loads and power factors obtained in practice, the influence of capacitance is beneficial in considerably reducing the losses of the system. Table 2 is calculated from the data given in Problem 3, and shows the actual copper losses expressed as a percentage of what they would have been if there had been no charging current present in the line.

TABLE 2.—*Effect of Capacitance on Line Losses of a Particular Transmission.*

Load Power Factor.	Actual I^2R Losses as a Percentage of those which would Exist if Charging Current were Zero.
80 % lagging	85·5 %
85 % ,,	87·5 %
90 % ,,	90·0 %
95 % ,,	93·3 %
Unity	101·9 %

38. Limitations of Localised-capacitance Methods of Solution.—The localised-capacitance methods of solution are capable of giving fairly accurate results for 50-cycle overhead

lines up to about 150 miles in length. That is to say the percentage error in the sending-end voltage, as calculated from the receiving-end data, by either of the methods described will be less than 1 $^{\circ}/_{\circ}$ for lines of this length. It is interesting to note that the nominal-T method over-compensates (that is, gives sending-end voltages which are too low), while the nominal-π method under-compensates.

A still higher degree of accuracy can be obtained by the three-condenser method suggested by Steinmetz,[2] in which the capacitance of the line is assumed to be concentrated across the line at three points, one-sixth being localised at each end and two-thirds midway along the line.

As a matter of fact, however, localised capacitance methods are now chiefly of academic interest as formulæ have been developed which take into exact account the distributed character of the capacitance and other line constants and at the same time do not involve much more labour in computation.

REFERENCES.

[1] J. A. Fleming, 'On Some Effects of Alternating-Current Flow in Circuits having Capacity and Self-Induction,' *Jour. I.E.E.*, Vol. 20, p. 362 (1891).

[2] C. P. Steinmetz, *Theory and Calculation of Alternating Current Phenomena*, p. 169 (5th Ed., 1916).

CHAPTER V.

PERFORMANCE OF LONG TRANSMISSION LINES (RIGOROUS METHODS OF SOLUTION).

39. Alternating-current Phenomena in a Line having Distributed Electrical Properties.—In order to obtain a physical conception of the voltage and current variation from point to point on a long transmission line it may be imagined that the line is divided into n sections each having a resistance, inductance and capacitance $\frac{1}{n}$th of those for the whole line. Also to take account of the energy losses occurring through leakage over the insulators, or due to corona effect between conductors, a conductance of value $\frac{G}{n}$ may be assumed to be connected across the line at each section. The line is then represented schematically by the upper part of Fig. 26.

Starting from one end of the line, the voltage and current at any point can be found by taking each section in turn and constructing a diagram showing the voltages and currents consumed by the components of each section.

For example, in Fig. 26, $OR(E_r)$ represents the voltage to neutral at the receiving end of the line, and $OD(I_r)$ the current in each conductor lagging ϕ_r degrees behind E_r.

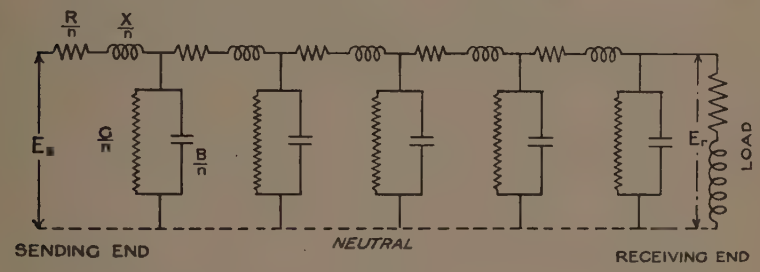
Consider first the section next to the receiving circuit.

A voltage RE_1, proportional to and in phase with OD, is consumed by the resistance, and a voltage E_1E_1', proportional to and 90 degrees ahead of OD, is consumed by the reactance. In the same section there is a current DI_1, proportional to and in phase with the voltage OR, representing the loss of current by leakage, and a current I_1I_1', proportional to and 90 degrees ahead of OR, being the charging current of the section.

Thus at the sending end of this section, the voltage and current are OE_1' and OI_1' respectively.

Passing now to the next section, voltages $E_1'E_2$ and E_2E_2' are consumed in phase and 90 degrees ahead of OI_1' respectively, and both proportional thereto, and also currents $I_1'I_2$ and I_2I_2' in phase and 90 degrees ahead of OE_1' respectively, and both proportional thereto.

In a similar manner, passing along from section to section, curves $RE_1'E_2'$ . . . and $DI_1'I_2'$ . . . are obtained which represent



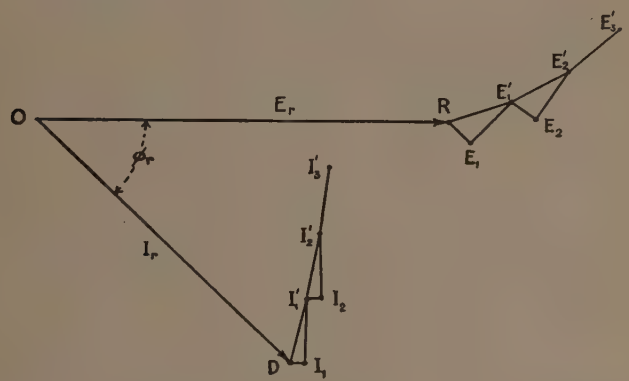


Fig. 26.—Transmission line with distributed electrical properties.

the loci of the voltage and current vectors of the line, and these can be traced out until the sending end of the line is reached.

When the number of line sections is infinite, the voltage and current loci can be readily calculated by means of hyperbolic functions, and then appear as smooth curves which have been called by Steinmetz 'Topographical circuit characteristics.' [1]

Fig. 27 shows the topographical circuit characteristics of a transmission line having the following constants per mile of conductor :—

Resistance = 0·275 ohms.
Reactance = 0·769 ohms.
Conductance = $0{\cdot}15 \times 10^{-6}$ mhos.
Susceptance = $5{\cdot}52 \times 10^{-6}$ mhos.
Frequency = 60 cycles per second.

The receiving-end voltage to neutral is 50 000 volts, and the load current is 25 amperes, lagging 25 degrees, behind the voltage.

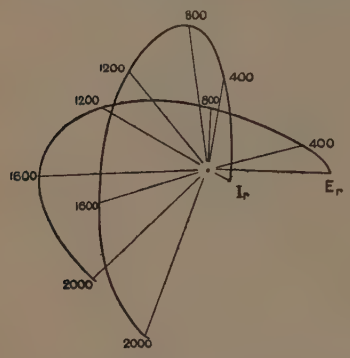


Fig. 27.—Topographical circuit characteristics for a long line.

In order to show clearly the nature of the phenomena which occurs, a line length of 2 000 miles is taken, the vectors for the end of each 400-mile section being drawn.

It will be observed that passing from the receiving end towards the sending end of the line, the voltage and current alternately rise and fall, while the angle of phase difference changes periodically between lag and lead. This is shown more clearly in Fig. 28 where the

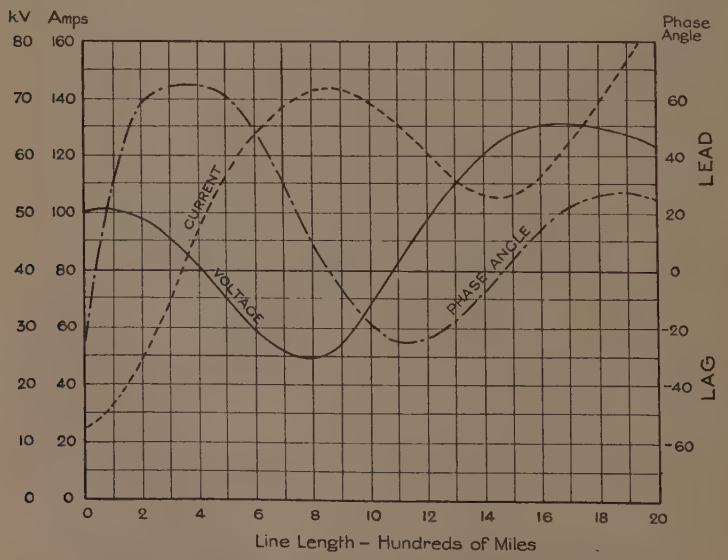


Fig. 28.—Voltage, current, and phase-angle in a long line.

values of voltage and current are plotted in rectangular co-ordinates with the distances from the receiving circuit as abscissæ.

40. Line Constants.—When the number of sections into which the line is supposed to be divided becomes infinite, the four electrical properties, *viz.*, resistance, inductance, capacitance and conductance, are evenly distributed throughout the length of the line, and thus conform to the conditions actually obtaining in practice. For this case rigorous mathematical formulæ can be applied to calculate the performance of the line.

Let

r = resistance per unit length of line,
x = reactance per unit length of line,
b = susceptance per unit length of line,
and g = conductance per unit length of line.

These constants must be regarded as effective values, since the apparent properties of a transmission system carrying alternating currents are dependent, to some extent, on the frequency of the system.

Then

$z = r + jx$ = impedance per unit length,
and $y = g + jb$ = admittance per unit length,

or in absolute values

$$z = \sqrt{r^2 + x^2}.$$
$$y = \sqrt{g^2 + b^2}.$$

41. Fundamental Differential Equations and Solutions.—The impedance of an element dl of the line is

$$z dl,$$

and the voltage $d\dot{E}$ consumed by the current $\dot{I}$ in this element

$$d\dot{E} = z\dot{I}dl.$$

The admittance of the line element dl is

$$y dl,$$

and the current $d\dot{I}$ consumed by the voltage $d\dot{E}$ of this element

$$d\dot{I} = y\dot{E}dl.$$

This gives the two equations

$$\frac{d\dot{E}}{dl} = z\dot{I}; \quad . \quad . \quad . \quad . \quad (59)$$

$$\frac{d\dot{I}}{dl} = y\dot{E}. \quad . \quad . \quad . \quad . \quad (60)$$

Differentiating the first equation and substituting therein the second gives

$$\frac{d^2\dot{E}}{dl^2} = zy\dot{E}, \quad . \quad . \quad . \quad . \quad (61)$$

and from the first equation follows

$$\dot{I} = \frac{1}{z}\frac{d\dot{E}}{dl}. \quad . \quad . \quad . \quad . \quad (62)$$

Equation (61) is integrated by

$$\dot{E} = A\epsilon^{Vl}, \quad . \quad . \quad . \quad . \quad (63)$$

and substituting (63) in (61) gives

$$V^2 = zy,$$

hence

$$V = \pm\sqrt{zy}.$$

Thus there exists two values of V which make (63) a solution of (61), and the most general solution therefore is

$$\dot{E} = A_1\epsilon^{+\sqrt{zy}\,l} + A_2\epsilon^{-\sqrt{zy}\,l}. \quad . \quad . \quad . \quad (64)$$

Substituting (64) in (62) gives

$$\dot{I} = \sqrt{\frac{y}{z}}\{A_1\epsilon^{+\sqrt{zy}\,l} + A_2\epsilon^{-\sqrt{zy}\,l}\}, \quad . \quad . \quad (65)$$

where l is counted from some point of the line as starting-point, as for instance from the receiving end.

If then, E_r and I_r are respectively the voltage and current at the receiving end of the line where $l = 0$ we have

$$\dot{E}_r = A_1 + A_2,$$

$$\dot{I}_r = \sqrt{\frac{y}{z}}\{A_1 - A_2\},$$

hence

$$A_1 = \tfrac{1}{2}\left\{\dot{E}_r + \sqrt{\frac{z}{y}}\dot{I}_r\right\}, \quad . \quad . \quad . \quad (66)$$

$$A_2 = \tfrac{1}{2}\left\{\dot{E}_r - \sqrt{\frac{z}{y}}\dot{I}_r\right\}, \quad . \quad . \quad . \quad (67)$$

and substituting (66) and (67) into (64) and (65) respectively

$$\dot{E} = \dot{E}_r\frac{\epsilon^{+\sqrt{zy}\,l} + \epsilon^{-\sqrt{zy}\,l}}{2} + \sqrt{\frac{z}{y}}\dot{I}_r\frac{\epsilon^{+\sqrt{zy}\,l} - \epsilon^{-\sqrt{zy}\,l}}{2}. \quad (68)$$

$$\dot{I} = \dot{I}_r\frac{\epsilon^{+\sqrt{zy}\,l} + \epsilon^{-\sqrt{zy}\,l}}{2} + \sqrt{\frac{y}{z}}\dot{E}_r\frac{\epsilon^{+\sqrt{zy}\,l} - \epsilon^{-\sqrt{zy}\,l}}{2}. \quad (69)$$

Replacing the quantities in brackets by their equivalent hyperbolic functions and noting that

$$\pm\sqrt{zy}\,l = \pm\sqrt{ZY},$$

where

$$Z = zl = \text{total line impedance},$$

and

$$Y = yl = \text{total line admittance},$$

gives the sending-end voltage and current as

$$\dot{E}_s = \dot{E}_r \cosh\sqrt{ZY} + \dot{I}_r\sqrt{\frac{Z}{Y}}\sinh\sqrt{ZY}, \quad . \quad (70)$$

$$\dot{I}_s = \dot{I}_r \cosh\sqrt{ZY} + \dot{E}_r\sqrt{\frac{Y}{Z}}\sinh\sqrt{ZY}, \quad . \quad (71)$$

which are the fundamental equations of voltage and current for a transmission line.

42. Introduction of Convergent Series.—Provided that tables or charts of complex hyperbolic functions are available, the above equations may be applied directly for the calculation of the line performance. If, however, this material is not available, the equations can be written in a form suitable for computation by using an infinite series the successive terms in which decrease rapidly.

The series for the hyperbolic cosine and sine are

$$\cosh\theta = 1 + \frac{\theta^2}{\lfloor 2} + \frac{\theta^4}{\lfloor 4} + \frac{\theta^6}{\lfloor 6} + \ldots,$$

$$\sinh\theta = \theta + \frac{\theta^3}{\lfloor 3} + \frac{\theta^5}{\lfloor 5} + \frac{\theta^7}{\lfloor 7} + \ldots.$$

Introducing these series in (70) and (71) with

$$\theta = \sqrt{ZY},$$

gives

$$\dot{E}_s = \dot{E}_r\left(1 + \frac{ZY}{2} + \frac{Z^2Y^2}{24} + \ldots\right) + \dot{I}_r\sqrt{\frac{Z}{Y}}\left(\sqrt{ZY} + \frac{(ZY)^{\frac{3}{2}}}{6} + \ldots\right), \quad (72)$$

$$\dot{I}_s = \dot{I}_r\left(1 + \frac{ZY}{2} + \frac{Z^2Y^2}{24} + \ldots\right) + \dot{E}_r\sqrt{\frac{Y}{Z}}\left(\sqrt{ZY} + \frac{(ZY)^{\frac{3}{2}}}{6} + \ldots\right), \quad (73)$$

and by combining factors in the second portion of each

$$\dot{E}_s = \dot{E}_r\left(1 + \frac{ZY}{2} + \frac{Z^2Y^2}{24} + \frac{Z^3Y^3}{720} + \ldots\right) + \dot{I}_rZ\left(1 + \frac{ZY}{6} + \frac{Z^2Y^2}{120} + \frac{Z^3Y^3}{5\,040} + \ldots\right), \quad (74)$$

$$\dot{I}_s = \dot{I}_r\left(1 + \frac{ZY}{2} + \frac{Z^2Y^2}{24} + \frac{Z^3Y^3}{720} + \ldots\right) + \dot{E}_rY\left(1 + \frac{ZY}{6} + \frac{Z^2Y^2}{120} + \frac{Z^3Y^3}{5\,040} + \ldots\right). \quad (75)$$

43. Auxiliary Constants of Circuit.—In the above equations, the three auxiliary constants, which take into account the distributed nature of the line properties, are represented by the quantities

$$A = \cosh\sqrt{ZY} = \left(1 + \frac{ZY}{2} + \frac{Z^2Y^2}{24} + \frac{Z^3Y^3}{720} + \ldots\right);$$

$$B = \sqrt{\frac{Z}{Y}}\sinh\sqrt{ZY} = Z\left(1 + \frac{ZY}{6} + \frac{Z^2Y^2}{120} + \frac{Z^3Y^3}{5\,040} + \ldots\right);$$

$$C = \sqrt{\frac{Y}{Z}}\sinh\sqrt{ZY} = Y\left(1 + \frac{ZY}{6} + \frac{Z^2Y^2}{120} + \frac{Z^3Y^3}{5\,040} + \ldots\right).$$

Equations (74) and (75) may therefore be expressed in terms of A, B, and C as follows :—

$$\dot{E}_s = \dot{E}_rA + \dot{I}_rB, \quad (76)$$

$$\dot{I}_s = \dot{I}_rA + \dot{E}_rC. \quad (77)$$

If the voltage and current values at the sending end are given, the corresponding values at the receiving end may be found by substituting $-l$ for l in equations (68) and (69), which gives

$$\dot{E}_r = \dot{E}_sA - \dot{I}_sB, \quad (78)$$

$$\dot{I}_r = \dot{I}_sA - \dot{E}_sC. \quad (79)$$

It should be noted that these auxiliary constants A, B, and C are functions of the physical properties of the line and of the frequency only, and not of the voltages and currents. In effect, all the various methods for transmission line solution are only schemes for determining the values of these constants. Having

these numerical values, they may then be applied directly to any numerical values of E and I for which a solution is desired. From this point on, the performance of the line may be determined either by mathematical or graphical methods.

44. Illustration of Convergence of Series.—When using the convergent series expressions for determining the auxiliary constants of the line, any desired degree of accuracy may be obtained by simply using a sufficient number of terms in the series.

To illustrate the manner of convergence of the series, Table 3 has been prepared, and shows the polar values of the auxiliary constants for a 300-mile, 50-cycle transmission having the line constants

$$R = 52{\cdot}5 \quad \text{ohms.}$$
$$X = 188{\cdot}4 \quad \text{ohms.}$$
$$G = 0$$
$$B = 0{\cdot}001411 \quad \text{mhos.}$$

TABLE 3.—*Convergence of Series Expressions for the Auxiliary Constants of 300-Mile Transmission Line.*

$R = 52{\cdot}5$ ohms. $X = 188{\cdot}4$ ohms. $G = 0$. $B = 0{\cdot}001411$ mhos.

No. of Terms.	A.	B.	C.
1	$1{\cdot}00000 \angle 0{\cdot}000°$	$195{\cdot}58 \angle 74{\cdot}429°$	$0{\cdot}001411 \mid 90{\cdot}000°$
2	$0{\cdot}86787 \angle 2{\cdot}446°$	$186{\cdot}93 \angle 75{\cdot}169°$	$0{\cdot}0013486 \angle 90{\cdot}740°$
3	$0{\cdot}87052 \angle 2{\cdot}330°$	$187{\cdot}03 \angle 75{\cdot}149°$	$0{\cdot}0013494 \angle 90{\cdot}720°$
4	$0{\cdot}87050 \angle 2{\cdot}332°$	$187{\cdot}03 \angle 75{\cdot}150°$	$0{\cdot}0013494 \angle 90{\cdot}721°$
Inf.	$0{\cdot}87050 \angle 2{\cdot}332°$	$187{\cdot}03 \angle 75{\cdot}150°$	$0{\cdot}0013494 \angle 90{\cdot}721°$

An inspection of the table shows that the use of three terms in the series expressions yields results which are sufficiently close to the exact values as obtained by the use of hyperbolic functions (infinite number of terms). In most cases occurring in practice, two terms will give a high degree of accuracy.

45. Solution of Problems by Convergent Series Method.—When the physical characteristics of the line and the frequency are stated, the form of solution and procedure will be on the following lines.

The total resistance, reactance, and susceptance for one of the

line conductors is first calculated. The conductance, which represents the energy loss in the form of leakage over insulators and of corona loss between conductors, is usually negligible under normal operating conditions, and it is common practice to ignore its effect in calculations.

The values of Z and Y can now be set down in the form of complex quantities, and by successive multiplication the second, third and fourth power of ZY can be found, according to the degree of accuracy required.

After the values of ZY, Z^2Y^2, Z^3Y^3, etc., have been calculated they are divided by 2, 24, 720, etc., respectively, set down and added to unity. This gives the value of the auxiliary constant A, which is a complex quantity and may be referred to as $A_1 + jA_2$. The absolute value of constant A is of course $\sqrt{A_1^2 + A_2^2}$, and its polar angle is $\tan^{-1}\frac{A_2}{A_1}$.

The solution for B is of the same form as that for A except that the values of ZY, Z^2Y^2, Z^3Y^3, etc., are divided by 6, 120, and 5 040, etc., respectively. After these results are added to unity they are multiplied by Z, the product being the value of the auxiliary constant B, or $B_1 + jB_2$. The absolute value of B is obtained in the same manner as that of A.

The solution for C is the same as that for B except that in place of the series terms being multiplied by Z, they are multiplied by Y, and the value of C, or $C_1 + jC_2$ obtained.

Supposing now that the voltage $\dot{E}_r$ and current $\dot{I}_r$ at the receiving end of the line are specified, the conditions at the sending end can be found directly by applying the above constants. It should be noted that, in general, both $\dot{E}_r$ and $\dot{I}_r$ are complex quantities, but since the receiving-end voltage is kept constant $\dot{E}_r$ is taken as a vector of reference, and $\dot{I}_r$ must be specified in relation thereto. Hence the factor $(\cos \phi_r \pm j \sin \phi_r)$ is introduced, the minus sign being used when the current is lagging and the plus sign when the current is leading.

The equations for the voltage and current at the sending end of the line may thus be expressed as

$$\dot{E}_s = \dot{E}_r(A_1 + jA_2) + \dot{I}_r(\cos \phi_r \pm j \sin \phi_r)(B_1 + jB_2), \quad (80)$$

$$\dot{I}_s = \dot{I}_r(\cos \phi_r \pm j \sin \phi_r)(A_1 + jA_2) + \dot{E}_r(C_1 + jC_2). \quad (81)$$

Similarly, if the voltage and current at the sending end are given, the corresponding quantities at the receiving end are

$$\dot{E}_r = \dot{E}_s(A_1 + jA_2) - \dot{I}_s(\cos \phi_s \pm j \sin \phi_s)(B_1 + jB_2), \quad (82)$$

$$\dot{I}_r = \dot{I}_s(\cos \phi_s \pm j \sin \phi_s)(A_1 + jA_2) - \dot{E}_s(C_1 + jC_2). \quad (83)$$

With zero load, the line being open-circuited at the receiving end, the term containing $\dot{I}_r$ disappears, and the conditions at the sending end are

$$\dot{E}_{so} = \dot{E}_{ro}(A_1 + jA_2), \quad . \quad . \quad . \quad (84)$$

$$\dot{I}_{so} = \dot{E}_{ro}(C_1 + jC_2). \quad . \quad . \quad . \quad (85)$$

At zero load sending end, the conditions are really not specified but what is usually meant is that the line is open-circuited at the receiving end. Of course the capacitance current will still have to be supplied to the line. If E_{so}, then, is the sending-end voltage when all the switches are open at the other end of the line, the other quantities can be obtained as follows :—

From equation (84) we have

$$\dot{E}_{ro} = \frac{\dot{E}_{so}}{A_1 + jA_2}, \quad . \quad . \quad . \quad . \quad (86)$$

and substituting for $\dot{E}_{ro}$ in (85) gives

$$\dot{I}_{so} = \dot{E}_{so}\frac{C_1 + jC_2}{A_1 + jA_2}. \quad . \quad . \quad . \quad (87)$$

46. Numerical Example.—In order to illustrate the convergent series method of solution the following problem will be taken :—

Problem 4.—Find the characteristics of the load at the sending end of a three-phase line 300 miles long, consisting of three 0·418 sq. in. steel-cored aluminium conductors, spaced in a 12-foot flat arrangement. Temperature taken as 20° C. Load conditions at receiving end assumed as 40 000 kVA. (36 000 kW. at 90 % power factor lagging), 154 000 volts, 50 cycles.

From wire tables the resistance per mile of conductor at 20° C. is found to be 0·175 ohms, and the overall diameter of the conductor is 0·931 inches.

The inductance and capacitance as calculated by the usual formulæ are respectively

$$L_0 = 0{\cdot}00200 \text{ henries per mile};$$

$$C_0' = 0{\cdot}01498 \times 10^{-6} \text{ farads per mile};$$

and the leakage losses are taken as negligible.

Hence the line constants (total value per conductor) are

$$R = 52{\cdot}5 \text{ ohms},$$
$$X = 188{\cdot}4 \text{ ohms},$$
$$G = 0,$$
$$B = 0{\cdot}001411 \text{ mhos}.$$

The method of procedure is then as indicated in the previous article :—

Multiplication of ZY.

$$\begin{array}{rrr} Y = & 0 & + 0{\cdot}001411j \\ Z = & 52{\cdot}5 & + 188{\cdot}4j \\ \hline & 0 & + 0{\cdot}074077j \\ & - 0{\cdot}265832 & + 0 \\ \hline ZY = & - 0{\cdot}265832 & + 0{\cdot}074077j \\ & - 0{\cdot}265832 & + 0{\cdot}074077j \\ \hline & + 0{\cdot}070667 & - 0{\cdot}019692j \\ & - 0{\cdot}005487 & - 0{\cdot}019692j \\ \hline Z^2Y^2 = & + 0{\cdot}065180 & - 0{\cdot}039384j \\ & - 0{\cdot}265832 & + 0{\cdot}074077j \\ \hline & - 0{\cdot}017327 & + 0{\cdot}010470j \\ & + 0{\cdot}002917 & + 0{\cdot}004828j \\ \hline Z^3Y^3 = & - 0{\cdot}014410 & + 0{\cdot}015298j \end{array}$$

Solution for A.

$$A = \left(1 + \frac{ZY}{2} + \frac{Z^2Y^2}{24} + \frac{Z^3Y^3}{720} + \ldots\right).$$

$$\begin{array}{rrr} & 1{\cdot}000000 & \\ \frac{ZY}{2} = & - 0{\cdot}132916 & + 0{\cdot}037038j \\ \frac{Z^2Y^2}{24} = & + 0{\cdot}002716 & - 0{\cdot}001641j \\ \frac{Z^3Y^3}{720} = & - 0{\cdot}000020 & + 0{\cdot}000021j \\ \hline A = & + 0{\cdot}869780 & + 0{\cdot}035418j \\ = & 0{\cdot}87050 & \angle 2{\cdot}332^\circ. \end{array}$$

Solution for B.

$$B = Z\left(1 + \frac{ZY}{6} + \frac{Z^2Y^2}{120} + \frac{Z^3Y^3}{5\,040} + \ldots\right).$$

$$1 \cdot 000000$$

$$\frac{ZY}{6} = -\ 0 \cdot 044305 + 0 \cdot 012349j$$

$$\frac{Z^2Y^2}{120} = +\ 0 \cdot 000543 - 0 \cdot 000328j$$

$$\frac{Z^3Y^3}{5\,040} = -\ 0 \cdot 000003 + 0 \cdot 000003j$$

$$+\ 0 \cdot 956235 + 0 \cdot 012024j$$

$$Z = \quad 52 \cdot 5 \qquad + 188 \cdot 4j$$

$$+\ 50 \cdot 2023 + \quad 0 \cdot 631j$$

$$-\ 2 \cdot 2653 + 180 \cdot 155j$$

$$B = +\ 47 \cdot 9370 + 180 \cdot 786j$$

$$= \quad 187 \cdot 03 \angle 75 \cdot 150°.$$

Solution for C.

$$C = Y\left(1 + \frac{ZY}{6} + \frac{Z^2Y^2}{120} + \frac{Z^3Y^3}{5\,040} + \ \ldots\right).$$

$$+ \quad 0 \cdot 956235 + 0 \cdot 012024j$$

$$Y = \quad 0 \qquad\qquad + 0 \cdot 001411j$$

$$C = -\ 0 \cdot 000017 + 0 \cdot 0013490j$$

$$= \quad 0 \cdot 0013494 \angle 90 \cdot 721°.$$

Solution for E_s.

$$E_s = E_r(A_1 + jA_2) + I_r(\cos \phi_r - j \sin \phi_r)(B_1 + jB_2). \quad (80)$$

$$A_1 + jA_2 = 0 \cdot 869780 + 0 \cdot 035418j$$

$$E_r = \quad 88\,912$$

$$E_r(A_1 + jA_2) = 77\,333 \cdot 8 + 3\,149 \cdot 08j$$

$$(\cos \phi_r - j \sin \phi_r) = \quad 0 \cdot 9 - \quad 0 \cdot 4359j$$

$$I_r = \quad 149 \cdot 96$$

$$I_r(\cos \phi_r - j \sin \phi_r) = \quad 134 \cdot 965 - \quad 65 \cdot 3680j$$

$$(B_1 + jB_2) = \quad 47 \cdot 9370 + 180 \cdot 786j$$

$$6\,469 \cdot 8 - \quad 3\,133 \cdot 5j$$

$$11\,817 \cdot 6 + 24\,399 \cdot 8j$$

$$I_r(\cos \phi_r - j \sin \phi_r)(B_1 + jB_2) = 18\,287 \cdot 4 + 21\,266 \cdot 3j$$

$$E_r(A_1 + jA_2) = 77\,333 \cdot 8 + \quad 3\,149 \cdot 08j$$

$$E_s = (E_s' + jE_s'') = 95\,621 \cdot 2 + 24\,415 \cdot 4j$$

$$E_s = 98\,689 \angle 14 \cdot 324° \text{ volts to neutral.}$$

Solution for I_s.

$$I_s = I_r(\cos\phi_r - j\sin\phi_r)(A_1 + jA_2) + E_r(C_1 + jC_2). \quad . \quad (81)$$

$$\begin{aligned}
I_r(\cos\phi_r - j\sin\phi_r) &= 134{\cdot}965 - 65{\cdot}3680j \\
(A_1 + jA_2) &= 0{\cdot}869780 + 0{\cdot}035418j \\
&\hline
&117{\cdot}390 - 56{\cdot}8558j \\
&2{\cdot}315 + 4{\cdot}7802j \\
&\hline
I_r(\cos\phi_r - j\sin\phi_r)(A_1 + jA_2) &= 119{\cdot}705 - 52{\cdot}0756j \\
(C_1 + jC_2) &= -\,0{\cdot}000017 + 0{\cdot}001349j \\
E_r &= 88\,912 \\
&\hline
&-\,1{\cdot}512 + 119{\cdot}942j \\
I_r(\cos\phi_r - j\sin\phi_r)(A_1 + jA_2) &= 119{\cdot}705 - 52{\cdot}076j \\
&\hline
I_s = (I_s' + jI_s'') &= 118{\cdot}193 + 67{\cdot}866j \\
I_s &= 136{\cdot}29 \angle 29{\cdot}864^\circ \text{ amperes.}
\end{aligned}$$

Having found the values of sending-end voltage and current, all the other electrical conditions at this end are readily determined as follows:—

$$\text{kVA}_s = \frac{98\,689 \times 136{\cdot}29}{1\,000}$$

$$= 13\,450 \quad \text{kVA. per phase.}$$

$$\text{kW}_s = \frac{E_s'I_s' + E_s''I_s''^{\,2}}{1\,000}$$

$$= \frac{(95\,621{\cdot}2 \times 118{\cdot}193) + (24\,415{\cdot}4 \times 67{\cdot}866)}{1\,000}$$

$$= 12\,959 \quad \text{kW. per phase.}$$

$$\text{Loss} = 12\,959 - 12\,000$$

$$= 959 \quad \text{kW. per phase.}$$

$$\text{Efficiency} = \frac{12\,000 \times 100}{12\,959}$$

$$= 92{\cdot}60\ \%$$

$$\text{Cos}\,\phi_s = \frac{12\,959 \times 100}{13\,450}$$

$$= 96{\cdot}35\ \% \quad \text{leading.}$$

$$\phi_s = 29{\cdot}864^\circ - 14{\cdot}324^\circ.$$

$$= 15{\cdot}54^\circ \quad \text{leading.}$$

Similarly if the line is open-circuited at the receiving end the electrical conditions at the sending end can be found as follows:—

$$E_{so} = E_{ro}(A_1 + jA_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad (84)$$
$$= 77\,333{\cdot}8 + 3\,149{\cdot}08j$$
$$= 77\,398 \angle 2{\cdot}332^\circ \text{ volts to neutral.}$$
$$I_{so} = E_{ro}(C_1 + jC_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad (85)$$
$$= -1{\cdot}512 + 119{\cdot}942j$$
$$= 119{\cdot}95 \angle 90{\cdot}722^\circ \text{ amperes.}$$
$$\text{kVA}_{so} = \frac{77\,398 \times 119{\cdot}95}{1\,000}$$
$$= 9\,284{\cdot}0 \text{ kVA. per phase.}$$
$$\text{kW}_{so} = \frac{(77\,333{\cdot}8 \times -1{\cdot}512) + (3\,149{\cdot}08 \times 119{\cdot}942)}{1\,000}$$
$$= 260{\cdot}8 \text{ kW. per phase.}$$
$$\cos \phi_{so} = \frac{260{\cdot}8 \times 100}{9\,284{\cdot}0}$$
$$= 2{\cdot}81\ \% \text{ leading.}$$
$$\phi_{so} = 90{\cdot}722^\circ - 2{\cdot}332^\circ$$
$$= 88{\cdot}390^\circ \text{ leading.}$$

All the above results are collected together in Table 4, and for purposes of comparison the numerical results obtained by using the two localised-capacitance methods of solution are also given.

47. Semi-graphical Solution for Various Receiving-end Conditions.—It is often preferable to adopt a semi-graphical method of solution, and this method is particularly advantageous in cases where the sending-end conditions are required for many different receiving-end currents and power factors. In order to show clearly the graphical treatment for various load conditions the numerical example already solved mathematically will be taken.

The first step in the work is to obtain the numerical values of the three auxiliary constants by calculation, and it will be found more convenient to have these expressed in polar rather than rectangular co-ordinates.

Referring to Table 3, these values are

$$A = 0{\cdot}87050 \angle 2{\cdot}332^\circ.$$
$$B = 187{\cdot}03 \angle 75{\cdot}150^\circ.$$
$$C = 0{\cdot}0013494 \angle 90{\cdot}721^\circ.$$

TABLE 4.—*Performance of 300-Mile Transmission Line Calculated by Various Methods.*

$R = 52{\cdot}5$ ohms. $X = 188{\cdot}4$ ohms. $G = 0$. $B = 0{\cdot}001411$ mhos.

Quantity.	Calculated Values Determined by		
	Rigorous Solution.	Nominal-T. Method.	Nominal-π Method.
Load Conditions.			
E_s	98 689 volts	97 836	99 754
I_s	136·29 amps.	140·39	134·16
kVA_s	13 450 kVA.	13 736	13 383
kW_s	12 959 kW.	13 108	12 956
Loss	959 kW.	1 108	956
Efficiency	92·60%	91·55%	92·62%
Cos ϕ_s	96·35%	95·43%	96·81%
ϕ_s	15·54°	17·39°	14·51°
Zero Load Conditions.			
E_{s0}	77 398 volts	77 164	77 164
I_{s0}	119·95 amps.	125·45	117·14
kVA_{s0}	9 284 kVA.	9 681	9 039
kW_{s0} = Loss on open-circuit	260·8 kW.	413	207
Cos ϕ_{s0} =	2·81%	4·27%	2·28%
ϕ_{s0}	88·39°	87·55°	88·69°

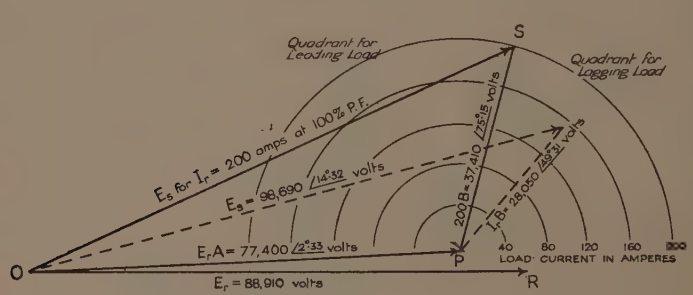


FIG. 29.—Voltage solution for various loads.

Hence

$$\underline{E}_s = \underline{E}_r A + \underline{I}_r B \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (76)$$
$$= \underline{E}_r(0{\cdot}87050 \angle 2{\cdot}332^\circ) + \underline{I}_r(187{\cdot}03 \angle 75{\cdot}150^\circ),$$

and

$$\underline{I}_s = \underline{I}_r A + \underline{E}_r C \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (77)$$
$$= \underline{I}_r(0{\cdot}87050 \angle 2{\cdot}332^\circ) + \underline{E}_r(0{\cdot}0013494 \angle 90{\cdot}721^\circ).$$

Fig. 29 is the solution for the sending-end voltage. Since the receiving-end voltage is supposed to be kept constant at 88 910

volts, the diagram is constructed on OR, which is drawn horizontally to represent $E_r = 88\ 910$ volts. The resultant at the sending end is made up of two parts, one of which is proportional to the receiving-end voltage, and the other to the receiving-end current. The vector OP representing $E_r A = 77\ 400 \angle 2{\cdot}33°$ is drawn at an angle of 2·33° from OR, and with a length to the same scale as OR, representing 77 400 volts. This vector, which is one of the two which go to make up the resultant vector for E_s, remains fixed regardless of the current in the line.

To complete the diagram, and make it applicable to any and all values of receiving-end current at any power factor, select a base value of current of, say, 200 amperes at unity power factor.

Then

$$\dot{I}_r' = 200 \angle 0°,$$

and

$$\begin{aligned} \dot{I}_r' B &= (200 \angle 0°)(187{\cdot}03 \angle 75{\cdot}15°) \\ &= 37\ 410 \angle 75{\cdot}15° \text{ volts.} \end{aligned}$$

The resultant sending-end voltage is then obtained by drawing PS at an angle of 75·15° with the vector of reference OR, and to a scale length of 37 410 volts.

This gives the solution for only one value of the receiving-end current, but it is readily seen that for any other value of current delivered at unity power factor the solution is obtained by taking the proportional part of the vector PS. For example, the line PS may be divided into five equal parts, giving thereby the values of sending-end voltage for $I_r = 0$, 40, 80, 120, 160, and 200 amperes. By continuing this subdivision on PS produced, solutions are found for receiving-end currents greater than 200 amperes.

The length of the vector PS depends only on the absolute value of I_r, while the angle it makes with the reference vector OR depends only upon the power factor of the load, being in fact equal to the algebraic sum of the receiving-end phase-angle and the angle of the auxiliary constant B. Thus for any given receiving-end power factor it is only necessary to turn the vector PS as drawn in the diagram for unity power factor through an angle equal to the phase-angle at the receiving end. By drawing arcs of circles through the points subdividing the line PS, solutions are provided for the intermediate values of current at any power factor.

The dotted lines in the figure indicate the voltage solution for

the load conditions of Problem 4, *i.e.* a total load of 40 000 kVA. delivered at 90 % power factor lagging, the current in this case being 150·0 amperes.

Fig. 30 is the corresponding solution for the sending-end current with $OR(E_r)$ again used as vector of reference. The fixed component of the sending-end current is

$$\dot{E}_r C = 88\,910(0{\cdot}0013494 \angle 90{\cdot}721°)$$
$$= 120{\cdot}0 \quad \text{amperes,}$$

and is represented in length and direction by the vector OM.

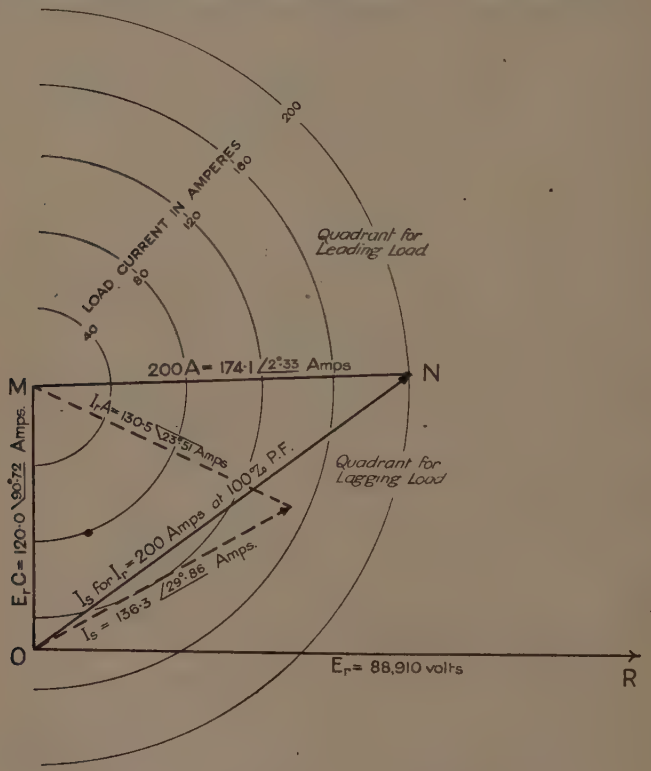


Fig. 30.—Current solution for various loads.

The other component I_rA has a variable value depending on the receiving-end current and power factor. Selecting then a base value of current of 200 amperes at unity power factor gives

$$\dot{I}_r A = (200 \angle 0°)(0{\cdot}87050 \angle 2{\cdot}332°)$$
$$= 174{\cdot}1 \quad \text{amperes.}$$

The vector *MN* representing this quantity is drawn to scale in the diagram making an angle of 2·33° with *OR*, and the sending-end current is found as *ON*.

In an analogous method to that used for the voltage diagram the sending-end current for any value of receiving-end current at unity power factor is obtained by taking the proportionate part of the vector *MN*. Circular arcs are drawn through the points of subdivision of *MN* and for other power factors than unity the vector *MN* is rotated through an angle equal to the receiving-end phase-angle.

The dotted lines in the figure indicate the current solution for Problem 4.

By drawing both the voltage and current diagrams on the same sheet of paper the sending-end phase-angle may be found by direct measurement, and thus the power input to the line, and efficiency of transmission may be readily determined.

48. Hyperbolic Method of Solution.—The fundamental equations of voltage and current for a long transmission line are generally expressed by means of hyperbolic functions, and providing that suitable tables of these functions are available their use leads to the most direct method of solution.

Such tables were, until recently, incomplete, but numerous tables and charts of complex hyperbolic functions have now been calculated and published by Kennelly.[3] Even now, with these latest tables, a considerable amount of interpolation is required, and thus the same refinement of calculation cannot be obtained as with the convergent series method using three or four terms. Nevertheless the hyperbolic method yields results with an accuracy which is amply sufficient for all calculations on ordinary power lines. The charts are particularly useful for rapid work as they enable the values of the functions to be read off directly and with an accuracy of at least three significant figures.

A discussion of the theory of real and complex hyperbolic functions is outside the scope of the present work, and for a detailed exposition of their derivation and employment in transmission line calculations reference should be made to the works of Dr. Kennelly. Fortunately, although a knowledge of hyperbolic trigonometry is desirable, it is by no means essential, and it is hoped that the information given in the following pages will enable the reader to apply the hyperbolic method successfully to any particular problem.

49. Real and Complex Hyperbolic Angles.—Real hyperbolic angles are derived from an equilateral hyperbola in a similar manner to the way circular angles are derived from a circle. Circular angles are generally expressed in radians, and hyperbolic angles in hyperbolic angles or hyps.

The hyperbolic functions sinh x, cosh x, tanh x, etc., also are analogous to the circular functions sin x, cos x, tan x, etc., and many of the trigonometrical formulæ used in connection with circular angles can be applied with very slight modification to the hyperbolic case. One important difference, however, is that circular functions have a period of 2π radians so that

$$\cos x = \cos (x + 2\pi n),$$

where n is an integer, while hyperbolic functions, having no true period, continually increase.

Mathematically the following relations exist and may be employed for purposes of calculation:—

$$\sinh x = \frac{\epsilon^x - \epsilon^{-x}}{2}$$

$$= x + \frac{x^3}{\underline{|3}} + \frac{x^5}{\underline{|5}} + \frac{x^7}{\underline{|7}} + \ldots$$

$$\cosh x = \frac{\epsilon^x + \epsilon^{-x}}{2}$$

$$= 1 + \frac{x^2}{\underline{|2}} + \frac{x^4}{\underline{|4}} + \frac{x^6}{\underline{|6}} + \ldots$$

$$\tanh x = \frac{\sinh x}{\cosh x}.$$

In alternating-current work, however, the quantities used for purposes of calculation are complex, and hence hyperbolic functions of *complex* variables are required.

A complex hyperbolic function is of the three-dimensional order and has two components, a real component relating to a hyperbolic angle, and an imaginary component relating to a circular angle. The angle may be expressed either in polar co-ordinates as $v\underline{/\theta_v}$, or in rectangular co-ordinates as $\alpha + j\beta$, where α is the real part of the angle expressed in hyps, and β is the imaginary part expressed in circular radians.

The published tables of complex hyperbolic functions, previously referred to, give functions of complex angles for polar values up to 3·0 by steps of 0·1 and for angles from 45° to 90° by steps of one

degree; also functions in terms of rectangular co-ordinates $\alpha + j\beta$ to $\alpha = 10$ by steps of 0·05 and of β virtually to infinity by steps of 0·05. Functions of angles lying between the values for angles given in the tables are obtained by simple proportion, the values thus obtained being sufficiently accurate for all practical calculations.

The charts cover exactly the same ground as the tables but are quicker in use as they reduce the work of interpolation to a minimum.

The following formulæ giving the relationship between the functions of the complex angle and those of its component parts may be employed if exact values are required in any particular case:—

$$\sinh(\alpha + j\beta) = (\sinh \alpha \cos \beta + j \cosh \alpha \sin \beta).$$
$$\cosh(\alpha + j\beta) = (\cosh \alpha \cos \beta + j \sinh \alpha \sin \beta).$$

50. Line Angle.—In the hyperbolic theory the part of the complete circuit formed by the line is said to subtend a certain complex angle $\sqrt{ZY} = \theta$. This quantity represents in a sense the electrical length of the line, and is an indication of how much distortion in the distribution of voltage and current is to be expected owing to the line impedance and admittance being distributed, instead of being localised at one point.

In the case of long-distance overhead transmission systems the conductors usually have a fairly high ratio of reactance to resistance, and the conductance representing the power loss by leakage is negligible at normal operation, hence the line angle is approximately proportional to $\sqrt{XB}$. In other words, the line angle is proportional to the physical length of the line and approximately proportional to the frequency of the supply.

As an indication of the values of line angle to be expected in practice, it may be stated that for 50-cycle overhead systems the modulus, or size of the angle, will vary from about 0·17 for a 100-mile line to about 0·50 for a 300-mile line. The argument, or slope of the angle, will have a value of about 80 degrees.

51. Determination of Auxiliary Constants.—As shown in Art. 43 the three auxiliary constants of the line may be expressed in terms of the complex angle θ as follows:—

$$A = \cosh \theta.$$
$$B = \sqrt{\frac{Z}{Y}} \sinh \theta.$$
$$C = \sqrt{\frac{Y}{Z}} \sinh \theta.$$

To determine their value by the hyperbolic method, the line angle θ is first calculated from the known values for the impedance and admittance of the line. The complex hyperbolic functions $\cosh\theta$ and $\sinh\theta$ are then obtained from the tables or charts, and substituted in the above formulæ, together with the calculated value of $\sqrt{\frac{Z}{Y}}$ or its reciprocal.

The operations of multiplication, division, and evolution of complex quantities are most readily carried out when the terms are expressed in polar co-ordinates, and hence in the hyperbolic method it is preferable to work throughout with polar values.

Thus to obtain the auxiliary constants for Problem 4:—

$$\begin{aligned} Z = R + jX &= 52{\cdot}5 + 188{\cdot}4j \\ &= 195{\cdot}58 \angle 74{\cdot}429^\circ. \\ Y = G + jB &= 0 + 0{\cdot}001411j \\ &= 0{\cdot}001411 \angle 90^\circ. \\ \theta = \sqrt{ZY} &= \sqrt{195{\cdot}58 \angle 74{\cdot}429^\circ \times 0{\cdot}001411 \angle 90^\circ} \\ &= \sqrt{0{\cdot}27596 \angle 164{\cdot}429^\circ} \\ &= 0{\cdot}52532 \angle 82{\cdot}214^\circ. \\ \sqrt{\frac{Z}{Y}} &= \sqrt{\frac{195{\cdot}58 \angle 74{\cdot}429^\circ}{0{\cdot}001411 \angle 90^\circ}} \\ &= 372{\cdot}30 \angle 7{\cdot}786^\circ, \\ \sqrt{\frac{Y}{Z}} &= \frac{1}{372{\cdot}30 \angle 7{\cdot}786^\circ} \\ &= 0{\cdot}0026860 \angle 7{\cdot}786^\circ. \end{aligned}$$

By interpolation from the tables is found

$$\begin{aligned} \cosh\theta &= \cosh 0{\cdot}52532 \angle 82{\cdot}214^\circ \\ &= 0{\cdot}8698 \angle 2{\cdot}36^\circ, \\ \sinh\theta &= \sinh 0{\cdot}52532 \angle 82{\cdot}214^\circ \\ &= 0{\cdot}5019 \angle 82{\cdot}94^\circ. \end{aligned}$$

Hence

$$\begin{aligned} A &= \cosh\theta \\ &= 0{\cdot}8698 \angle 2{\cdot}36^\circ, \\ B &= \sqrt{\frac{Z}{Y}} \sinh\theta \\ &= 372{\cdot}30 \angle 7{\cdot}786^\circ \times 0{\cdot}5019 \angle 82{\cdot}94^\circ \\ &= 186{\cdot}9 \angle 75{\cdot}15^\circ, \end{aligned}$$

$$C = \sqrt{\frac{Y}{Z}} \sinh \theta,$$
$$= 0{\cdot}0026860 \angle 7{\cdot}786^\circ \times 0{\cdot}5019 \angle 82{\cdot}94^\circ$$
$$= 0{\cdot}001348 \angle 90{\cdot}73^\circ.$$

Having found the auxiliary constants the remainder of the solution proceeds along the lines of the calculation given in Art. 46, or the graphical method of treatment may be employed.

52. Comparison of Methods.—Now that the chief methods of transmission line solution have been described it is interesting to evaluate the three auxiliary constants corresponding to the approximate localised-capacitance or localised-admittance methods, and see how they compare with their true value. Loss of power by leakage was not mentioned in connection with the approximate methods, but it can be allowed for if necessary by supposing a high resistance shunting each condenser.

In the nominal-T method

$$\dot{E}_s = \dot{E}_r\left\{1 + (R + jX)\left(G_1 - jB_1 + j\frac{B}{2}\right) + j\frac{B}{4}(R + jX)^2(G_1 - jB_1)\right\}, \quad (47)$$

$$\dot{I}_s = \dot{E}_r\left[G_1 - jB_1 + jB\left\{1 + \frac{(R + jX)(G_1 - jB_1)}{2}\right\}\right]. \quad (46)$$

Substituting $\dot{I}_r$ for $\dot{E}_r(G_1 - jB_1)$, Z for $R + jX$ and Y for jB gives

$$\dot{E}_s = \dot{E}_r + Z\left(\dot{I}_r + \frac{\dot{E}_r Y}{2}\right) + \frac{Y}{4}Z^2\dot{I}_r$$
$$= \dot{E}_r\left(1 + \frac{ZY}{2}\right) + \dot{I}_r Z\left(1 + \frac{ZY}{4}\right). \quad (88)$$

Also

$$\dot{I}_s = \dot{I}_r + \dot{E}_r Y + \frac{ZY\dot{I}_r}{2}$$
$$= \dot{I}_r\left(1 + \frac{ZY}{2}\right) + \dot{E}_r Y. \quad (89)$$

Hence the auxiliary constants corresponding to the nominal-T method are

$$A = 1 + \frac{ZY}{2}.$$
$$B = Z\left(1 + \frac{ZY}{4}\right).$$
$$C = Y.$$

In the nominal-π method

$$\dot{E}_s = \dot{E}_r\left\{1 + \left(G_1 - jB_1 + j\frac{B}{2}\right)(R + jX)\right\}, \quad . \quad (48)$$

$$\dot{I}_s = \dot{E}_r\left\{G_1 - jB_1 + jB + \frac{jB}{2}\left(G_1 - jB_1 + \frac{jB}{2}\right)(R + jX)\right\}. \quad (49)$$

Substituting $\dot{I}$ for $\dot{E}_r(G_1 - jB_1)$ and making the other substitutions as before gives

$$\dot{E}_s = \dot{E}_r + \left(\dot{I} + \frac{\dot{E}_r Y}{2}\right)Z$$

$$= \dot{E}_r\left(1 + \frac{ZY}{2}\right) + \dot{I}Z, \quad . \quad . \quad . \quad . \quad (90)$$

and

$$\dot{I}_s = \dot{I} + \dot{E}_r Y + \frac{Y}{2}\left(\dot{I} + \frac{\dot{E}_r Y}{2}\right)Z$$

$$= \dot{I}\left(1 + \frac{ZY}{2}\right) + \dot{E}_r Y\left(1 + \frac{ZY}{4}\right). \quad . \quad . \quad (91)$$

Hence the auxiliary constants corresponding to the nominal-π method are

$$A = 1 + \frac{ZY}{2}$$

$$B = Z$$

$$C = Y\left(1 + \frac{ZY}{4}\right).$$

The numerical values of these constants for the 300-mile line of Problem 4 have been calculated, and are given in Table 5 for purposes of comparison with the values obtained by the more rigorous methods of solution.

TABLE 5.—*Comparison of Auxiliary Constants for 300-Mile Transmission Line, as Determined by Various Methods.*

$R = 52{\cdot}5$ ohms. $X = 188{\cdot}4$ ohms. $G = 0$. $B = 0{\cdot}001411$ mhos.

Method of Calculation.	A.	B.	C.
Convergent series method (Exact Values) . .	$0{\cdot}87050 \angle 2{\cdot}332°$	$187{\cdot}03 \angle 75{\cdot}150°$	$0{\cdot}0013494 \angle 90{\cdot}721°$
Hyperbolic method (values obtained from Tables for cosh θ and sinh θ) . . .	$0{\cdot}8698 \angle 2{\cdot}36°$	$186{\cdot}9 \angle 75{\cdot}15°$	$0{\cdot}001348 \angle 90{\cdot}73°$
Nominal-T method . .	$0{\cdot}8679 \angle 2{\cdot}45°$	$182{\cdot}6 \angle 75{\cdot}57°$	$0{\cdot}001411 \mid 90{\cdot}00°$
Nominal-π method . .	$0{\cdot}8679 \angle 2{\cdot}45°$	$195{\cdot}6 \angle 74{\cdot}43°$	$0{\cdot}001317 \angle 91{\cdot}14°$

53. Electrical Conditions at Intermediate Points.—So far, only the electrical conditions at the termini of the line have been considered. In many cases, however, and particularly with long lines, it is necessary to determine the voltage and current at points along the line between the termini. One way of doing this would be by the step-by-step method.

In this method (supposing the receiving-end conditions to be specified) the values of voltage and current are calculated as though the sending end were successively at, say, 30, 60, 90, . . . 300 miles from the receiving end. The results obtained are then plotted against distance from one of the line termini and a smooth curve drawn through the various points. It is readily seen that this method involves a large number of tedious calculations and a much shorter solution is provided by the employment of hyperbolic functions.

In using hyperbolic functions for this purpose advantage is taken of the fact that the potentials of any and all points on a smooth transmission line vary as the sines, and the currents as the cosines, of the corresponding position angles. This relationship is expressed in equation form as

$$\frac{E_\rho}{E_c} = \frac{\sinh \delta_\rho}{\sinh \delta_c}, \quad . \quad . \quad . \quad . \quad (92)$$

$$\frac{I_\rho}{I_c} = \frac{\cosh \delta_\rho}{\cosh \delta_c}, \quad . \quad . \quad . \quad . \quad (93)$$

where the subscripts c and ρ relate to points along the line, c being some point where the electrical conditions are known, and ρ the point for which they are to be computed.[4]

54. Position Angles.—Reference has been made to the line as subtending a certain complex hyperbolic angle θ. The remaining part of the circuit formed by the impedance which represents the load may be said to subtend another complex hyperbolic angle θ_r, so that the receiving end of the line occupies an angular position $\delta_r = \theta_r$. The total angle of the complete circuit (line and load) is thus $\delta_s = \theta_r + \theta$, as shown in Fig. 31.

By similar reasoning, all points lying between the receiving and sending ends of the line will occupy, or assume, an angular position intermediate between δ_r and δ_s. Thus at the point P distant $\frac{1}{n}$th of the total line length from the receiving end, the position angle is $\delta_\rho = \delta_r + \frac{\theta}{n}$.

The following cases will be considered :—

1. *Line Earthed.*—If the line is short-circuited or earthed at the receiving end, the load angle vanishes, and the distribution of position angles along the line is purely a linear function of the total line angle. In such a case $\delta_s = \theta$.

2. *Line Supplying Load.*—Before the position angle of any intermediate point on the line can be found, it is first necessary to determine the angle subtended by the terminal load. This is obtained from the following relationship :—

$$\theta_r = \tanh^{-1}\frac{\sigma}{Z_0}, \quad . \quad . \quad . \quad . \quad (94)$$

where σ is the impedance load to earth, or neutral, at the receiving

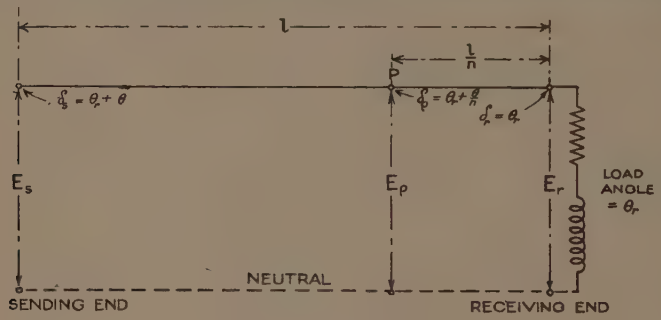


Fig. 31.—Position angles.

end of the line, and Z_0 is the surge impedance of the line to neutral. The two latter quantities are both complex and are given by

$$\sigma = \frac{E_r}{I_r}, \quad . \quad . \quad . \quad . \quad . \quad (95)$$

and

$$Z_0 = \sqrt{\frac{Z}{Y}}. \quad . \quad . \quad . \quad . \quad (96)$$

After $\frac{\sigma}{Z_0}$ has been calculated, the corresponding angle may be obtained with sufficient accuracy from the chart of complex hyperbolic tangents.

The change in the position angle from point to point along the line due to the line impedance and admittance is purely a linear function of the line angle θ. This is the case whether the receiving end is earthed, loaded, or open-circuited. Thus to obtain the

position angle of an intermediate point, it is necessary to add to the load angle a part of the line angle proportional to the distance of the point from the receiving end. The point P, distant $\frac{1}{n}$th of the line length from the receiving end, has thus a position angle of $\delta_p = \delta_r + \frac{\theta}{n}$.

3. *Line Open-circuited.*—In this case there is no load, and consequently no real part to the load angle. The impedance of the load may be regarded as infinite, that is $\sigma = \infty$, and substituting in formula (94),

$$\theta_{ro} = \tanh^{-1} \frac{\infty}{Z_o}.$$

Thus, the effect of the infinite impedance existing at the receiving end of an open-circuited line is to cause a phase rotation of 90 degrees, or $j\frac{\pi}{2} = 1{\cdot}5708$ circular radians.

Consequently at zero load $\delta_{ro} = (0 + 1{\cdot}5708j)$, and to find the position angle of a point P on the line δ_{ro} must be added to the part of the line angle proportional to the distance of the point from the receiving end. In equation form $\delta_{po} = \delta_{ro} + \frac{\theta}{n}$.

The method of procedure will be seen clearly from a numerical example. Referring to Problem 4,

$$\sigma = \frac{\dot{E}_r}{\dot{I}_r} = \frac{88\,912}{149{\cdot}96 \angle 25{\cdot}84^\circ}$$
$$= 592{\cdot}90 \angle 25{\cdot}84^\circ,$$

and from Art. 51,

$$Z_o = \sqrt{\frac{\overline{Z}}{\overline{Y}}} = 372{\cdot}30^\circ \angle 7{\cdot}79^\circ.$$

Hence

$$\tan \delta_r = \frac{\sigma}{Z_o} = \frac{592{\cdot}90 \angle 25{\cdot}84^\circ}{372{\cdot}30 \angle 7{\cdot}79^\circ}$$
$$= 1{\cdot}5925 \angle 33{\cdot}63^\circ,$$

and by interpolation from the table of complex hyperbolic tangents the angle subtended by the load is found as

$$\delta_r = 0{\cdot}4855 + 1{\cdot}144j.$$

The angle subtended by the line

$$\theta = \sqrt{ZY} = 0{\cdot}0712 + 0{\cdot}520j.$$

Hence the position angle of the point P, distant, say, 60 miles from the receiving end of the line is

$$\delta_\rho = \delta_r + \frac{\theta}{5}$$

$$= 0{\cdot}4855 + 1{\cdot}144j + \frac{1}{5}(0{\cdot}0712 + 0{\cdot}520j)$$

$$= 0{\cdot}4997 + 1{\cdot}248j,$$

and at zero load the position angle of the same point is

$$\delta_{\rho o} = \delta_{ro} + \frac{\theta}{5}$$

$$= 0 + 1{\cdot}571j + \frac{1}{5}(0{\cdot}0712 + 0{\cdot}520j)$$

$$= 0{\cdot}0142 + 1{\cdot}675j.$$

55. Voltage and Current Distribution along Line.—Having found the position angles and corresponding values of the sines and cosines for various points along the line, and knowing the absolute electrical conditions at one of the points, the voltage and current distribution is easily determined from the relationship given in equations (92) and (93).

For example, under the load conditions of Problem 4

$$E_r = 88\,910 \quad \text{volts to neutral,}$$
$$I_r = 150{\cdot}0 \angle 25{\cdot}84^\circ \quad \text{amperes,}$$

and the functions of the load angle are

$$\sinh \delta_r = \sinh(0{\cdot}4855 + 1{\cdot}144j)$$
$$= 1{\cdot}0405 \angle 78{\cdot}39^\circ,$$
$$\cosh \delta_r = \cosh(0{\cdot}4855 + 1{\cdot}144j)$$
$$= 0{\cdot}6538 \angle 44{\cdot}77^\circ.$$

At the point P, distant 60 miles from the receiving end,

$$\sinh \delta_\rho = \sinh(0{\cdot}4997 + 1{\cdot}248j)$$
$$= 1{\cdot}081 \angle 81{\cdot}19^\circ,$$
$$\cosh \delta_\rho = \cosh(0{\cdot}4997 + 1{\cdot}248j)$$
$$= 0{\cdot}6104 \angle 54{\cdot}06^\circ.$$

Hence the voltage and current existing at this point are respectively

$$\dot{E}_\rho = \frac{\dot{E}_r \sinh \delta_\rho}{\sinh \delta_r}$$
$$= \frac{88\,910(1{\cdot}081 \angle 81{\cdot}19°)}{1{\cdot}0405 \angle 78{\cdot}39°}$$
$$= 92\,370 \angle 2{\cdot}80° \text{ volts to neutral,}$$

and

$$\dot{I}_\rho = \frac{\dot{I}_r \cosh \delta_\rho}{\cosh \delta_r}$$
$$= \frac{(150{\cdot}0 \angle 25{\cdot}84°)(0{\cdot}6104 \angle 54{\cdot}06°)}{0{\cdot}6538 \angle 44{\cdot}77°}$$
$$= 140{\cdot}0 \angle 16{\cdot}55° \text{ amperes.}$$

The power factor is the cosine of the angular difference between these two vectors or

$$\cos \phi_\rho = \cos (2{\cdot}80° + 16{\cdot}55°)$$
$$= \cos 19{\cdot}35°$$
$$= 94{\cdot}35\ \%\ \text{lagging.}$$

The intermediate electrical conditions at zero load are obtained, of course, in an exactly analogous manner.

The voltage and current distribution for Problem 4 has been determined, using functions of position angles as described above, and values of voltage, current and power factor for points 30 miles apart are given in Tables 6 and 7. The graphs shown in Fig. 32 have been plotted from these results.

It will be noted that when the specified load is being delivered, the dotted graph representing the current at intermediate points along the line has a minimum value about 100 miles from the sending end. This is due to the fact of the charging current of the system being greater than the lagging quadrature component of the load current. Starting at the receiving end the power factor is 90 % lagging. As the centre of the line is approached, the increasing charging current neutralises an increasing portion of the lagging component of the load current. At a point about 100 miles from the sending end of the line this lagging component is entirely neutralised, and the actual line current has a minimum value, the power factor being unity. Passing the centre, and approaching the sending end, the increasing charging current now causes a decreasing leading power factor which, when the sending end is reached, becomes 96·35 %.

TABLE 6.—*Voltage and Current Distribution (Load Conditions) on 300-Mile Line.*

$R = 52{\cdot}5$ ohms. $X = 188{\cdot}4$ ohms. $G = 0$. $B = 0{\cdot}001411$ mhos.

Distance from Sending End.	Position Angle, δ_ρ.	Sinh δ_ρ.	E_ρ.	Cosh δ_ρ.	I_ρ.	Power Factor Cos ϕ_ρ.
300 miles	$0{\cdot}4855 + 1{\cdot}144j$	$1{\cdot}0405 \angle 78{\cdot}39°$	$88\,910 \angle 0{\cdot}00°$	$0{\cdot}6538 \angle 44{\cdot}77°$	$150{\cdot}0 \angle 25{\cdot}84°$	90·00 % lagging
270 ,,	$0{\cdot}4926 + 1{\cdot}196j$	$1{\cdot}062 \angle 79{\cdot}80°$	$90\,750 \angle 1{\cdot}41°$	$0{\cdot}6309 \angle 49{\cdot}23°$	$144{\cdot}7 \angle 21{\cdot}38°$	92·19 ,,
240 ,,	$0{\cdot}4997 + 1{\cdot}248j$	$1{\cdot}081 \angle 81{\cdot}19°$	$92\,370 \angle 2{\cdot}80°$	$0{\cdot}6104 \angle 54{\cdot}06°$	$140{\cdot}0 \angle 16{\cdot}55°$	94·35 ,,
210 ,,	$0{\cdot}5069 + 1{\cdot}300j$	$1{\cdot}098 \angle 82{\cdot}57°$	$93\,820 \angle 4{\cdot}18°$	$0{\cdot}5941 \angle 59{\cdot}29°$	$136{\cdot}3 \angle 11{\cdot}32°$	96·36 ,,
180 ,,	$0{\cdot}5140 + 1{\cdot}352j$	$1{\cdot}114 \angle 83{\cdot}96°$	$95\,190 \angle 5{\cdot}57°$	$0{\cdot}5804 \angle 64{\cdot}74°$	$133{\cdot}1 \angle 5{\cdot}87°$	98·02 ,,
150 ,,	$0{\cdot}5211 + 1{\cdot}404j$	$1{\cdot}127 \angle 85{\cdot}40°$	$96\,300 \angle 7{\cdot}01°$	$0{\cdot}5703 \angle 70{\cdot}65°$	$130{\cdot}8 \angle 0{\cdot}04°$	99·26 ,,
120 ,,	$0{\cdot}5282 + 1{\cdot}456j$	$1{\cdot}137 \angle 86{\cdot}81°$	$97\,160 \angle 8{\cdot}42°$	$0{\cdot}5662 \angle 76{\cdot}65°$	$129{\cdot}9 \angle 6{\cdot}04°$	99·91 ,,
90 ,,	$0{\cdot}5353 + 1{\cdot}508j$	$1{\cdot}145 \angle 88{\cdot}24°$	$97\,840 \angle 9{\cdot}85°$	$0{\cdot}5657 \angle 82{\cdot}68°$	$129{\cdot}8 \angle 12{\cdot}07°$	99·93 leading
60 ,,	$0{\cdot}5425 + 1{\cdot}560j$	$1{\cdot}151 \angle 89{\cdot}69°$	$98\,350 \angle 11{\cdot}30°$	$0{\cdot}5703 \angle 88{\cdot}73°$	$130{\cdot}8 \angle 18{\cdot}12°$	99·29 ,,
30 ,,	$0{\cdot}5496 + 1{\cdot}612j$	$1{\cdot}153 \angle 91{\cdot}17°$	$98\,520 \angle 12{\cdot}78°$	$0{\cdot}5804 \angle 94{\cdot}65°$	$133{\cdot}1 \angle 24{\cdot}04°$	98·08 ,,
0 ,,	$0{\cdot}5567 + 1{\cdot}664j$	$1{\cdot}155 \angle 92{\cdot}74°$	$98\,690 \angle 14{\cdot}35°$	$0{\cdot}5943 \angle 100{\cdot}57°$	$136{\cdot}3 \angle 29{\cdot}96°$	96·31 ,,

TABLE 7.—*Voltage and Current Distribution (Zero-load Conditions) on 300-Mile Line.*

$R = 52{\cdot}5$ ohms. $X = 188{\cdot}4$ ohms. $G = 0$. $B = 0{\cdot}001411$ mhos.

Distance from Sending End.	Position Angle $\delta_{\rho o}$.	Sinh $\delta_{\rho o}$.	$E_{\rho o}$.	Cosh $\delta_{\rho o}$.	$I_{\rho o}$.	Power Factor Cos $\phi_{\rho o}$.
300 miles	$0{\cdot}0000 + 1{\cdot}571j$	$1{\cdot}000 \mid 90{\cdot}00°$	$88\,910 \angle 0{\cdot}00°$	$0{\cdot}0000 \mid 90{\cdot}00°$	$0{\cdot}0 \angle 7{\cdot}79°$	—
270 ,,	$0{\cdot}0071 + 1{\cdot}623j$	$0{\cdot}998 \angle 90{\cdot}02°$	$88\,730 \angle 0{\cdot}02°$	$0{\cdot}0525 \angle 172{\cdot}21°$	$12{\cdot}6 \mid 90{\cdot}00°$	$0{\cdot}03$ % leading
240 ,,	$0{\cdot}0142 + 1{\cdot}675j$	$0{\cdot}996 \angle 90{\cdot}08°$	$88\,550 \angle 0{\cdot}08°$	$0{\cdot}1048 \angle 172{\cdot}23°$	$25{\cdot}1 \angle 90{\cdot}02°$	$0{\cdot}10$,,
210 ,,	$0{\cdot}0214 + 1{\cdot}727j$	$0{\cdot}988 \angle 90{\cdot}19°$	$87\,840 \angle 0{\cdot}19°$	$0{\cdot}1568 \angle 172{\cdot}27°$	$37{\cdot}5 \angle 90{\cdot}06°$	$0{\cdot}23$,,
180 ,,	$0{\cdot}0285 + 1{\cdot}779j$	$0{\cdot}979 \angle 90{\cdot}34°$	$87\,040 \angle 0{\cdot}34°$	$0{\cdot}2084 \angle 172{\cdot}32°$	$49{\cdot}8 \angle 90{\cdot}11°$	$0{\cdot}40$,,
150 ,,	$0{\cdot}0356 + 1{\cdot}831j$	$0{\cdot}967 \angle 90{\cdot}54°$	$85\,930 \angle 0{\cdot}54°$	$0{\cdot}2595 \angle 172{\cdot}38°$	$62{\cdot}0 \angle 90{\cdot}17°$	$0{\cdot}65$,,
120 ,,	$0{\cdot}0427 + 1{\cdot}883j$	$0{\cdot}953 \angle 90{\cdot}79°$	$84\,730 \angle 0{\cdot}79°$	$0{\cdot}3099 \angle 172{\cdot}46°$	$74{\cdot}1 \angle 90{\cdot}25°$	$0{\cdot}94$,,
90 ,,	$0{\cdot}0498 + 1{\cdot}935j$	$0{\cdot}935 \angle 91{\cdot}09°$	$83\,130 \angle 1{\cdot}09°$	$0{\cdot}3595 \angle 172{\cdot}55°$	$86{\cdot}0 \angle 90{\cdot}34°$	$1{\cdot}31$,,
60 ,,	$0{\cdot}0570 + 1{\cdot}987j$	$0{\cdot}916 \angle 91{\cdot}44°$	$81\,440 \angle 1{\cdot}44°$	$0{\cdot}4081 \angle 172{\cdot}66°$	$97{\cdot}6 \angle 90{\cdot}45°$	$1{\cdot}73$,,
30 ,,	$0{\cdot}0641 + 2{\cdot}039j$	$0{\cdot}895 \angle 91{\cdot}85°$	$79\,570 \angle 1{\cdot}85°$	$0{\cdot}4556 \angle 172{\cdot}78°$	$108{\cdot}9 \angle 90{\cdot}57°$	$2{\cdot}23$,,
0 ,,	$0{\cdot}0712 + 2{\cdot}031j$	$0{\cdot}8705 \angle 92{\cdot}33°$	$77\,400 \angle 2{\cdot}33°$	$0{\cdot}5019 \angle 172{\cdot}92°$	$120{\cdot}0 \angle 90{\cdot}71°$	$2{\cdot}83$,,

Taking now the zero-load graphs, and assuming that the normal voltage of 88 910 is maintained constant at the receiving end of the line, the full curve showing the voltage at intermediate points gradually falls due to the Ferranti effect to a sending-end value of 77 400 volts. The rate of change of voltage is greater as the sending end is approached, owing to the larger charging current at this end.

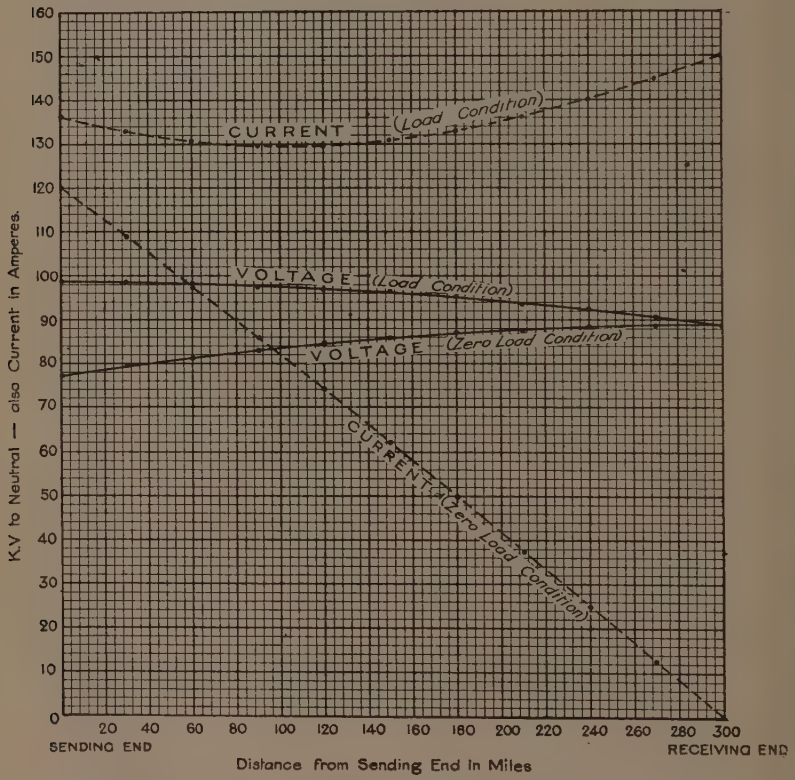


Fig. 32.—Voltage and current distribution on a 300-mile line.

From the dotted current graph under zero-load conditions, it will be seen that the charging current at the sending end has a value not much below that established when the line is supplying full load of 12 000 kW. per phase. As the receiving end of the line is approached, the current falls lower and lower until at the terminus of the line its value is, of course, zero.

56. Effect of Non-sinusoidal Sending-end Voltage.—So far in the treatment of the subject, all equations and data have been derived on the assumption that the voltage wave impressed on the line was sinusoidal in shape. If this is not the case, a rigid line solution involves the separation of the distorted wave into its constituent terms, consisting of a fundamental wave and its odd harmonics. Calculations of the effect due to each separate frequency must then be made as though the others did not exist, and the final result obtained by the method of superposition. Where only the R.M.S. value of the resultant wave is required this may be obtained by perpendicular summation. Thus if E_1, E_3, E_5, etc., are the R.M.S. values of the various harmonics of the receiving-end voltage as obtained by calculation, the R.M.S. value of the receiving-end voltage wave is

$$E_r = \sqrt{E_1^2 + E_3^2 + E_5^2 + \ldots} \quad . \quad (97)$$

A similar method of treatment must be applied in the case of plural-frequency currents.

The effect of a distorted sending-end wave is brought most into prominence when the line is open-circuited at the far end. Both the susceptance ωC and reactance ωL of the line increase in proportion to the frequency of the various harmonics. Hence a high-voltage harmonic of quite small value may produce charging currents of the same order of magnitude as those produced by the fundamental voltage wave. Furthermore, these high charging currents, flowing through the correspondingly increased reactance of the line, magnify the distortion of the voltage wave at points along the line and at the receiving end.

For purposes of illustration, a problem worked out in detail by Pernot will be taken.[5] In this example, a voltage having a wave shape whose equation as a function of $\theta = 2\pi ft$ is

$$\begin{aligned} E_s = {} & 100\,000 \sin \theta \\ & + 8\,000 \sin (3\theta - 40^\circ) \\ & + 6\,000 \sin (5\theta - 295^\circ) \\ & + 5\,000 \sin (7\theta - 110^\circ) \\ & + 2\,000 \sin (9\theta - 50^\circ) \\ & + 500 \sin (11\theta - 170^\circ) \end{aligned}$$

is assumed to be impressed on a 100-mile open-circuited line having the following constants:—

$R = 27{\cdot}5$ ohms.
$X = 76{\cdot}9$ ohms.
$G = 0{\cdot}15 \times 10^{-4}$ mhos.
$B = 5{\cdot}52 \times 10^{-4}$ mhos.
$f = 60$ cycles.

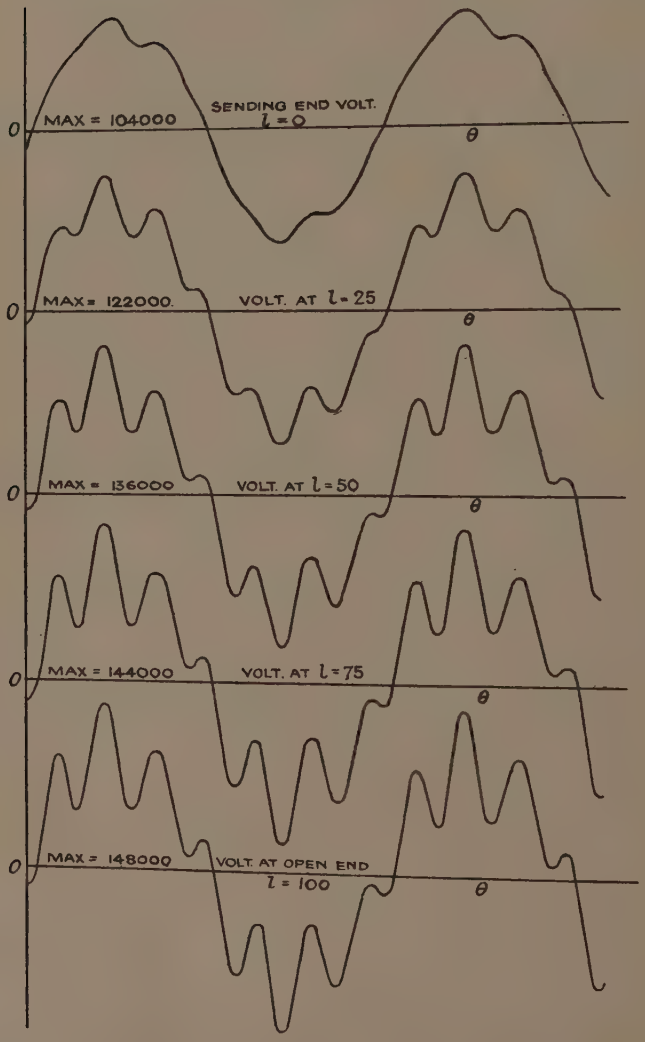


FIG. 33.—Voltage wave shapes along a 100-mile open-circuited line.

The resultant voltage at the receiving end is

$$\begin{aligned} E_r = 102\,100 \sin (\theta - 0{\cdot}47^\circ) \\ + 9\,808 \sin (3\theta - 41{\cdot}61^\circ). \\ + 11\,604 \sin (5\theta - 298{\cdot}76^\circ) \\ + 36\,835 \sin (7\theta - 126{\cdot}81^\circ) \\ + 7\,134 \sin (9\theta - 222{\cdot}20^\circ) \\ + 782 \sin (11\theta - 347{\cdot}27^\circ), \end{aligned}$$

and the intensification of the various harmonics should be noted.

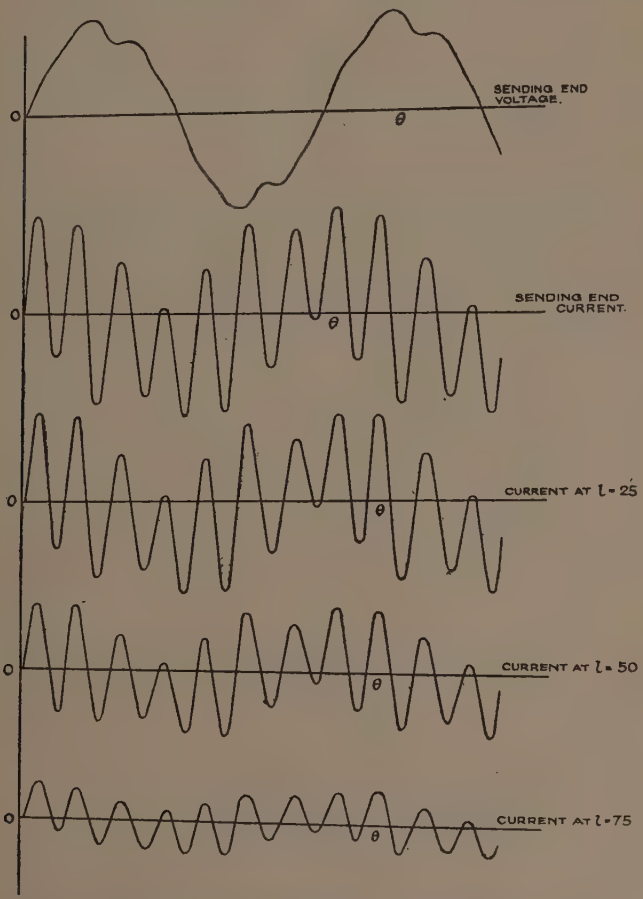


FIG. 34.—Current wave shapes along a 100-mile open-circuited line.

The line is approximately in resonance for the seventh harmonic, so that although the seventh harmonic in the impressed voltage is only 5 % of the fundamental, the seventh harmonic in the voltage at the receiving end of the line is 36 % of the fundamental.

The charging current at the sending end of the line is

$$\begin{aligned} I_c = {} & 55\,800 \sin (\theta + 88{\cdot}12^\circ) \\ & + 15\,184 \sin (3\theta + 48{\cdot}35^\circ) \\ & + 26\,604 \sin (5\theta - 208{\cdot}23^\circ) \\ & + 97\,725 \sin (7\theta - 35{\cdot}76^\circ) \\ & + 18\,346 \sin (9\theta - 130{\cdot}50^\circ) \\ & + 1\,611 \sin (11\theta - 254{\cdot}53^\circ). \end{aligned}$$

Here again, the seventh harmonic component is magnified the most, its value being approximately twice as great as the fundamental.

The wave shapes of voltage and current at various points along the line are shown in Figs. 33 and 34. Special attention should be given to the former curves as on these have been indicated the maximum instantaneous values of the voltage wave. The phase relations of the several component harmonics is such that the maximum value of the impressed voltage wave is only a little greater than the maximum value of the fundamental—104 000 maximum, with 100 000 volts as maximum value of the fundamental. At the open end of the line, the maximum value of the voltage wave is 148 000 volts, or an increase of nearly 50 % over the impressed voltage, which might be serious in its effect upon the line insulation. Furthermore, in a line designed to operate normally without corona formation at the voltage assigned to the fundamental frequency in this illustrative example, the presence of such large superimposed harmonics, which increase the maximum voltage by nearly 50 %, might be sufficient to start corona formation, and thus introduce an additional power loss in the system.

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CHAPTER VI.

LINE CONDUCTORS AND SUPPORTING STRUCTURES.

57. Properties of Stranded Conductors.—All conductors employed on overhead lines are preferably stranded, on account of the increased flexibility thereby obtained. Solid wires, except in the smaller sizes, are difficult to handle, and when used for long spans tend to crystallise at the points of support due to swinging in the wind.

In stranded conductors there is generally one central wire, and round this, successive layers of wires containing 6, 12, 18, 24 . . . wires. Thus if there are n layers, the total number of individual wires employed is

$$N = 3n(n + 1) + 1. \quad . \quad . \quad . \quad (98)$$

In the process of manufacture, the consecutive layers of wires are twisted or spiralled in opposite directions, the effect being to bind all the layers together. This method of construction is known as 'concentric lay.'

With very large sections of conductor, however, another method of stranding called 'rope lay' is sometimes used as it gives a more flexible conductor.

When a current enters a stranded conductor it divides among the wires, and each separate current, for all practical purposes, remains in its own wire throughout the length of the conductor. This is because the individual wires being circular, touch only along lines, and the surface resistance, due to dirt and the formation of oxide or sulphide, has a fairly high value. The result is that each current, in general, pursues a spiral path of greater length than the length of the conductor as a whole, and this effective increase of the path length correspondingly increases the resistance. The precise magnitude of this effect depends on the *lay* adopted for the conductor, meaning by this term the axial length of one complete turn of any wire. The lay is usually

expressed numerically in terms of the mean diameter of the layer containing the wire.

There is no fixed lay used by all manufacturers, but in wire tables the assumption is usually made that the length, and corresponding resistance, of all wires except the straight central one, is increased by 2 % above the values for the central one. This is equivalent to assuming that every twisted wire has a lay ratio of about 15·6.

Another effect of stranding is to modify slightly the fundamental formula for inductance which is based on a solid round conductor. According to Dwight,[1] the inductance per mile of concentric-lay conductor is as follows:—

$$\text{7-strand conductor, } L_0 = \left(0{\cdot}103 + 0{\cdot}741 \log_{10} \frac{d}{r}\right)10^{-3} \text{ henries,} \quad (99)$$

$$\text{19-strand conductor, } L_0 = \left(0{\cdot}089 + 0{\cdot}741 \log_{10} \frac{d}{r}\right)10^{-3} \text{ henries,} \quad (100)$$

$$\text{37-strand conductor, } L_0 = \left(0{\cdot}085 + 0{\cdot}741 \log_{10} \frac{d}{r}\right)10^{-3} \text{ henries,} \quad (101)$$

$$\text{61-strand conductor, } L_0 = \left(0{\cdot}083 + 0{\cdot}741 \log_{10} \frac{d}{r}\right)10^{-3} \text{ henries,} \quad (102)$$

where d is the interaxial distance between conductors, and r is the overall radius of the conductor, both measured in the same units.

For conductors having more than sixty-one strands, the formula for solid conductors

$$L_0 = \left(0{\cdot}080 + 0{\cdot}741 \log_{10} \frac{d}{r}\right)10^{-3} \text{ henries per mile,} \quad (9)$$

is used.

Spiralling the individual wires of a stranded conductor also tends to increase the inductance. It is difficult to calculate the spirality effect for various cases, but it may be taken as negligible for overhead lines using non-magnetic conductors.

58. Conductivity.—The resistivity of all conductor materials is usually referred to the standard for annealed copper as adopted by the International Electrotechnical Commission in 1913 (corresponding for the first four significant figures with the older Matthiessen Standard). The International Annealed Copper Stan-

dard is expressed in terms of mass resistivity as 0·15328 ohms, this being the resistance of a uniform round wire one metre long, weighing one gramme, at the temperature of 20° C.

Since the density of copper at this temperature is taken as 8·89 grammes per cubic centimetre, the standard may be alternatively expressed in terms of volume resistivity as $1 \cdot 7241 \times 10^{-6}$ ohms, representing the resistance between opposite faces of a centimetre cube of the metal at 20° C.

The most convenient method of comparing different materials is then to express their conductivity (reciprocal of volume resistivity) at 20° C. as a percentage of the conductivity of the International Standard taken as 100 per cent.

The resistance of all pure metals increases with increase of temperature, and the variation can be expressed by a linear relation for the temperature changes occurring in transmission line operation. Hence

$$R_t = R_0(1 + at), \quad . \quad . \quad . \quad (103)$$

where R_0 is the resistance at 0° C., R_t is the resistance at a temperature of t° C., and a is the temperature coefficient of resistance. For example, the value of a for copper of 100 °/₀ conductivity is 0·00427.

If the temperature t_1° C. is chosen as a standard of reference, the formula may be written

$$R_t = R_{t_1}\{1 + a(t - t_1)\}, \quad . \quad . \quad . \quad (104)$$

where a is not the same as before but is smaller, because the increase of resistance is expressed as a fraction of the resistance at t_1° C. which is greater than the resistance at 0° C. Hence

$$a_{t_1} = a_0\frac{R_0}{R_{t_1}} = \frac{a_0R_0}{R_0(1 + a_0t_1)}$$

$$= \frac{1}{\frac{1}{a_0} + t_1}$$

$$= \frac{1}{234\cdot5 + t_1}. \quad . \quad . \quad (105)$$

At a temperature of 20° C. the temperature coefficient for copper of 100 °/₀ conductivity is therefore

$$a_{20} = \frac{1}{234\cdot5 + 20} = 0\cdot00393.$$

59. **Current Distortion Effects.**—In previous discussions it has been taken for granted that an alternating current distributes itself uniformly over the conductor cross-section. In actual practice, however, the distribution of current is more or less distorted due to: 1. Skin effect, 2. Proximity effect, and 3. Spirality effect.

1. *Skin Effect.*—A solid round conductor may be thought of as composed of a large number of separate filaments or strands, each carrying a small finite part of the total current. The inductance of the individual filaments will vary according to their position, due to the fact that those filaments near the centre of the conductor are surrounded by a greater magnetic flux than are the filaments near the outer surface. The self-induced back e.m.f. is, therefore, greater in the filaments located near the centre of the conductor. The higher reactance of the inner filaments causes the current to distribute in such a manner that the current density is less in the interior than at the surface. This crowding of the current towards the surface of the conductor is known as skin effect, and its result is to apparently increase the resistance, and to a smaller extent decrease the reactance, of the conductor. The same phenomenon occurs also in the case of stranded conductors.

Maxwell[2] and Rayleigh[3] have shown that for solid cylindrical conductors the effective values of resistance and internal reactance (slightly changing the notation) are given by

$$R' = R\left(1 + \frac{1}{12}m^2 - \frac{1}{180}m^4 + \ldots\right), \quad . \quad (106)$$

$$X'_2 = X_2\left(1 - \frac{1}{24}m^2 + \frac{13}{4320}m^4 - \ldots\right), \quad . \quad (107)$$

where

R = resistance of conductor to a steady current.

R' = effective resistance of conductor to an alternating current.

$m = \left(\frac{2\pi^2 r^2 f \mu}{\rho} 10^{-9}\right)$

r = radius of conductor in centimetres.

f = frequency of alternation in cycles per second.

μ = permeability of conductor material.

ρ = resistivity of conductor material in ohms per centimetre cube.

$X_2 = \pi f l \mu$ = internal reactance of conductor with a steady current (see formula 7).

X_2' = effective internal reactance of conductor with an alternating current.

l = length of conductor in centimetres.

The magnitude of the effect depends largely on the frequency of alternation. As this is gradually raised the resistance steadily rises, and the internal reactance (instead of increasing directly with f like the external reactance), increases also but at a slower rate than f. When the frequency is very great, the effective resistance and the effective internal reactance of the conductor are equal, and are given by

$$R' = X'_2 = \sqrt{\frac{l^2 \rho \mu f}{r^2}} \text{ ohms,} \quad . \quad . \quad (108)$$

which steadily increases without limit with f.

It should be noted that the value of μ for magnetic conductors is difficult to assign in the above formulæ. It varies throughout each cycle according to the instantaneous value of the current flowing, and only in a rough sense can it be taken as constant.

Considering now the usual case of non-magnetic overhead conductors, the internal reactance is always small compared with the external reactance produced by the necessary separation of the conductors. Since skin effect can only alter the value of the former component, its influence on the total reactance of a conductor is negligible.

The increase of resistance due to skin effect is noticeable at commercial frequencies with large sections of conductor, and assumes great importance in the case of high-frequency currents such as are induced in overhead lines by lightning disturbances.

For the purpose of calculating the increase of resistance in solid round conductors one of the following methods should be employed:—

(1) For values of m less than 1, formula (106) gives results to a three-figure accuracy.

(2) For values of m over 6, formula (106) is too slowly convergent, and can be written in a form more suitable for numerical computation as

$$R' = R\left(\frac{\sqrt{m}}{\sqrt{2}} + \frac{1}{4} + \frac{3}{32\sqrt{2m}}\right), \quad . \quad . \quad (109)$$

which for large values of m reduces to the form given in formula (108).

(3) For values of m from 1 to 6 inclusive, the effective resistance can be readily determined by multiplying the steady current resistance R by the factors shown in Table 8.[4]

TABLE 8.—*Skin-effect Ratios for Values of* m *from* 1 *to* 6.

m.	$\frac{R'}{R}$.	m.	$\frac{R'}{R}$.	m.	$\frac{R'}{R}$.
1·0	1·078	2·8	1·437	4·6	1·785
1·2	1·109	3·0	1·470	4·8	1·819
1·4	1·144	3·2	1·521	5·0	1·853
1·6	1·182	3·4	1·562	5·2	1·885
1·8	1·222	3·6	1·601	5·4	1·916
2·0	1·264	3·8	1·641	5·6	1·947
2·2	1·307	4·0	1·678	5·8	1·977
2·4	1·351	4·2	1·715	6·0	2·007
2·6	1·394	4·4	1·750	—	—

In order to give some idea of the magnitudes involved, values of $\frac{R'}{R}$ have been calculated for various frequencies and conductor sizes, and are given in Table 9.

TABLE 9.—*Increase of Effective Resistance of Solid Copper Conductors Due to Skin Effect.*

Temp., 20° C. $\rho = 1{\cdot}77 \times 10^{-6}$ ohms.

Diameter in Cms.	Frequency.	m.	Method of Calculation.	$\frac{R'}{R}$.
1·0	50	0·139	Formula (106)	1·002
2·0	50	0·558	—	1·025
3·0	50	1·255	Table 8	1·118
2·0	1 000	11·15	Formula (109)	2·63
2·0	10 000	111·5	—	7·72
2·0	100 000	1 115	—	23·86
2·0	1 000 000	11 150	—	74·92

It is interesting to note that when aluminium is used in place of copper the larger diameter necessary to give the same conductivity does not lead to a greater power loss through skin effect. As a matter of fact, equivalent sections of all non-magnetic conductors have exactly the same percentage increase of resistance; the increased cross-section being exactly compensated for by the increased resistivity.

2. *Proximity Effect.*—Proximity effect is the crowding of the current to one side of a conductor due to the proximity of another current-carrying conductor. Thus considering two solid round conductors A and B, lying comparatively close together, the flux due to B cutting the near half of A is greater than the flux cutting the remote half of A, and consequently the current distribution over the cross-section of A is distorted. Proximity effect, like skin effect, increases the resistance and decreases the internal inductance of a conductor. The phenomenon is more pronounced for large conductors, high frequencies and close proximity. The magnitude of the effect, in the case of the wide spacings of conductors required for overhead transmission lines, is so small that it can be ignored.

In the case of stranded conductors, due to the spiralling of the wires, each wire traverses alternately weaker and stronger portions of the magnetic field produced by an external current-carrying conductor. Hence the average value of the field along the path of any wire will be the same, and assuming that the currents in the conductor follow closely the paths of the individual wires, the proximity effect must be practically eliminated.

3. *Spirality Effect.*—The spirality effect, first mentioned by Stirnimann,[5] tends to increase both the resistance and internal reactance of a stranded conductor. The phenomenon is caused by the current following, to a large extent, the spiral path of the individual wires, so that the conductor may be regarded as built up of a series of coaxial solenoids. Each of the solenoids formed by the consecutive layers produces a magnetic field whose direction is along the axis of the solenoid, *i.e.* the axis of the conductor. The outermost layer contains the greater number of wires so that its field will be stronger than that due to the other layers. The next layer is spiralled in the opposite direction and contains fewer wires than the outer layer, so that there is produced a weaker field in the opposite direction. Owing to the reversal of spirality, the layer below gives rise to a still weaker field in the original direction, and so on. The resultant effect is, that travelling in a radial direction from the surface of the conductor, the axial magnetic field alternates in magnitude although it does not actually change in direction on passing from one layer to another. As the potential difference applied to the ends of all the layers is the same, this variation in the axial magnetic flux leads to phase differences among the currents

in consecutive layers. Hence the sum of the R.M.S. values of the currents in the various layers may be appreciably larger than the R.M.S. value of the total current passing along the conductor, or in other words, the resistance of the conductor has been apparently increased.

The magnitude of the effect depends on the size and method of construction of the conductor. At present there is a lack of numerical data relating to the phenomenon, but it appears to be small, and can quite safely be ignored in the case of overhead non-magnetic conductors.

60. Conductor Materials.—The materials used at the present time for overhead transmission line conductors include copper, aluminium, steel-cored aluminium, copper-clad steel, and to some extent steel alone. It is very difficult to make hard-and-fast comparisons between these various metals, because not only the electrical, but also the mechanical, properties have to be taken into account, and the requirements in the latter respect depend to a large extent on local legislative restrictions affecting the design of the line. The question of first cost, also, except when based on current quotations, is of little value, and must be gone into for each particular case. In the following articles, however, leading technical particulars and data will be given regarding the materials mentioned above. Tables of resistances, sizes and weights will not be given, as they are readily accessible in all the handbooks and manufacturers' catalogues.

61. Copper.—Copper ranks first in importance among transmission line conductors, owing to its high electrical conductivity and great tensile strength. The conductivity of the metal depends on its degree of chemical purity, and also to a lesser degree on its physical state. Hard-drawn copper as used for overhead lines has a conductivity somewhere about 97 %, the reduction being due to the operations of cold rolling and drawing used in manufacturing. The tensile properties, on the other hand, are considerably improved by this cold working.

It is found, by experiment, that the temperature coefficient of commercial copper at a given temperature varies with the conductivity of the copper, being greater the purer the copper. Dellinger [6] has shown that if n be the fractional conductivity of the sample as compared with standard annealed copper, then its temperature coefficient is na_t where a_t is the temperature coefficient

for pure copper. If, for example, the percentage conductivity of a sample of copper is 96 %, its temperature coefficient at 20° C. will be

$$n\alpha_t = 0{\cdot}96 \times 0{\cdot}00393 = 0{\cdot}00377.$$

As previously mentioned, line conductors should preferably be stranded, but solid copper wires may be used with care up to No. 000 S.W.G., which is about the practical limit.

62. Aluminium.—Aluminium possesses many advantages as a conductor material, and a considerable saving in the first cost of the line is sometimes possible by the employment of this metal in place of copper.

Cold rolling and drawing, as in the case of copper, slightly decreases the electrical conductivity of the metal and improves the tensile properties. At 20° C. the conductivity of the hard-drawn wire is about 60 %, its temperature coefficient of resistance is 0·00390, and its density is 2·71 grammes per cubic centimetre.

The smaller conductivity of the metal necessitates that, for any particular efficiency of transmission, the sectional area of the conductor must be larger in aluminium than in copper. Comparing aluminium of 60 % with copper of 97 % conductivity, the diameter of the aluminium conductor would be 1·26 times the diameter of a copper conductor of equal resistance. In spite of the greater section of aluminium necessary, the density of the metal is so low, that an aluminium conductor has almost exactly one-half the weight of the equivalent copper conductor. The tensile strength is much lower than that of copper, but the larger sectional area of metal neutralises the difference to some extent, and an aluminium conductor has about 75 % of the ultimate strength of the equivalent copper conductor.

On the other hand, the increased cross-section of conductor exposes a greater surface to wind pressure, and so the supporting towers must be designed for a greater transverse strength. In many cases also, higher towers must be used with aluminium conductors than would be required with copper conductors having the same length of span. This is in consequence of the greater sag of aluminium conductors, due to the decreased working stress permissible, and also owing to the fact that the linear coefficient of expansion of aluminium is 1·4 times that of copper. When considering an aluminium installation, it is therefore necessary to set the additional cost of towers against any saving which may be

effected on the conductors. Apparently the chief field for aluminium is for heavy-current transmissions where the conductor size is large and its cost forms a large proportion of the total cost of the complete installation.

63. Steel-cored Aluminium.—Steel-cored aluminium conductors have a central core of galvanised steel wires with one or more layers of aluminium wires outside. This type of conductor has been developed to permit of longer spans being employed with the ordinary height of tower. The breaking load of such a composite conductor is considerably larger than that of the equivalent copper conductor, and its weight is still about 25 % smaller. The importance of this lies in the fact that the sag in an overhead conductor is dependent upon the ratio of the loading in the span to the strength of the conductor. The loading factor is the resultant of the weight of the conductor and the wind pressure, ice deposits and other artificial loads, and though the diameter of a steel-cored aluminium conductor is larger than that of any other conductor of equal resistance, and the artificial loads are therefore greater, the weight factor is so small that no other conductors can equal steel-cored aluminium in the smallness of the ratio of total loading to strength. The result is that the sag with steel-cored aluminium is much less than with copper, and the supporting towers may therefore be shorter, or alternatively the span length longer.

Since the composite type of conductor is more costly than the all-aluminium type, it is chiefly used in those cases where an all-aluminium conductor would necessitate a much higher and more costly support than copper. This occurs principally where spans of 300 feet and over are involved, steel structures generally being used to support the line conductors.

In many long-distance transmission systems, the use of copper conductors necessitates a lower line voltage than would be otherwise desirable, in order to avoid excessive corona losses. In these cases, the use of the equivalent steel-cored aluminium conductor with its greater diameter has the advantage of raising the critical voltage limits of the line. The magnitude of this increase may be of the order of 30 to 50 %, and this point will be raised again in the course of the following chapter.

For the purpose of obtaining the resistance and reactance of a steel-cored conductor, Dwight[7] has shown that the resistance may be taken as equal to that of the core and the outer wires connected

in parallel, and the reactance as equal to that of a non-magnetic conductor of the same outside diameter. Since the resistance of the steel core to alternating currents is very high compared to that of the aluminium wires, it may generally be neglected, and the resistance of the composite conductor taken as equal to that of the aluminium wires alone.

64. Copper-clad Steel.—A composite wire known as hard-drawn, copper-clad steel wire is obtained by welding a coating of copper on to a steel wire core. Line conductors made of this material are preferably stranded, and have a considerably greater tensile strength than the equivalent all-copper conductors. The proportion of copper and steel is so chosen that the composite wire has a conductivity either 30 or 40 % of a copper conductor of equal diameter. This material appears to be very suitable for river crossings, or other places where an exceptionally long span is necessary.

65. Iron and Steel.—Galvanised iron and steel conductors have been used to advantage for abnormally long spans, or for short line sections exposed to abnormally high stresses due to climatic conditions. Recently, however, investigations have been made into the characteristics of these materials, and it appears that in some cases they can be economically employed for the whole installation, and particularly when the prices of copper and aluminium are high. Generally speaking, the use of iron or steel is most advantageous for short-distance small-power transmissions, where the size of the copper conductor desirable from economic considerations would be smaller than No. 8 S.W.G., and would thus be unsuitable for use owing to its lack of mechanical strength.

As an example of an actual transmission may be cited the case of a 22 000-volt, three-phase line in America. This line transmits 95 kVA. a distance of 31 miles over galvanised iron wires, the line current being about 2·5 amperes, and the power loss 9 %.[8]

With magnetic conductors, the skin effect causes a material increase in resistance which is difficult to calculate, as it depends not only on the frequency, but also on the magnitude, of the currents flowing. The permeability of the material increases with the current density within certain limits, and as the skin effect is due to a greater inductance (a function of the permeability) at the centre of the conductor than at the surface, an increase in current produces an increase in resistance.

The inductance of a magnetic conductor also varies according to the current flowing as it is composed of two factors, the external inductance, depending only on the diameter and spacing of the conductors, and the internal inductance, which is a function of the permeability.

Unfortunately, the various electrical and magnetic constants for any particular grade of iron or steel wire cannot be given with

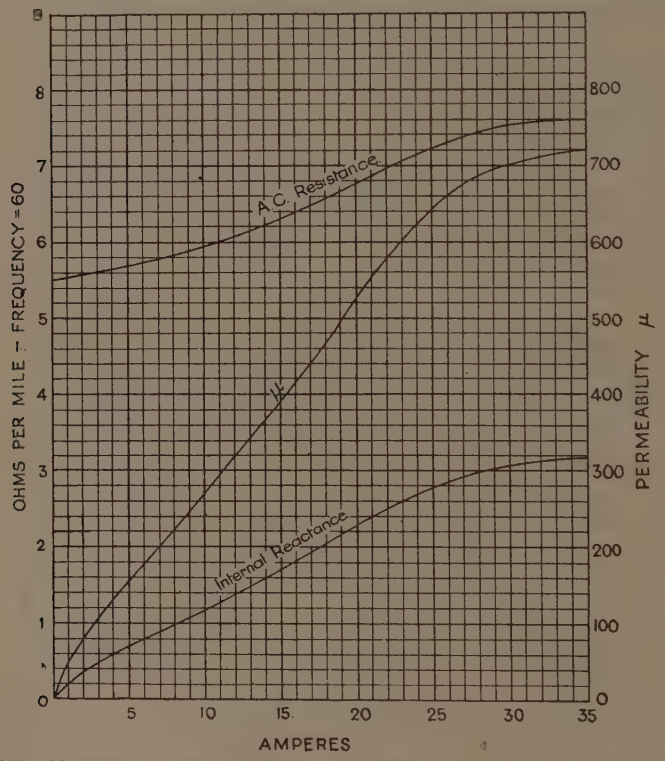

FIG. 35.—Alternating-current electrical properties for 7-strand $\frac{5}{16}$-in. galvanised steel conductor.

any degree of accuracy, because of the method of manufacture. A slight change in the composition of the wire, that does not affect the mechanical properties, often causes a material change in the electrical and magnetic characteristics. For this reason, all calculations should be based on test data, obtained from sizes and grades of wire approximating to those of the conductor it is proposed to use.

Fig. 35, for example, which is abstracted from material given in an article by Oakes and Eckley,[9] shows the alternating-current resistance, internal reactance and permeability for a 7-strand, $\frac{5}{16}$-in. diameter, ordinary galvanised steel conductor. The data were obtained from tests made on an experimental line, and were corrected to a temperature of 20° C. Thus the value of resistance given for zero current is the continuous-current resistance at 20° C.

Owing to the high internal reactance of iron and steel conductors, the pressure drop per mile of line is far greater than would occur if the equivalent copper or aluminium conductors were used. To illustrate the method of calculation, assume a transmission line consisting of the above-mentioned $\frac{5}{16}$-in. steel wires, spaced 30 inches apart (delta arrangement), and carrying a current of 30 amperes at a frequency of 60. Temperature taken as 20° C.

From Fig. 35 the alternating-current resistance per mile is 7·5 ohms, so the ohmic pressure drop in each wire is

$$7{\cdot}5 \times 30 = 225 \text{ volts per mile.}$$

Also the internal reactance per mile is 3·06 ohms, hence the total reactance of each wire is

$$3{\cdot}06 + \left(0{\cdot}080 + 0{\cdot}741 \log_{10} \frac{30}{0{\cdot}156}\right) 10^{-3} \times 2\pi \times 60$$
$$= 3{\cdot}06 + 0{\cdot}67$$
$$= 3{\cdot}73 \text{ ohms per mile,}$$

and the reactive pressure drop is

$$3{\cdot}73 \times 30 = 112 \text{ volts per mile.}$$

Given the receiving-end power factor as 80 % lagging, the resultant fall of pressure in each conductor is

$$IR \cos \phi_r + IX \sin \phi_r$$
$$= 225 \times 0{\cdot}8 + 112 \times 0{\cdot}6$$
$$= 247 \text{ volts per mile.}$$

66. Cadmium Copper.—A material now being considered is copper alloyed with 1 to 2 % of cadmium. This increases the tensile strength by over 80 %, and as the conductivity is only reduced by about 20 % below that of pure copper, and the cost is very little more, cadmium copper shows considerable economy on lines of small or medium cross-section, due to the saving in the number of supports necessary.

67. Line Supporting Structures.—The supporting structures

for overhead power conductors are of various kinds, including wood, steel, and reinforced-concrete poles, and steel towers either of the rigid or flexible type. A few typical designs from present-day

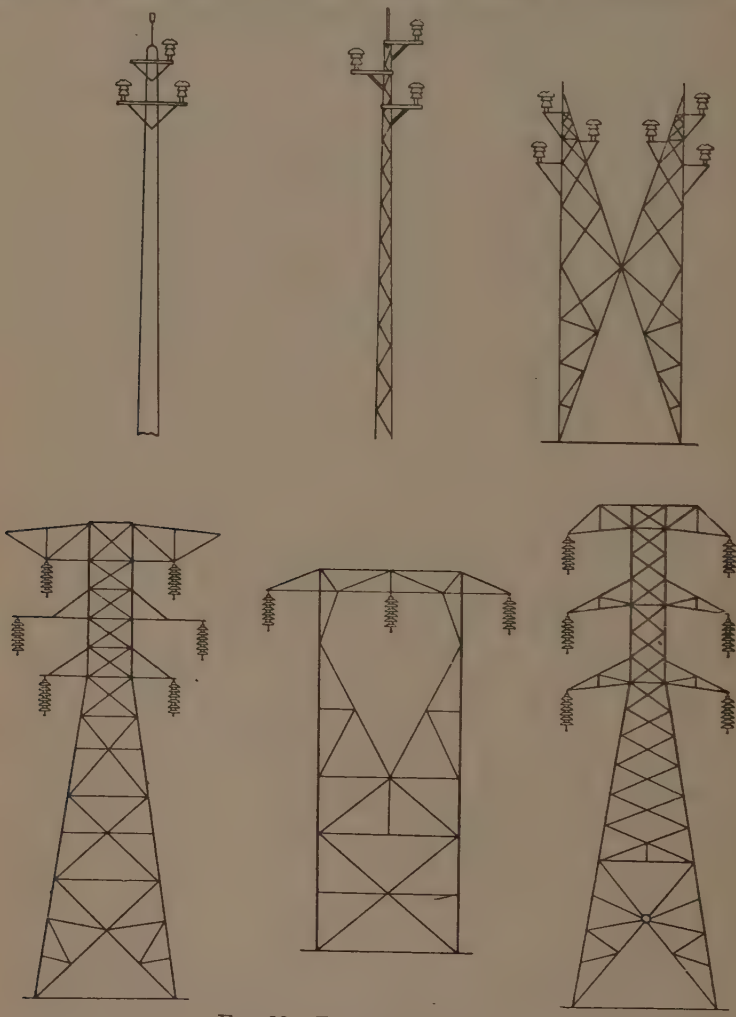

FIG. 36.—Typical line supports.

practice are exhibited in Fig. 36, and the drawings also show the various methods of spacing the conductors.

The design of a transmission line support depends very materially upon whether the support is rigid, or has a certain

amount of flexibility in the direction of the line. Wooden poles and certain special types of steel structures fall within the latter category, and in designing these it is usual to consider only the transverse wind pressure which occurs upon the conductors and upon the support itself. The longitudinal pull of the conductors is normally balanced on either side of the support, but in the event of the breaking of one or more conductors on one side, there will be an unbalanced load which may be greatly in excess of the transverse wind pressure. With flexible supports this unbalanced load is quickly absorbed into the system. The supports on each side of the wrecked span bend towards the adjoining spans, and this movement of the pole tops produces a reduction of tension in the wires of the adjoining span owing to the increased sag of these wires. There will be an appreciable deflection of the second and third poles beyond the break, but the magnitude of these successive deflections decreases rapidly, and is rarely noticeable beyond the fifth pole. The greater the flexibility of the supports in the direction of the line, the smaller will be the extra load which any one support may be called upon to withstand. In any case, a certain general rigidity in the longitudinal direction is provided for, by employing at intervals of a mile or so rigid anchoring towers, designed to withstand the failure of one conductor in three on one side, as well as the transverse load.

In the case of rigid supports, such as the lattice-steel broad-base structures which are typical of American and Canadian practice, it is usual to provide equal strength in both the longitudinal and transverse directions, and to design every tower to withstand the unbalanced load due to the breaking of one conductor in three, on the same side. In addition, anchor towers are often provided, in which the support is capable of withstanding the failure of two conductors out of three, or even of the whole of the conductors, on one side.

68. Wood Poles.—The use of wood poles is chiefly limited to comparatively low pressures, and spans having a maximum length of 250 or 300 feet. There are a few notable exceptions, however, existing in districts having a plentiful supply of suitable timber, and where the cost of transporting steel towers would be high. Under these special conditions, single and 'H' poles have been employed for lines working at voltages up to 130 000, and average span lengths of 500 feet.

Wood poles are very elastic, and lines using this type of support are often designed throughout for the transverse load; longitudinal strength at terminals and for anchor supports being provided by means of guys. Double-pole structures, of the 'A' or 'H' type, are used in many situations to obtain a higher transverse strength than could be economically provided by means of the single poles. In the transverse direction, 'A' poles are about five times, and 'H' poles about three times, as strong as one of the single poles composing the legs.

The chief objection to wood supports is the tendency to rot at the ground level, and the life of a wood pole under normal conditions can be taken as about twenty years, provided that the wood has been well impregnated with a preserving compound such as creosote.

69. Steel Poles.—Steel poles of tubular construction, or narrow-base lattice-steel masts, are often used as a substitute for wood poles. They enable a rather longer span to be used, at the same time taking up little more base room than a wood pole, and may therefore be employed for crossing cultivated fields or for running along the roadside. In the flexible system of line construction, 'A' or 'H' supports are generally used.

All steel supports should be well galvanised, and a life of at least thirty years can be counted on. There is apparently no reason why the life should not be indefinitely prolonged, if the structure is regularly scraped and painted after the galvanising fails.

70. Reinforced-concrete Poles.—Reinforced-concrete poles have been employed as line supports in some recent installations, using spans up to 250 feet. These poles are very heavy and have to be moulded on site, otherwise the cost of transportation would generally be prohibitive. Their most attractive quality is their practically unlimited life, and the fact that maintenance charges are exceedingly low.

71. Steel Towers.—Broad-base, lattice-steel towers are employed for the majority of long-distance transmission lines running across open country, although their first cost is from two to four times greater than that of wooden poles. Steel towers have the advantage of a long life, which may be extended almost indefinitely by a reasonable amount of attention to their maintenance. Furthermore, their reliability is much greater than that of wooden supports.

The substantial construction of the towers renders them capable of withstanding the most severe climatic conditions, and immune from destruction by forest fires. The risk of interrupted service, due to broken or punctured insulators, is considerably reduced owing to the longer spans rendered possible by the stronger and taller steel supports. Lightning troubles are also minimised as each tower is a lightning conductor, whereas on wood pole lines shattered poles and wrecked line sections are not infrequent. At a moderate additional cost double-circuit towers can be provided, thus giving a further insurance against discontinuity of supply. In case of break-down to one circuit it is then possible to carry out repairs while maintaining the supply on the other circuit.

72. Spacing and Arrangement of Conductors.—There is no general agreement among engineers as to the best arrangement of

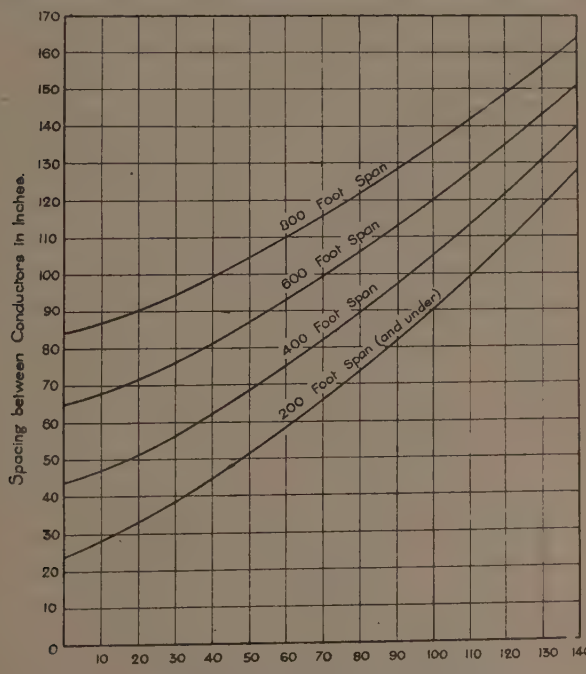

FIG. 37.—Spacing of overhead conductors.

the conductors on the support. Apparently there is no special advantage to be obtained by using the symmetrical-delta arrangement, and in most cases flat-horizontal or vertical spacing is adopted

from mechanical considerations, particularly when suspension insulators are employed. In unsymmetrical arrangements, the conductors are usually transposed at regular intervals, in order to balance the electrical characteristics of the several phases, and prevent inductive interference with neighbouring telephone circuits.

The spacing of the conductors is determined by considerations partly electrical and partly mechanical. Increased spacing increases the inductance of the line and hence the pressure drop, so that to keep the latter within a reasonable value the conductors should be as close together as is consistent with the prevention of corona. In many cases, however, the most powerful consideration which governs the spacing is the necessity for preventing the conductors from swinging together in the wind. Usually wires swing synchronously in the wind, but with the longer spans there is always a possibility of non-synchronous swinging taking place, and so the spacing should increase somewhat with the span length. According to Still,[10] present-day practice in this respect is given by Fig. 37.

73. **Span Lengths.**—Neglecting the deciding influence upon the span length of such local conditions as the necessity for following the configurity of roads, canals, or railways, it is interesting to note that there is one definite value for the span length which will give the minimum overall cost of the line. It will be obvious that this must be so, because, as the length of span increases, the total number of supports and insulators decreases, but the height of the support and the conductor spacing must be increased to allow for the greater sag. It can readily be seen that the higher the operating voltage of the system, the greater will be the economical length of span, owing to the higher relative cost of insulators to supports. Again, where line supports are placed on private property, the cost of pole or tower rights is generally reckoned as a fixed amount per support irrespective of the size of support, and this is another factor which operates in favour of longer spans and should receive consideration. It is not possible, therefore, to give any hard-and-fast rules as to the best span length to be employed, and the only way of determining this is to calculate the total line cost per mile for a number of different span lengths, and plot the results.

In this connection it should be noted that with small sizes of conductors the length of span is often unduly restricted, owing to the mechanical requirements of the line. For example, with No. 8 S.W.G. copper conductors it is not advisable to employ spans over

150 feet in length as the sag then becomes excessive. Thus, when the section of conductor as determined from electrical calculations comes out rather small, it is often possible to reduce the total line cost by using a larger and stronger conductor, and increasing the spacing of the line supports.

In the case of long-distance transmission systems employing the steel-tower form of construction, spans of 600 to 800 feet are commonly used, but there is a definite tendency towards longer span lengths. One reason for this is that the insulators constitute the weakest part of a transmission line, and a reduction in the number of towers per mile provides a fewer number of points of potential break-down. Modern views on this point are typified in a recently constructed line where steel-cored aluminium conductors supported on high tensile limit steel towers have been adopted, the average span length being 1 250 feet.

For river or ravine crossings exceptionally long single spans are sometimes necessary. Under these special circumstances spans of 4 000 to 6 000 feet have been satisfactorily employed, the conductor material being steel-cored aluminium, copper-clad steel, or simply galvanised steel.

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CHAPTER VII.

INSULATION OF OVERHEAD LINES AND CORONA EFFECT.

74. Insulator Materials.—The successful operation of a transmission system depends to a great extent on the proper maintenance of the line insulation, and hence the selection of suitable insulators should be made very carefully. The material most commonly used for insulators on overhead lines is porcelain, but glass and special composition materials are also used to a limited extent.

Glass is cheaper than porcelain in the simpler shapes, and when properly annealed it has a high resistivity and dielectric strength. Owing to its transparency, flaws in the material can be readily detected by visual examination. On the other hand, moisture readily condenses on its surface and facilitates the accumulation of dirt deposits, thus giving a high surface leakage. Glass is also less reliable than porcelain in the irregular-shaped pieces forming the make-up of large high-voltage insulators, on account of internal strains resulting from irregular cooling after casting. Nevertheless, glass insulators are well adapted for use on installations employing pressures up to about 25 000 volts, and under specially favourable climatic conditions they have been used for very much higher voltages. In some respects, glass is superior to porcelain as an insulating material, and if the same amount of study and research devoted to the development of porcelain insulators had been devoted to the improvement of glass insulators, these might now have a far wider field of application.

Special composition insulators are on the market, and have the advantage that they may be readily moulded to the most complicated shapes without any internal stresses being caused. Metallic fittings can also be firmly embedded in the material without any difficulty. Some of these materials, however, deteriorate fairly rapidly under bad climatic conditions, or may yield by softening in the sun when subjected to high mechanical stresses. The insulator

surface also is liable to be carbonised, and conducting paths formed, if subjected to flash-over.

Porcelain is produced by firing at a high temperature a mixture which consists essentially of kaolin, feldspar, and quartz. The resulting material is stronger mechanically than glass, gives less trouble from leakage, and is less affected by changes of temperature. On the other hand, it is not so homogeneous as glass, owing to the fact that each component shell of a porcelain insulator is glazed during the process of manufacture, and the satisfactory performance of the insulator in service depends to a considerable extent on the preservation of this glaze which is only of the order of one-thousandth of an inch in thickness. Also it is not possible to detect flaws in the material by simple inspection, and each separate insulator shell has to be subjected to pressure tests as part of the manufacturing routine.

In the manufacture of porcelain, under-firing produces good mechanical qualities but causes porosity in the material, which results in rapid deterioration and failure when put in service. Over-firing, on the other hand, minimises porosity, but makes the material brittle. The quality of electrical porcelain has been greatly improved in recent years due to new methods of manufacture, and particularly to close control of the kiln temperature.[1]

75. Factors Involved in Insulator Design.—Insulators are required to withstand both mechanical and electrical stresses, in addition to which the surface leakage path must have sufficiently high resistance to prevent any appreciable current flowing to earth. Electrical breakdown may occur either by flash-over or puncture. In a flash-over, an arc occurs between the line conductor and earth (the latter being represented by the supporting pin of the insulator), and the discharge jumps across the air gaps in its path. In a puncture, the discharge occurs from conductor to pin through the body of the insulator. When a break-down of the former type is involved, the insulator will continue to act in its proper capacity after the event unless fractured by the heat of the arc, but after a puncture it is permanently ruined. It is thus of importance to provide sufficient thickness of porcelain in the insulator to resist puncture by the combined effect of the line voltage and any probable transient pressure rises on the line. The ratio of puncture strength to flash-over voltage may be termed the factor of safety of the insulator against puncture, and this ratio should be high so that

a good margin is obtained to protect the insulator from complete failure.

Fig. 38 shows the flash-over distances as usually measured on a pin-type insulator. The sum of $A + B + C$, or $D + B + C$ (depending on whether A or D is the shorter distance), gives the length of path on a dry flash-over. When the exposed upper surfaces of the insulator are wet with rain they are practically conducting, and thus the wet flash-over distance is the sum of $F + G + H$. The exact path of the arc will depend somewhat on the shape of the parts, and will tend to lie between 45 degrees and a normal from one shell to the outer surface of the next lower shell.

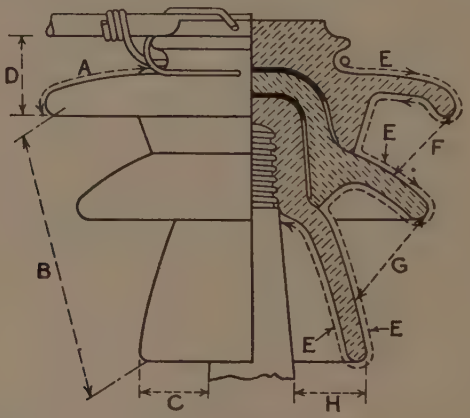

FIG. 38.—Flash-over and leakage distances of a pin-type insulator.

The leakage distance is generally measured from tie wire to pin taken radially along the surfaces as shown by the line E, but obviously this measurement only gives a rough idea of the leakage resistance as it neglects the varying width of the leakage path.

76. Pin-type Insulators.—One-piece, pin-type insulators can be employed satisfactorily for operating voltages up to about 25 000 volts. An adequate length of leakage path is obtained by providing the insulator with two or three petticoats or rain sheds. These are so designed, that even when the outer surface of the insulator is wet due to rain, sufficient leakage resistance is still given by the inner dry surfaces.

In the past, it has been found difficult to vitrify porcelain properly if the thickness was considerable. Hence for higher

voltages, where the total thickness of the material required would be greater than desirable from a manufacturing point of view, multi-part insulators are employed. These consist of shells of moderate thickness fastened firmly together by neat Portland cement, the cemented surfaces being unglazed and corrugated in order to obtain a good bond. Multi-part, pin-type insulators are supplied for all voltages up to 80 000 volts, but the suspension

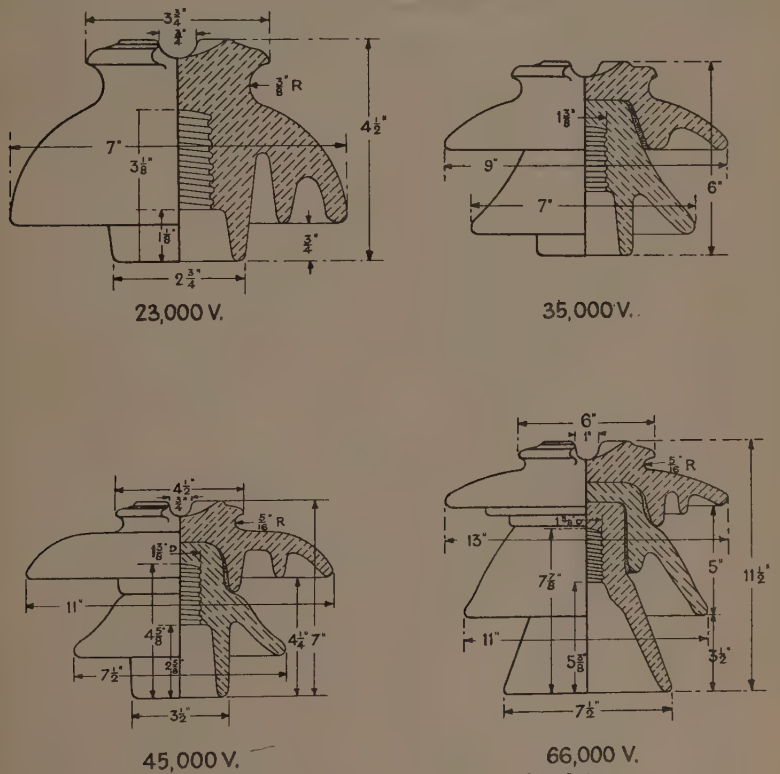

FIG. 39.—Typical designs of pin-type insulator.

type of insulator is generally more satisfactory, and also more economical for pressures above 50 000 volts. Typical pin-type insulators as manufactured by the R. Thomas & Sons Co. are illustrated in Fig. 39.

For supporting the insulator, steel pins are generally used, these being screwed direct into the inner shell of the insulator, or fastened by means of a lead thimble. Wooden supporting pins

impregnated with paraffin are occasionally employed on lines working at the lower pressures. They have the advantage of cheapness, but are mechanically weak and liable to give trouble by burning due to leakage currents.

The commonest method of attaching the conductor to the insulator is to tie it down with soft copper, or soft aluminium, binding wire according to the conductor material. This holds the conductor firmly, but the restraining force is gradually applied, and the swinging of the conductor in the wind does not cause continuous bending backwards and forwards at one point. Mechanical clamps have been employed for such fastenings but suffer from lack of flexibility.

In their electrical behaviour, pin-type insulators may be compared to a complicated series of condensers with resistances in series and shunt. The petticoats, with the intervening air spaces, form the condenser system, and the leakage paths over the surface and through the body of the material are represented by the resistances. Now considering any two consecutive insulator shells, the two surfaces which are opposite to each other are at different potentials except where they are cemented together. Also the potential difference between corresponding points on opposite surfaces is greater, the further they are removed from the joint at the centre. Hence in designing the insulator the shells must be arranged so that they diverge, so as to increase the distance between the shells as the potential difference increases. It should also be noted that owing to the higher permittivity of porcelain (the value of which is 4—6) the greater part of the electrical stress occurs in the air spaces. If the air spaces are of insufficient length corona tends to form, reducing both the arcing distance and the leakage path.

As regards the leakage resistance, this quantity is directly proportional to the length and inversely proportional to the width of the leakage path. Hence after a certain point is reached, increasing the diameter of the petticoats does not appreciably increase the leakage resistance. Increasing the number of petticoats increases the length of path without necessarily increasing the width, and therefore forms the best means of securing a high leakage resistance.

During the last few years the shape of pin-type insulators has undergone a radical change, mainly on the lines suggested by

Gilchrist and Klinefelter.[2] This improved design comprises the following features:—

1. The surfaces of the petticoats or rain sheds conform to the equipotential surfaces of the electrostatic field between the pin and the tie wire or cap.

2. The body of the insulator conforms to the lines of the electrostatic field.

3. The leakage resistance for each shell is about the same value, and the capacitance per shell is also approximately constant.

These improvements have the effect of minimising the formation of corona, and increasing the flash-over voltage and also the mechanical strength of the insulator.

Fig. 40 shows the electrostatic flux distribution between pin

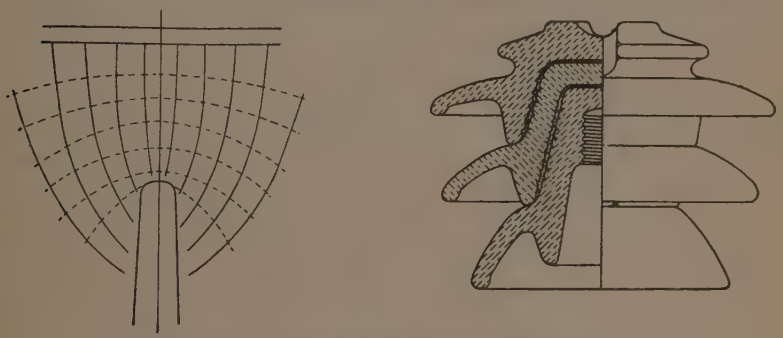

FIG. 40.—Improved design of pin-type insulator.

and cap of a pin-type insulator, and also illustrates the actual design of insulator used on 33 000-volt installations.

77. Suspension-type Insulators.—With the suspension type of insulator, the line conductor is hung below the point of support at the end of a string of insulator units, or discs, connected to each other by metal links. There are three main designs in use at the present time, the Hewlett or interlinking type, the cemented-cap type, and the core-and-tine type.

The Hewlett suspension insulator, Fig. 41 (*a*), was the first type to be successfully developed and is characterised by great simplicity of design. Each disc consists only of one piece of porcelain, the central bulbous portion of which is provided with two curved tunnels lying in planes at right angles to each other. The short steel strips forming the connection between individual discs are threaded through these tunnels and thus loop through

each other, being separated by a layer of porcelain which is wholly in compression. This method of construction gives a high mechanical strength and has the further advantage that should the porcelain between the links be accidentally broken, the links still hold the other units together and thus prevent any interruption to

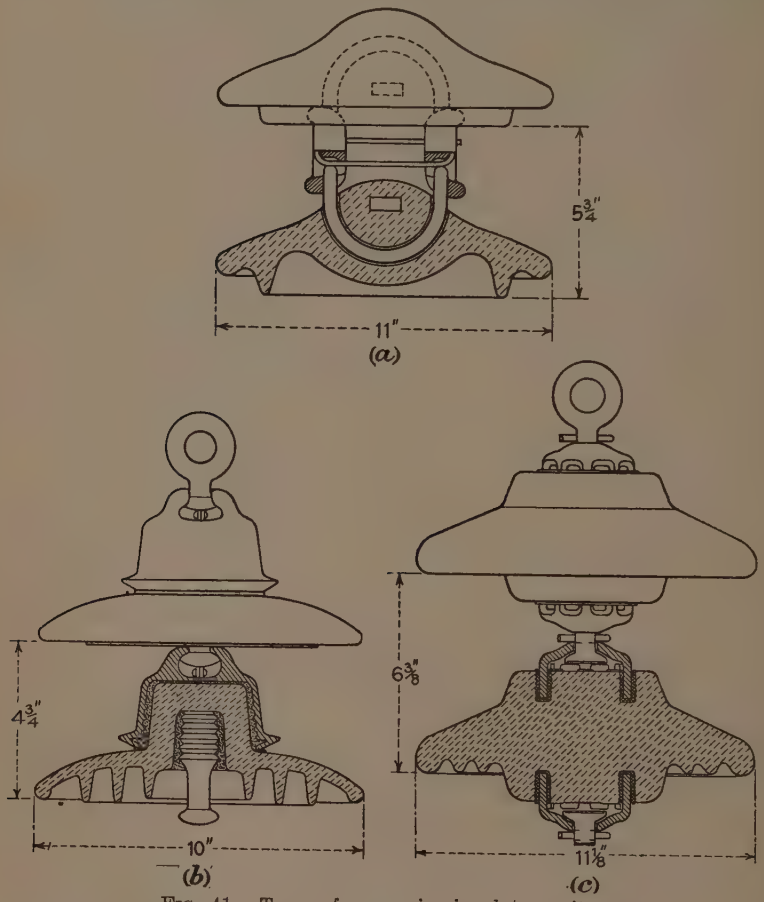

FIG. 41.—Types of suspension-insulator units.

the service. There is also no risk of breakage due to the difference in expansion or contraction of the connecting links and the insulating material. On the other hand, the Hewlett insulator appears to be rather more liable to puncture than other types of suspension insulator, owing to the high electrostatic stress in the material between the links.

In the cemented-cap type of insulator shown in Fig. 41 (*b*), each disc is covered by a metal cap cemented in place, the upper end of each cap terminating in a lug to which the pin of the unit above is fastened. The pins also are firmly cemented in place, and consecutive discs are joined together either by pin-and-clevis or ball-and-socket joints, the porcelain being mainly in tension. A very uniform distribution of the electrostatic stress in the material between the connecting-links is obtained in this type of insulator. The disadvantage of the cemented-cap type hitherto has been the high rate of deterioration,[2] insulators failing in service at rates varying up to 20 % per year. This appears to have been mainly due to the insulator being cemented together very rigidly and no provision being made for expansion. The three materials, porcelain, cement, and steel, have different coefficients of cubical expansion, and the sudden temperature changes occurring in service were sufficient to set up internal stresses which ultimately cracked the porcelain, leading to electrical failure. Furthermore, the cement itself, which is subject to volumetric changes depending on its moisture content, has no doubt often materially assisted in the process of fracturing the insulator. These causes of ultimate failure are now clearly recognised by insulator manufacturers and modern designs show a great improvement so far as reliability of service is concerned.

In the core-and-tine type of insulator manufactured by the Jeffrey-Dewitt Insulator Co., and illustrated in Fig. 41 (*c*), it is claimed that several important advantages exist. Each insulator disc is symmetrical, and conforms as far as possible to the lines of the electrostatic field, thus avoiding placing materials of different permittivities in series. The shape is such that the equipotential planes between the electrodes of the insulator intersect, for equal increments of potential, fairly equal zone widths on the insulator surface. The metal work consists of pressed steel spiders, the legs of which are fastened into the porcelain by an alloy having approximately the same coefficient of cubical expansion as the porcelain. By this method of construction high mechanical stresses on the porcelain, whether due to sudden temperature variations or to the employment of cement, are entirely avoided. The manufacturers state that the difficult problem of producing thick porcelain discs has been solved, and the discs actually used

have a maximum thickness of 4·25 inches, with a correspondingly high puncture strength.

78. Limits of Pin-type Insulators.—Pin-type insulators are available for all pressures up to about 80 000 volts, but generally speaking, the suspension type is preferable on systems operating at above 50 000 volts. Some of the points in favour of the latter type are :—

1. Suspension insulators are usually found to be cheaper than pin-type insulators for pressures over 50 000 volts.

2. Pin-type insulators become very heavy and cumbersome when designed for the higher voltages, and owing to the great length of the supporting pin the bending moment near the point of attachment to the cross-arm tends to become excessive.

3. In cases where the load supplied by the transmission line develops rapidly, it is often found more satisfactory to supply the greater demand by raising the line voltage than to provide another set of conductors. With suspension insulators it is an easy matter to obtain the additional line insulation required, by adding one or more discs to the string.

4. From a mechanical point of view the suspension insulator greatly improves the flexibility of the line. The conductor is secured by a clamp, but the connection at the cross-arm is such that the insulator string is free to swing in any direction, and thus takes up a position where it experiences only a pure tensile stress.

5. The suspension arrangement, when used in conjunction with steel supporting structures, has the advantage of rendering the conductor less liable to be affected by lightning disturbances. At every point of support the wire is hung below the earthed cross-arm, thus enabling the tower to function as a lightning rod.

79. Strain Insulators.—Special mechanically-strong suspension insulators are used on all lines (irrespective of the voltage) where the conductors are dead ended, or at intermediate anchor towers. They are also installed on sharp curves, where the conductors would tend to break the pin of a pin-type insulator, or pull a string of suspension insulators too far from the vertical. On extra long spans, as at river crossings, two or three strings of strain insulators arranged in parallel are often used.

80. Arcing Horns and Rings.—In case of insulator flash-over the porcelain is often cracked or broken up by the power arc

which follows the initial potential discharge. To protect against this trouble, the arcing horns or rings originally introduced by Nicholson[3] are installed on some overhead systems. One of the horns or rings is earthed, and the other is in electrical connection with the line conductor, and the arrangement is such that the arc is taken by the electrodes and held at a sufficient distance from the porcelain to prevent damage by the heat of the arc. The advantage of the ring design lies in the fact that the arc can form at any point around the insulator, and in case of formation on the windward side, it may be blown round without injury to the insulator. With definite terminals, on the other hand, the arc may be blown under the insulator and injure it. It should be noted that the application of these devices lowers the flash-over voltage of the insulator, but this disadvantage is held to be more than counterbalanced by the protective value of the equipment. Fig. 42 shows the arrangement of the horns on a 4-unit string of suspension insulators.

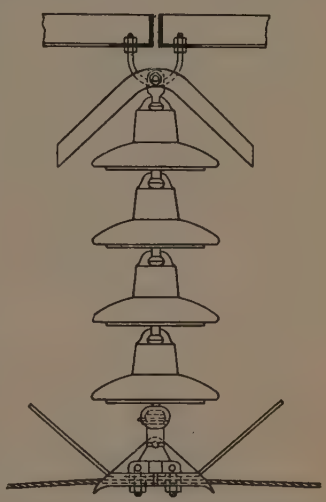

FIG. 42.—Arrangement of arcing horns on suspension insulator.

81. Distribution of Potential over a String of Suspension Insulators.—The voltage impressed on a string of suspension insulators does not distribute itself uniformly across the individual discs owing to the proximity of earth (represented by the supporting tower). The metal-work (cap and connecting-link) of each disc has a capacitance c_2 relative to the metal-work of the next disc, and also a capacitance c_1 relative to earth. Hence the arrangement may be represented by a series of condensers as shown diagrammatically in Fig. 43 for the specific case of a 4-unit string. A greater capacitance current flows through the line disc 1 than through disc 2, hence the voltage across disc 1 is greater than the voltage across disc 2. Similarly the voltage across disc 2 is greater than that across disc 3, and so on, the least share of the total voltage being taken by the cross-arm unit. The inequality of the voltage distribution is accentuated both by an increase in the

number of units in the string, and a decrease in the value of the ratio $\frac{c_2}{c_1}$.

Assuming that $\frac{c_2}{c_1}$ is known, Peek [4] has shown that the voltage stresses on the discs can be calculated by the following formulæ :—

Volts across 1st or line unit,

$$e_1 = \frac{E}{k}\left(\frac{C_n}{c_1} - 1\right).$$

Volts across 2nd unit,

$$e_2 = e_1 - \frac{E - e_1}{k}.$$

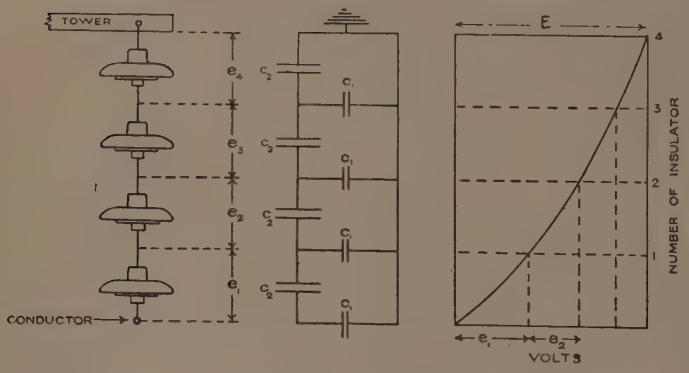

FIG. 43.—Potential distribution on string of suspension insulators.

Volts across mth unit,

$$e_m = e_{m-1} - \frac{\{E - (e_1 + e_2 + e_3 + \ldots e_{m-1})\}}{k} \qquad (110)$$

where E = total voltage across complete string of n units,

$$k = \frac{c_2}{c_1} = \frac{\text{mutual capacitance}}{\text{earth capacitance}},$$

C_n = total capacitance of complete string

$$= c_1 + c_2 - \cfrac{c^2{}_2}{2c_2 + c_1 - \cfrac{c^2{}_2}{2c_2 + c_1 - \cfrac{c^2{}_2}{2c_2 + c_1 - \cfrac{c^2{}_2}{2c_2 + c_1 -}}}}$$

the function to be carried to $(n - 1)$ of the $2c_2 + c_1$ terms.

With long insulator strings the above formulæ are rather tedious to apply, and the voltage distribution can be obtained more conveniently by using hyperbolic functions. In this method, the potential at the cross-arm end of the mth unit is given by

$$v_m = E \frac{\sinh \{(n - m)\sqrt{k}\}}{\sinh (n\sqrt{k})} \quad . \quad . \quad (111)$$

and after having calculated this value for the various units, the actual voltage across any particular unit can be seen at once.

Table 10 shows the voltage distribution over a 10-unit string as calculated by the latter formula, and assuming $k = 16$. It is seen that the line disc takes 22·5 °/₀ of the applied pressure, while the disc at the cross-arm end of the string only takes about 4·2 °/₀.

TABLE 10.—*Voltage Distribution over a 10-Unit Suspension Insulator.*

$k = 16.$ $E = 100.$

No. of Unit.	Potential at Line End of Unit.	Voltage across Unit.
1	100	22·5
2	77·5	17·6
3	59·9	13·8
4	46·1	10·9
5	35·2	8·7
6	26·5	7·1
7	19·4	5·8
8	13·6	5·0
9	8·6	4·4
10	4·2	4·2

Supposing now that the applied pressure is gradually increased, flash-over of the string will occur when the stress across the line disc exceeds the flash-over voltage of a single disc. When this happens, the flash-over voltage of the string

$$E_a = \frac{e_a}{e} E, \quad . \quad . \quad . \quad . \quad (112)$$

where e is the voltage taken by the line unit when a voltage E is applied to the insulator, and e_a is the flash-over voltage of a single unit.

The term 'string efficiency' has been applied by Peek to the ratio

$$\frac{\text{Flash-over voltage of string of } n \text{ units}}{n \text{ times flash-over voltage of one unit}} = \frac{E_a}{ne_a}. \quad (113)$$

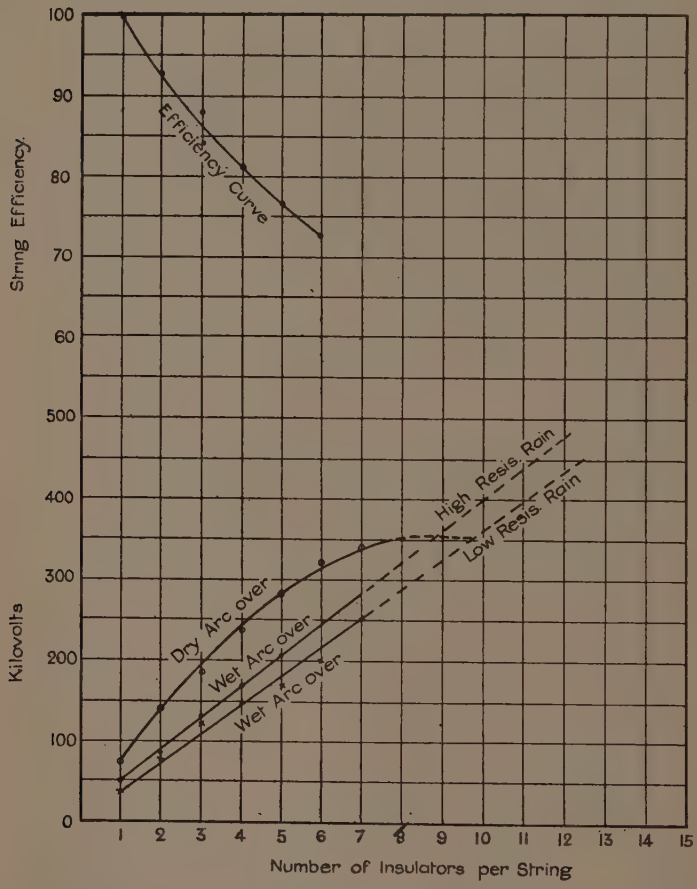

FIG. 44.—Effect of leakage in modifying potential distribution on suspension-insulator string.

In the particular example considered, the string efficiency would be about

$$\frac{100}{22{\cdot}5 \times 10} = 44\ \%.$$

It is thus important when installing suspension-type insulators to consider carefully not only the electrical tests made on individual discs, but also the spacing between the discs, and between the insulator string and the tower. By attention to these details, a short insulator string will often give better results than a much longer series of discs installed with no special regard to the question of high string efficiency.

So far the presence of surface leakage has been ignored, but its effect is to equalise to some extent the voltage drops across the individual units. Fig. 44, taken from actual measurements,[5] exhibits this phenomenon clearly. When the insulator is dry the voltage distribution is unequal, and the string efficiency for varying numbers of discs is shown by the upper curve in the diagram. When the leakage surfaces of the discs are covered with moisture the flash-over voltage is lowered, but the distribution of voltage is equalised completely, the string efficiency under these conditions being unity. This is owing to the heavy leakage current swamping the effect of the capacitance current. For these wet tests the insulator was heavily sprayed with water for a considerable period and the mist formed damped the underside of the discs. When only the top surfaces of the discs are wet, the flash-over curve occupies an intermediate position to the dry and wet curves shown, and the voltage distribution is unequal but not to the same extent as with a dry insulator.

82. Shielding and Grading of Suspension-type Insulators.—It will now be readily recognised that the voltage rating of an insulator string is determined by the electrical stresses on the line disc. With the usual ten-inch discs and spacings employed in practice it is found that for strings of five or more units the line unit takes about one-quarter of the total applied voltage. The voltage distribution is most unequal on insulators of the Hewlett and Jeffrey-Dewitt type, in both of which the line disc takes about 25 to 30 °/₀ of the applied pressure. In the cemented-cap type of insulator the metal cap secures a higher value for k, the line disc taking approximately 20 to 25 °/₀ of the applied pressure.

In order to show clearly how this phenomenon limits the rating of the insulator, the voltage across the line unit has been calculated for strings of 5, 6, . . . 10 units, assuming as before that $k = 16$, and the results are given in Table 11.

TABLE 11.—*Voltage across Line Disc for Various Lengths of Suspension Insulator.*

$k = 16$.

Length of String.	Percentage Voltage across Line Disc.
5-unit	26·6 %
6-unit	24·8 %
7-unit	23·7 %
8-unit	23·1 %
9-unit	22·7 %
10-unit	22·5 %

Supposing now that the safe operating limit for a single disc is 20 000 volts, the voltage rating for a string of five units is

$$20\,000 \times \frac{100}{26{\cdot}6} = 75\,200 \text{ volts},$$

and for a string of ten units is

$$20\,000 \times \frac{100}{22{\cdot}5} = 88\,900 \text{ volts}.$$

Hence, increasing the number of discs by 100 % only results in an 18·2 % increase of voltage rating. Furthermore, even with a 10-unit string the maximum line pressure is

$$88\,900 \times \sqrt{3} = 154\,000 \text{ volts}.$$

With the advent of very much higher line pressures the problem of equalising the electrical stresses has become of considerable importance. Various methods have been proposed for this purpose such as:—

1. Increasing the c_2 capacitances without increasing the c_1 capacitances so that the effect of the earth current is relatively less. This may be done by fitting sheet-metal caps or rings in intimate contact with the porcelain.

2. Increasing the c_2 capacitances of the discs on the line half of the string by using larger units.

3. Increasing the c_2 capacitances of the insulator along the string in proportion to the current flowing through the unit. To effect this, metal caps or rings are employed, the size of these being varied for the different units so as to give the greatest c_2 capacitance for the line unit, a less c_2 capacitance for unit 2, and so on. This is generally known as grading.

4. Elimination of the earth capacitances by means of an antenna shield from the line conductor. This is called shielding, and Fig. 45 illustrates the principle of action of the device.

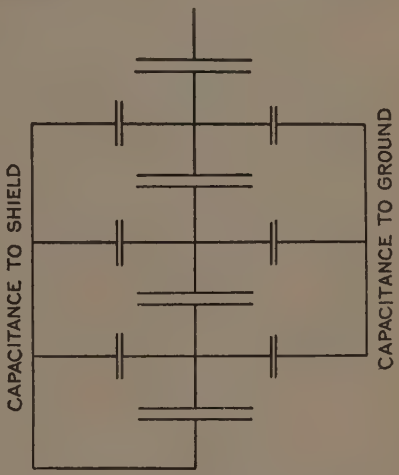

FIG. 45.—Principle of action of antenna shield.

Good results can be obtained by all these methods, but method 4 has the special advantage of not requiring different types or kinds of insulator discs to be installed, and has been adopted on the 220 000-volt system of the Southern California Edison Co.[6] The antenna shield in this case consists of a metal ring 28 inches outside diameter, surrounding the line unit and electrically connected to the conductor. At the cross-arm end of the string, arcing horns or a light ring is fitted, so that in the event of flash-over the arc is held between

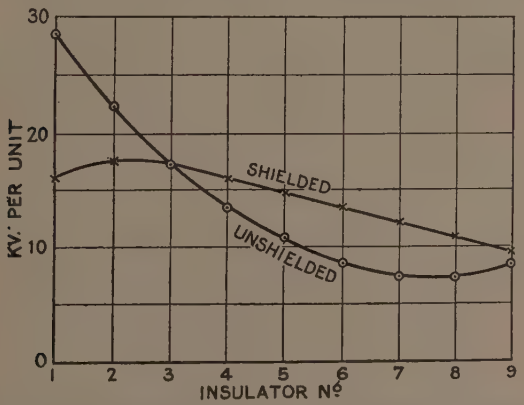

FIG. 46.—Effect of grading on a 9-unit suspension insulator.

the horn and the shield ring and prevented from injuring any of the discs. The curves in Fig. 46 show the effect of the shield ring in equalising the voltage distribution over a 9-unit string of

cemented-cap type units. Contrary to what might be expected, the flash-over voltage of an insulator string instead of being increased by shielding is usually slightly decreased, but not to a serious extent.

83. Recent Developments in the Design of Suspension-type Insulators.—The advance in transmission line voltages during the last thirty years has been continuous, and there seems no reason to doubt that before long pressures of 300 000 or 400 000 volts will be used for the transmission of very large amounts of power. The length and cost of a conventional type suspension insulator designed for pressures of this magnitude would be prohibitive, and there is great need of an insulator which would be better adapted to increasing line voltages.

Recent research work by Smith[7] has resulted in the development of a suspension-type insulator of simple construction, and possessing greatly improved electrical and mechanical characteristics. The form finally adopted for a series of insulators which are being tried out under actual service conditions is shown in Fig. 47. The mechanical strain member consists of a special impregnated - wood spindle about 2 inches in diameter. Metallic flux-distributing terminals surround the spindle at each end so that the electrostatic field produced in the neighbourhood of the spindle conforms closely to the surface of the latter. The upper flux terminal which is a ring of about 45 inches diameter runs round the edge of a hood or water shed.

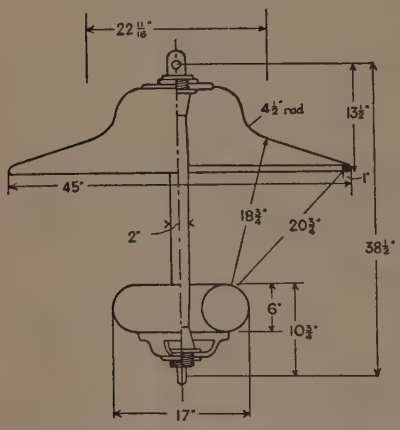

FIG. 47.—Experimental design for 110-kV. suspension insulator.

The flash-over voltage of the insulator has a value of 280 kV. or over, dry, and 200 kV. or over, wet, under standard wet-spray test. It is designed for use on 110 000 volt lines, two units in series would be sufficient for 220 000 volt lines, and when necessary three units in series on 330 000 volt lines. No corona is visible on any point of the insulator until within about 5 % below

break-down voltage. Owing to the absence of arcing conditions on the surface of the insulating member the use of treated-wood spindles can be permitted, and it is claimed that the mechanical properties of the latter are far superior to any porcelain yet available.

84. **High-frequency Effects.**—When an insulator, either of the pin or suspension type, is subjected to the action of high-frequency potentials, the electrical stresses are usually of a different character than when the potential is applied at the ordinary line frequency. The distribution of potential among any arrangement of pure capacitances is independent of frequency, but an insulator besides consisting of many capacitances may be considered to have a certain amount of leakage path in series thereto. Even if this leakage path can be considered non-inductive, the distribution of potential across such an arrangement is not the same at high as at low frequencies. This is probably one of the chief reasons why insulators, that will flash-over rather than puncture on tests conducted at the line frequency, will sometimes fail through puncture during atmospheric electrical disturbances. Another reason for this phenomenon is to be found in the fact that at high frequencies corona has no time to form and relieve the stress by rupture of the air.

In considering high-frequency effects care should be taken to differentiate between :—

1. Continuous high-frequency oscillations.
2. Damped high-frequency oscillations.
3. Impulses with steep wave front.

1. Continuous or undamped high-frequency oscillations such as might be obtained from an Alexanderson alternator, when applied to an insulator cause high dielectric losses and heating, resulting in failure at comparatively low voltages. This is a condition that does not occur in the case of transmission lines.

2. Damped high-frequency oscillations occur frequently on transmission lines due to switching operations or arcing grounds.

3. Impulses are often produced in overhead systems by a lightning discharge in the proximity of the line. The voltage induced thereby, travels over the line as a disturbance with a steep wave front. At one instant an insulator may be subjected to the normal voltage, and a millionth of a second later, to the normal voltage plus or minus the induced voltage. If the voltage

impressed on the insulator due to the impulse rises from zero to a maximum value in t seconds, the impulse may be said to have an equivalent frequency of $\frac{1}{4t}$ cycles per second.

It is found that both with impulses and single trains of damped high-frequency oscillations, a much higher voltage can be impressed on the insulator than the 50-cycle flash-over voltage. This is owing to the fact that the disruptive discharge through a dielectric requires not only a sufficiently high voltage but also a definite minimum amount of energy. Thus, on applying voltage rapidly to an insulator as by an impulse, flash-over does not occur when the continuously-applied break-down voltage is reached. The voltage overshoots the mark for such a time as will enable this amount of energy to be supplied to the dielectric, and rupture then takes place. The excess voltage is greater, the greater the rate of application. Also for a given rate of increase of pressure, the rupturing voltage is greater, the greater the ununiformity of the dielectric field. With a parallel-plate condenser, or spheres of large diameter compared with their distance apart, the impulse-frequency flash-over voltage is practically equal to the 50-cycle value. For ununiform fields, such as exist between needle points, the impulse-frequency flash-over voltage may be two or three times the 50-cycle value. Peek[8] has applied the term 'impulse ratio' to the ratio of the flash-over voltage at impulse frequency to that at commercial frequency. For insulators the ratio is of the order of 1·2 to 1·6 for the suspension type, and 1·3 to 2·3 for the pin type.

85. Causes of Insulator Failure.—The chief causes of insulators failing in service are:—

1. Deterioration by cracking of the porcelain.
2. Porosity.
3. Puncture of weak porcelain.
4. Shattering by power arcs.
5. Flash-over caused by dust deposits.
6. Failure from mechanical stresses.
7. Short-circuits by birds and similar objects.

1. Failure by cracking of the insulator after a certain period of service has given great trouble both with pin-type insulators and the cemented-cap suspension type. The continued action of alternate and sometimes rapidly varying heat and cold, dryness and

dampness, on the complete assembly of porcelain, steel, and cement, produces high stresses on the porcelain and ultimate fracture. This may be due to the differential expansion of the component parts which have different coefficients of cubical expansion, or to volume changes in the cement produced by hydration. Improvements have been made in later designs of insulators by providing for the relative expansion of the different materials. In some cases elastic cushions have been used between the shells, and the steel pins fixed into the insulator by means of lead thimbles or tarred-hemp wrappings.

2. Porosity in the porcelain, due to under-firing or other causes, always leads to failure after a comparatively short period of service. The porous material absorbs water, either from the atmosphere or the cement, thus decreasing its electrical resistance. The leakage current which then flows through the porcelain tends to heat up a localised portion to a high temperature. This local heat may then cause mechanical failure followed by the passage of a power arc through the insulator shell, or under certain circumstances the leakage current may gradually increase with increasing temperature until the porcelain is punctured. In this connection it should be noted that good glazing may postpone the deterioration of porous porcelain but it cannot eliminate it.

3. Puncture of insulators occurs occasionally due to improperly vitrified material. Severe routine electrical tests during the course of manufacture are the best form of insurance against this trouble.

4. The shattering of insulators by the intense heat of the power arc which usually follows a lightning flash-over is one of the most common interruptions to service. Arcing horns or rings are satisfactorily employed on many systems to reduce this particular operating hazard.

5. Flash-over caused by salt, cement-dust, or soot deposits may give trouble on localised line sections. These deposits are partially conducting and produce a considerable reduction in the flash-over voltage of the insulator, particularly, when in addition to the dust deposits on the interior surfaces, the exterior surfaces of the shells are wet. It is often recommended to install larger insulators on line sections where the atmospheric conditions are bad, but this is only a palliative measure, and trouble due to this cause can only be eliminated by periodically cleaning the insulators.

6. Failure of insulators by mechanical stresses which cause

actual pulling apart is very rare as defective material is usually weeded out by routine factory tests.

7. Short-circuiting of conductors by large birds or similar objects gives trouble in certain localities. This can often be prevented by increasing the spacing of conductors and using suspension, instead of pin-type, insulators.

86. Safety Factors.—Insulators are generally so designed that the wet and dry flash-over voltages are several times the

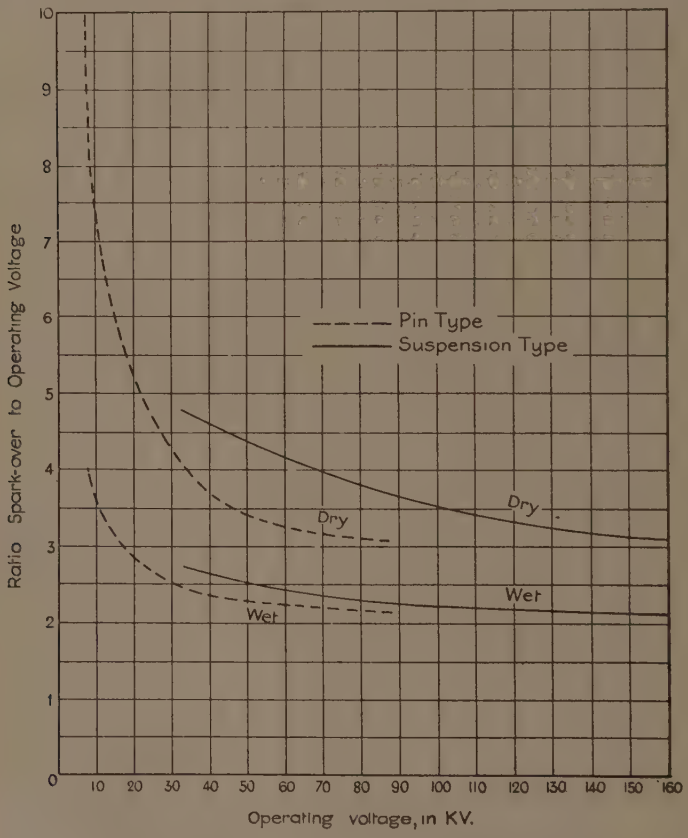

FIG. 48.—Typical safety factors for insulators.

operating voltage of the line, so as to make provision for abnormal pressure rises. The ratio between the flash-over voltage of the insulator at normal frequency, and the operating voltage is known as the safety factor. Average values of this factor for pin- and

suspension-type insulators are exhibited by the curves of Fig. 48. It will be noticed that the safety factor is far higher on the low-voltage lines. This is as it should be, seeing that all overhead lines in a given locality are subject to virtually the same pressure rises due to lightning. The high-voltage lines are thus relatively less affected. In any case it does not appear desirable to use safety factors less than 2, as low-frequency surges of double voltage may appear in the line caused by normal switching operations.

The severest electrical stresses on insulators are caused by steep wave front impulses and other high-frequency potentials produced by lightning disturbances. Fortunately, the impulse flash-over voltage of an insulator is always higher than the 50-cycle flash-over voltage, and has the important advantage of not being lowered by rain or moisture. Hence the protective value of an insulator against high-frequency disturbances is greater than indicated by the safety factor as above defined. The puncturing power of high-frequency impulses is, however, greater than that of 50-cycle potentials, and so, beside a high impulse ratio, insulators should possess a correspondingly high dielectric strength.

The flash-over voltage of insulators is lowered by a decrease of air density produced by atmospheric temperature or pressure changes. In a uniform dielectric field the reduction of voltage would be strictly proportional to the decrease in the air density, but with the non-uniform dielectric field of an insulator the reduction depends somewhat upon the particular type and design. In any case, however, the departure from the theoretical relation for uniform dielectric fields is small, and the correction for atmospheric conditions can safely be made by assuming the flash-over voltage to be directly proportional to the air density. For instance, if the flash-over voltage of a pin-type insulator is found to be 100 kilovolts at sea level (barometric pressure 29·9 inches), then at an elevation of 5 000 feet (barometric pressure 24·6 inches) it would be liable to flash-over at a pressure of

$$100 \times \frac{24{\cdot}6}{29{\cdot}9}$$
$$= 82{\cdot}3 \quad \text{kilovolts.}$$

Due allowance must be made for this reduction of flash-over voltage when selecting insulators for lines erected at a high altitude.

87. Testing of Insulators.—In the course of manufacture insulator tests are made for one of two purposes:—

1. Design tests to ascertain the electrical and mechanical performance of a particular type of insulator.

2. Routine tests to weed out defective insulators.

These two categories are indicated by the letters 'D.' and 'R.' respectively in the list given below. At present there is a lack of uniformity among manufacturers both as to the tests employed and the precise methods of carrying out the same. Some or all of the following tests are used:—

50-*cycle Dry Flash-over Voltage* (*D. and R.*).—The dry flash-over voltage is determined by applying a gradually increasing voltage between the electrodes of the insulator until the surrounding air breaks down with formation of a sustained arc. As a routine test, vigorous flash-over voltage is impressed for two or three minutes on all shells before assembly, and afterwards on the complete insulator.

50-*cycle Wet Flash-over Voltage* (*D.*).—This is determined by gradually applying voltage between the electrodes of the insulator, the latter being sprayed with water. The standard precipitation conditions are 0·2 inches per minute at an angle of 45 degrees, the water having a resistance at 15° C. of less than 20 000 ohms per centimetre cube.

Puncture Voltage (*D. and R.*).—For this test the insulator is totally immersed in oil, the voltage being applied gradually until puncture occurs. In a routine test, the average puncture strength of each batch of insulators is estimated by selecting a small number of units (about 5 per thousand) out of those which have passed the flash-over voltage test successfully, and carrying them to destruction under oil.

Impulse Ratio (*D.*).—To obtain the flash-over voltage at impulse frequency, and hence the impulse ratio, the lightning generator developed by Peek is employed. By means of this apparatus a steep wave front voltage having an equivalent frequency of several hundred kilocycles per second can be impressed on the insulator, the voltage being measured by a sphere gap.

High-frequency Oscillator (*R.*).—This is a valuable method of testing introduced by Creighton,[9] in which a potential having a frequency of the order of 200 kilocycles per second is impressed

on the insulator. The oscillator itself consists of a resonating circuit with a spark gap, which produces a damped high-frequency wave train every time the 50-cycle supply voltage passes through its maximum value. Due to time lag, a voltage about 30 % higher than the 50-cycle flash-over voltage can be applied to the insulators, and the defective units are rapidly picked out. Furthermore, the discharge tends to spread over the insulator surfaces and may thus detect flaws in the petticoats more readily than the 50-cycle test. The high-frequency test has also a logical basis inasmuch as the electrical stresses due to lightning are of a similar high-frequency nature. Owing, however, to the difficulty of standardisation, it does not appear so suitable for a design test as the test at operating frequency.

Mechanical Strength (*D. and R.*).—To ascertain the ultimate mechanical strength of a pin-type insulator it is mounted on a steel pin, and a gradually increasing load applied to the tie-wire groove by means of a steel bridle until failure occurs. Suspension insulators are tested in pure tension. Routine mechanical tests are usually made by subjecting all insulators to, say, 50 % of their rated ultimate strength. In addition about 5 out of every thousand units are tested to destruction.

Porosity (*R.*)—The best method of determining the degree of vitrification of the porcelain is by the fuchsine penetration test. Samples of every batch of recently-fired insulators are broken into pieces, and immersed under pressure in a solution of fuchsine dye in alcohol. After a sufficient period of time has elapsed the specimens are removed from the testing vessel and examined. The slightest degree of porosity is indicated by a deep penetration of the dye into the body of the specimen.

The actual conditions under which the various insulator tests are carried out has an important influence on the results obtained, and it appears highly desirable to standardise such conditions. Efforts are being made to accomplish this, and so facilitate comparison between the products of different manufacturers. In Great Britain a most practical and concise specification on high-tension insulators has been published by the British Engineering Standards Association.[10]

88. Formation of Corona.—When the pressure between the conductors of an overhead transmission system exceeds a certain definite value—the visual critical voltage of the line—a hissing

noise is heard, and the conductors are found to be surrounded by a luminous envelope to which the name of corona has been applied. The luminous envelope is composed of air which has broken down and become temporarily conducting under the high electrostatic stress, and its effect is equivalent to increasing the diameter of the conductors. The break-down starts first near the surface of the conductor, as the electrostatic stress or potential gradient has its maximum value there, and the thickness of the conducting layer of air increases with increase of pressure. If the conductors are very close together (the interaxial distance being less than fifteen times the diameter of the conductors) the formation of corona involves an increase in the potential gradient between them; the corona spreads farther, and so on, until flash-over occurs. In this case immediate disruptive discharge takes place, and no stable corona can be formed. With the wide spacings adopted for commercial lines, the formation of corona has the precisely opposite effect of decreasing the stress between the conductors. The corona extends until the increase in the effective diameter of the conductor is sufficient to bring the potential gradient at the edge of the corona down to the disruptive gradient of air, and the corona can then spread no farther at the particular voltage under consideration.

Corona is accompanied by a loss of energy which increases very rapidly once the visual critical voltage is exceeded. This energy is dissipated in the form of light, heat, sound, and chemical action.

89. Dielectric Strength of Air and Disruptive Critical Voltage.—The potential gradient at which complete disruption of a dielectric occurs, is called the disruptive strength or dielectric strength of the material. With air at a barometric pressure of 76 cm., and temperature of 25° C., it is $g_0 = 30$ kilovolts per centimetre. The dielectric strength of air is proportional to its density over a very wide range, and thus directly proportional to the barometric pressure, and inversely proportional to the absolute temperature. Taking the density of air under the above standard pressure and temperature conditions as being equal to unity, the relative density at a pressure of b cm., and temperature of t' C., is

$$\delta = \frac{3{\cdot}92b}{273 + t'}$$

and the dielectric strength under these conditions is $g_0\delta$.

In a uniform dielectric field such as exists between parallel

plates the potential gradient has a constant value at all points. If the voltage is gradually increased, as soon as the break-down gradient of 30 kilovolts per centimetre is attained (assuming standard pressure and temperature conditions), the air breaks down, and flash-over occurs short-circuiting the plates. If the dielectric field is not uniform, as for instance the field between two parallel wires or two spheres, then with increasing voltage the break-down gradient will not be reached simultaneously throughout the entire field, but is first attained at the surface of the wires or spheres.

In the case of parallel wires it has been shown that the potential gradient at the surface of the conductors is given by

$$g_{max} = \frac{E}{r \log_e \frac{d}{r}}, \quad . \quad . \quad . \quad (21)$$

where E is the voltage to neutral, both g_{max} and E being expressed in R.M.S. values.

When the disruptive gradient of air is reached at the conductor surface

$$g_0 = \frac{E_0}{r \log_e \frac{d}{r}}$$

or

$$E_0 = g_0 r \log_e \frac{d}{r},$$

E_0 being known as the disruptive critical voltage.

In practice, corrections have to be applied to this formula for air density and surface conditions of the conductor. Noting also that for sine waves the R.M.S. value of g_0 is 21·1 kilovolts per centimetre, the complete formula is

$$E_0 = 21{\cdot}1 m_0 r \delta \log_e \frac{d}{r} \text{ kV. to neutral (R.M.S)}, \quad . \quad (114)$$

where

δ = air density factor (1 at 76 cm. barometer, 25° C.).

$$= \frac{3{\cdot}92b}{273 + t}$$

b = barometric pressure in centimetres.

t = temperature in degrees C.

r = radius of conductor in centimetres.

d = spacing between centres of conductors in centimetres.
m_0 = 1 for polished wires.
= 0·98 – 0·93 for roughened or weathered wires.
= 0·87 – 0·80 for stranded conductors (concentric lay).

In the case of stranded conductors the variation in m_0 is due to the number of strands employed, and the varying ratio between the strand diameter and overall diameter in different sizes of conductors. It also depends on the surface conditions of the conductor, often improving after a conductor has been in service for a time, and become weathered and the roughness and abrasions oxidised away. Any mutilation of the conductor during construction or stringing lowers m_0 considerably, but in the absence of any special features 0·85 seems a safe value to use in calculations.

The value of E_0 as given above is the disruptive critical voltage for fair-weather conditions. Bad atmospheric conditions, such as fog, sleet, rain, and snow-storms, lower E_0 considerably and increase the corona losses.

90. Visual Critical Voltage.—In the case of parallel wires it is found that visual corona does not begin at the voltage E_0 at which the disruptive gradient of air g_0 is reached, but at a higher voltage E_v called the visual critical voltage. As a result, when corona begins, the potential gradient g_v at the conductor surface is higher than the disruptive gradient g_0. Contrary to what might be expected, g_v, the apparent strength of air, is not a constant but depends on the size of the conductors, air being apparently stronger at the surface of small conductors than large ones. Peek[11] has shown that the relation between the apparent strength of air and the radius of the conductor can be expressed by the formula

$$g_v = g_0\left(1 + \frac{0{\cdot}3}{\sqrt{r}}\right).$$

Since we have also

$$g_v = \frac{E_v}{r \log_e \frac{d}{r}},$$

it follows that

$$g_0 = \frac{E_v}{(r + 0{\cdot}3\sqrt{r}) \log_e \frac{d}{r}}.$$

Comparing this with formula (20) it is evident that g_0 is the gradient $0{\cdot}3\sqrt{r}$ centimetres from the surface. Hence the stress

at the conductor surface when corona appears is not the same for all diameters, as already stated, but is always constant and equal to g_0 at a distance $0{\cdot}3\sqrt{r}$ centimetres from the surface. This is shown diagrammatically in Fig. 49.

According to Peek, this phenomenon is due to the fact that dielectric break-down of air requires a finite volume of over-stressed air. In other words, a finite amount of dielectric energy is necessary to cause rupture or start corona. Hence the potential gradient at the conductor surface must exceed the elastic limit g_0, or be increased to g_v, in order to supply the necessary rupturing energy between the conductor surface and the finite radial distance in space away ($0{\cdot}3\sqrt{r}$ cm.) where the stress is g_0 and break-down occurs.

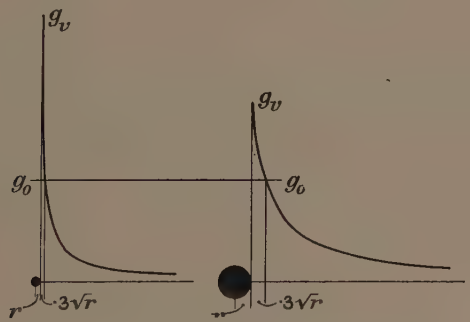

FIG. 49.—'g_v' and 'g_0' for small and large wires.

The equation for the visual critical voltage may now be written

$$E_v = g_v r \log_e \frac{d}{r}$$

$$= g_0\left(1 + \frac{0{\cdot}3}{\sqrt{r}}\right) r \log_e \frac{d}{r},$$

where E_v is the R.M.S. value in kilovolts to neutral, g_0 having the value of 21·1.

If the energy-distance theory is true, g_v, unlike g_0, should not vary directly with the relative air density δ, and as a matter of fact it is found by experiment that

$$g_v = g_0\delta\left(1 + \frac{0{\cdot}3}{\sqrt{\delta r}}\right).$$

The visual critical corona point is quite sharp and definite for wires, but not so in the case of stranded conductors. With the latter, corona, after it first appears, increases gradually for a considerable range of voltage until a certain definite voltage is reached, after which the increase is very rapid. The first point is called the local corona point, and the second the decided corona point.

The complete formula for the visual critical voltage is thus

$$E_v = 21{\cdot}1 m_v \delta r \left(1 + \frac{0{\cdot}3}{\sqrt{\delta r}}\right) \log_e \frac{d}{r} \text{ kV. to neutral (R.M.S.)} \quad (115)$$

where

$m_v = m_0$ for wires $(1 - 0{\cdot}93)$,
$= 0{\cdot}72$ for local corona on stranded conductors,
$= 0{\cdot}82$ for decided corona on stranded conductors,

the other constants being expressed as in formula (114).

91. Corona Power Loss.—The formula for loss of power in fair weather as derived from experimental data by Peek [12] is

$$P_c = \frac{390}{\delta}(f + 25)\sqrt{\frac{r}{d}}(E - E_0)^2 10^{-5} \text{ kW. per mile per phase,} \quad (116)$$

where E = R.M.S. kilovolts to neutral,
f = frequency in cycles per second,

the other constants having their former values.

The approximate loss under storm conditions is obtained by taking E_0 as 0·8 times its fair-weather value.

As a matter of fact, with perfectly smooth and cylindrical conductors no loss occurs until the visual critical voltage is reached, when the loss suddenly takes a definite value equal to that calculated by the quadratic law with E_v as the applied voltage. It then follows the quadratic law for all higher voltages. With the roughened and weathered conductors used in practice, however, the quadratic law is approximately followed over the whole range of voltage starting at E_0.

92. Practical Corona Formulæ and Their Application.—The formulæ required for the determination of the corona characteristics of transmission lines expressed in terms of inch units and common logarithms are:—

Disruptive Critical Voltage,

$$E_0 = 123 m_0 r \delta \log_{10} \frac{d}{r} \text{ kV. to neutral (R.M.S.).} \quad (117)$$

Visual Critical Voltage,

$$E_v = 123 m_v \delta r \left(1 + \frac{0{\cdot}189}{\sqrt{\delta r}}\right) \log_{10} \frac{d}{r} \text{ kV. to neutral (R.M.S.)} \quad (118)$$

Power Loss,

$$P_c = \frac{390}{\delta}(f + 25)\sqrt{\frac{r}{d}}(E - E_0)^2 10^{-5} \text{ kW. per mile per phase} \quad (119)$$

where

E = R.M.S. kilovolts to neutral.
δ = air density factor (1 at 29·9 in. barometer, 77° F).
$= \frac{17{\cdot}9b}{459 + t}$.
b = barometric pressure in inches.
t = temperature in degrees F.
r = radius of conductor in inches.
d = spacing between conductor centres in inches.
f = frequency in cycles per second.

For approximating storm loss consider E_0 = 0·8 of its fair-weather value.

m_0 = 1 for polished wires,
= 0·98 – 0·93 for roughened or weathered wires,
= 0·87 – 0·80 for stranded conductors (concentric lay).
$m_v = m_0$ for wires (1 – 0·93),
= 0·72 for local corona on stranded conductors,
= 0·82 for decided corona on stranded conductors.

In order to illustrate the use of the formulæ the following example will be taken:—

Problem 5.—Find the corona characteristics of a three-phase line 100 miles long, consisting of three 0·10 sq. in., stranded copper conductors, spaced in an eight-foot delta arrangement. Temperature taken as 80° F. Altitude of line 1 000 feet, corresponding to an approximate barometric pressure of 28·8 inches. Frequency 50 cycles.

From wire tables the diameter of the conductor is found to be 0·408 inches.

Then

$$\frac{d}{r} = \frac{96}{0{\cdot}204} = 470{\cdot}6$$

$$\log_{10}\frac{d}{r} = 2{\cdot}673$$

$$\sqrt{\frac{r}{d}} = 0{\cdot}0461$$

$$\delta = \frac{17{\cdot}9 \times 28{\cdot}8}{459 + 80} = 0{\cdot}956$$

$$m_0 = 0{\cdot}85$$

$$m_v = 0{\cdot}72 \quad \text{or} \quad 0{\cdot}82$$

$$1 + \frac{0{\cdot}189}{\sqrt{\delta r}} = 1 + \frac{0{\cdot}189}{\sqrt{0{\cdot}956 \times 0{\cdot}204}} = 1{\cdot}428.$$

The disruptive critical voltage is

$$E_0 = 123 m_0 r \delta \log_{10}\frac{d}{r} \qquad (117)$$

$$= 123 \times 0{\cdot}85 \times 0{\cdot}204 \times 0{\cdot}956 \times 2{\cdot}673$$
$$= 54{\cdot}5 \quad \text{kV. to neutral.}$$

Visual corona first appears at

$$E_v = 123\, m_v \delta r\left(1 + \frac{0{\cdot}189}{\sqrt{\delta r}}\right)\log_{10}\frac{d}{r} \qquad (118)$$

$$= 123 \times 0{\cdot}72 \times 0{\cdot}956 \times 0{\cdot}204 \times 1{\cdot}428 \times 2{\cdot}673$$
$$= 65{\cdot}9 \quad \text{kV. to neutral,}$$

and more decidedly at

$$E_v = 123 \times 0{\cdot}82 \times 0{\cdot}956 \times 0{\cdot}204 \times 1{\cdot}428 \times 2{\cdot}673$$
$$= 75{\cdot}1 \quad \text{kV. to neutral.}$$

Supposing now that the line is operated at 110 000 volts (63·5 kV. to neutral) the fair-weather power loss is

$$P_c = \frac{390}{\delta}(f + 25)\sqrt{\frac{r}{d}}(E - E_0)^2\, 10^{-5} \qquad (119)$$

$$= \frac{390}{0{\cdot}956} \times 75 \times 0{\cdot}0461(63{\cdot}5 - 54{\cdot}5)^2\, 10^{-5}$$
$$= 1{\cdot}14 \quad \text{kW. per mile per phase,}$$

and the approximate storm loss is

$$P_c = \frac{390}{\delta}(f + 25)\sqrt{\frac{r}{d}}(E - 0{\cdot}8E_0)^2\ 10^{-5}$$

$$= \frac{390}{0{\cdot}956} \times 75 \times 0{\cdot}0461(63{\cdot}5 - 43{\cdot}6)^2\ 10^{-5}$$

$$= 5{\cdot}59 \quad \text{kW. per mile per phase.}$$

In actual practice, the voltage, and therefore the corona loss, varies at different parts of the line. In many cases also, the altitude of the line varies considerably at different points. These effects can be allowed for in a long line by calculating the loss per mile at a number of points. A curve is then plotted with the loss per mile as ordinates, and length of line as abscissæ; the area of the curve representing the total loss on the line.

When the line conductors are not spaced in an equilateral triangle, but as is often the case, unsymmetrically in a plane, corona will start at a lower voltage on the centre conductor where the stress is greatest than on the outside conductors. The exact calculation then becomes more complicated, but, considering the other variables in the problem such as altitude or atmospheric conditions, the use of a mean value $d = \sqrt[3]{abc}$ will usually give results of sufficient accuracy.[13]

93. Voltage Limitations of Line.—The critical voltage limit of a line can be raised by increasing either the spacing or the size of the conductors, but the latter method is preferable as the spacing must be kept down to a minimum value in order to save tower costs, and avoid excessive reactance drop in the line. For increasing the size of the conductors, stranded conductors with hemp centres have occasionally been employed, but have not proved satisfactory from a mechanical point of view owing to the hemp deteriorating rapidly. Steel-cored aluminium conductors have a much greater diameter than copper ones of the same conductivity, and this consideration often leads to the choice of steel-cored aluminium for systems operating near the corona limit.

In a new and improved conductor construction introduced by the Anaconda Copper Mining Co., one or more layers of copper strands are spiralled round a core of twisted copper I-beam in shape. Thus strength is added to the hollow conductor without the addition of dead weight or sacrifice of conductivity or durability.

In general, it is not advisable to operate a line above its fair-weather disruptive critical voltage E_0 (determined for 25° C. and average barometric conditions). If the operating voltage happens to be just below this value the corona losses in fine weather will be negligible. They may, however, have a fairly high value under storm conditions, but, since storms are only experienced at intervals in most districts, it is usually more economical to pay for these losses for small parts of the year than try to eliminate them absolutely by using heavy conductors.

94. Triple-harmonic Currents Due to Corona.—The practice of designing the line so as to avoid corona formation under normal conditions is also desirable in order to eliminate triple-harmonic voltages or currents which may be introduced in the system thereby. The formation of corona starts at a point on the ascending portion of the voltage wave, and disappears when the wave has passed through a maximum and reached the same value on the descending portion. This occurs every half-cycle, and produces a pulsation in the voltage wave having three times the generator frequency. In an earthed system this triple-frequency voltage causes a triple-frequency current to flow through the capacitance of the system to earth and back through the earthed neutral. The effect is accentuated by the fact that the capacitance of each conductor is not constant but also pulsates at triple-frequency due to the increase and decrease of effective conductor diameter caused by the corona. In tests made by Lewis [14] the magnitude of the triple harmonics found were of the order of 40 %, with neutral earthed at one end of the line. In such a case the capacitance current in the conductors, and the rise in pressure along the line at times of light load, may be much greater than their calculated values.

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CHAPTER VIII.

VOLTAGE CONTROL BY SYNCHRONOUS PHASE MODIFIERS.

95. Importance of Voltage Control.—One of the most troublesome features associated with the operation of overhead transmission systems is the inherent variation of voltage at the receiving end, due to changes in the load. These fluctuations of voltage must be prevented from reaching the distributing network for the following reasons:—

1. In the case of a lighting load, the lamp characteristics are very sensitive to changes of pressure. For example, a 5 °/₀ decrease in the applied voltage results in a decrease of 15-20 °/₀ in the illuminating power of metallic-filament lamps. On the other hand, if the voltage is 5 °/₀ above the correct value, the lamps deteriorate at a rapid rate, their life being shortened by about 60 °/₀.

2. When a power load consisting mainly of induction motors is supplied, rather larger variations may be permitted than on a lighting circuit, and in order to take advantage of this it is customary to supply power loads and lighting loads from two distinct local circuits. Even in this case, however, it is distinctly undesirable to have more than a small percentage voltage variation. For, if the voltage is above normal, the motor operates with a saturated magnetic circuit, with consequent large magnetising current and heating and lower power factor. If the voltage, on the other hand, is low, its effect is to very considerably reduce both the starting torque and pull-out torque.

3. If the exciting voltage impressed on the distributing transformers is carried above normal, the exciting currents and core losses begin to increase rapidly, and excessive heating is thereby occasioned. Also, due to saturation of the iron, very prominent third harmonics are introduced either in the voltage or current.

96. Methods of Voltage Control.—In many systems the method adopted to deliver a steady voltage at the consumer's

terminals is to change the voltage impressed on the line at the sending end *pari passu* with the load. The line is designed so that the regulation or inherent voltage variation at the receiving end, from no load to full lagging load and including the effect of raising and lowering transformers, is kept to a value not exceeding, say, 20 to 25 %. The voltage at the sending end is then altered from hour to hour, so as to keep the fluctuations of the receiving-end voltage quite small, say, within 5 %. Finally, the voltage in the distributing network is kept steady by using potential regulators, or boosting transformers, on the feeders of the distributing system. These potential regulators consist of hand- or automatically-controlled variable-ratio transformers connected with their primary in shunt with the feeder and their secondary in series therewith, so that they add or subtract a certain low voltage to the feeder voltage. They may be of the static-transformer type with tappings taken out to multiple-contact switches, but are preferably of the inductor-regulator type. The main function of these boosting transformers is to compensate for the pressure drop in the feeders, but they may be designed to take care of a small variation of receiving-end line voltage also. If, however, the line voltage at the receiving end has too great a variation for the capacity of the boosting transformers installed, there must be variation in the voltage supplied to the consumers.

It is easily seen that the above method of working is very unsatisfactory, inasmuch as most of the adjustment has to be performed at the sending end of the line where the receiving-end voltage cannot be directly observed. The difficulty of operation is accentuated where, as in many systems, hand regulation of the generator voltage is used rather than automatic regulation. It should be noted, moreover, that by this method of control the variations of pressure are not eliminated, but merely transferred from one end of the line to the other. In some cases, however, a considerable amount of power has to be supplied to consumers in the vicinity of the generating station, and so voltage variation at this end involves the same disadvantage as variation at the receiving end. It is only in recent years that the problem of voltage control has been adequately solved by the introduction of the constant-voltage system of transmission. In this method, specially designed synchronous motors are installed at the receiving end of the line to change the power factor of the system as the load changes and

thus always preserve a constant pressure drop along the line. By this means, not only can the sending- and receiving-end voltages be held absolutely constant, but other important advantages are secured.

97. Synchronous Motors.—It is well known that by altering the excitation of a synchronous motor it may be made to take

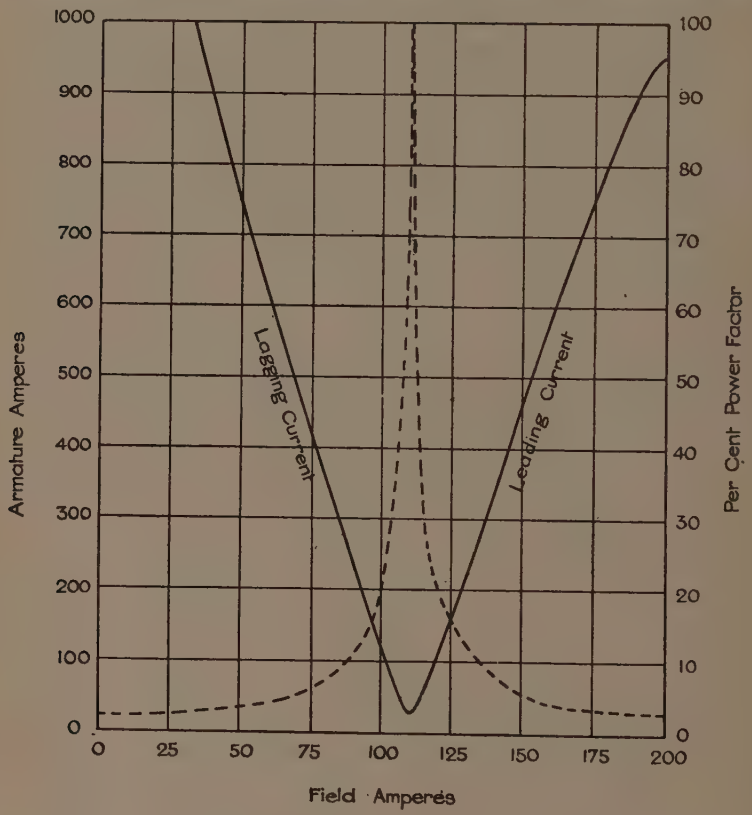

FIG. 50.—'V' curves of a synchronous motor.

either a lagging or leading current from the line. These changes are clearly shown by the '*V*' curves of the machine. The curve given in Fig. 50 relates to a particular synchronous motor when running light or carrying no mechanical load. The minimum value of armature current, between the lagging-current and leading-current branches of the curve, indicates that the motor is then working at unity power factor, and the power input is just sufficient

to meet the no-load losses of the machine. For this particular motor the excitation must be increased from 112 amperes at unity power factor to 155 amperes at full kVA. output leading; the range of excitation being 1·4 to 1. For operation between full lagging and full leading kVA. output, the range of excitation is 67 to 155 amperes or 2·3 to 1.

Idle-running synchronous motors were first employed in connection with power plants to correct for low power factor of the load, and thus reduce the current and power losses in the generators and feeders. For this purpose they should be installed on the distributing system network as near the centre of gravity of the load as possible, so that the benefits of power-factor correction may be felt over the entire generating, transforming and transmitting apparatus. When required for this duty synchronous machines are usually referred to as 'synchronous condensers' since they are always required to take a leading current and hence have the electrical characteristics of a condenser. In the case under consideration, where idle-running synchronous machines are used for voltage-regulating purposes, they are required to run part of the time with leading current and part of the time with lagging current, and thus the term ' synchronous phase modifier ' appears to be more appropriate.

Synchronous phase modifiers differ from synchronous motors intended to be used for mechanical loads inasmuch as they are built for the highest economical speeds, and equipped with smaller shafts and bearings. Furthermore, special attention is given to securing a high overall efficiency. With modern improvements and refinements of design, the total losses at full-load leading power factor expressed in terms of the kVA. output are approximately 4 °/₀ for a 2 000 kVA. machine, falling down to 2 °/₀ for machines of 20 000 kVA. and over. As usually built, synchronous phase modifiers are designed to give their full-load output at leading power factor, and can then carry about 50 °/₀ of their rated capacity lagging. Larger lagging loads result in unstable operation on account of the weakened field. Machines can, however, be designed to operate at full rating both leading and lagging, but they are larger, have a greater loss, and are more expensive than standard machines.

98. Power Factor Control of Line Voltage.—The action of synchronous phase modifiers in controlling the terminal voltage of a line can be easily understood by considering a modification of the

fundamental vector diagram which was given in Fig. 14. Fig. 51 is the same as Fig. 14 except that the current in the line at the receiving end is split up into its two components, I_p in phase with E_r, and I_q in quadrature with E_r and lagging behind it for usual loads. Then we have

RC = ohmic pressure drop due to I_p (drawn parallel to OR).
CH = reactive pressure drop due to I_p (drawn 90° ahead of OR).
HG = ohmic pressure drop due to I_q (drawn parallel to ND).
GS = reactive pressure drop due to I_q (drawn 90° ahead of ND).

The voltage drop along the circuit is, to a first approximation,

$$I_pR + I_qX. \quad . \quad . \quad . \quad . \quad . \quad (120)$$

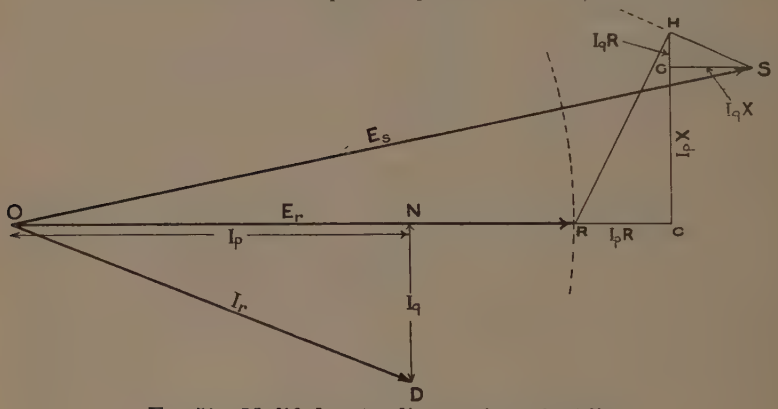

FIG. 51.—Modified vector diagram for a short line.

Now the component I_p cannot be altered as this represents the actual power taken by the receiving-end load. By drawing a leading wattless current from the line, however, I_q can be decreased or reversed if necessary. This alters the size of the impedance triangle HGS. For as I_q is reduced more and more by the leading current taken by the synchronous phase modifiers, the point S travels along SH until it reaches H. If the leading current taken by the synchronous phase modifiers is further increased, I_q is reversed and the impedance triangle also, the point S travelling along SH produced. It is easy to see that by this means, no matter what the load or power factor, the difference between the sending- and receiving-end pressures can be adjusted so as to be absolutely constant.

For instance, assume a line with a constant impressed voltage of 110 000 volts at the sending end. Under normal conditions the

receiving-end pressure at no load rises to, say, 120 000 volts due to capacitance effects, while on full lagging load the receiving-end pressure may fall to about 80 000 volts. Synchronous phase modifiers might be installed at the receiving end of the line to maintain a constant terminal pressure of 100 000 volts. At times of light load the machines would be operated under-excited, drawing a lagging current to pull down the voltage from 120 000 to 100 000, while under load they would be over-excited and draw a leading current to raise the terminal pressure from 80 000 to 100 000 volts. This constant voltage regulation is usually made automatic by installing voltage regulators, such as the Tirrell regulator, operating on the field circuit of the synchronous phase modifiers, and having their control magnets actuated by the phase voltage of the circuit in which the pressure is to be maintained constant.

99. Influence of Synchronous Phase Modifiers on Carrying Capacity of Line.—One of the greatest advantages of the constant-voltage system is that it allows the load of a transmission line to be made two or three times as great as could otherwise be carried, thus reducing line costs per kilowatt transmitted. In any non-regulated overhead system, the voltage variation at the line termini fixes a limit to the amount of power that can be transmitted. With the lagging loads obtained in practice, this voltage variation is determined chiefly by the reactance of the line as shown by formula (120). Now with high-voltage lines, the reactance is usually very much greater than the resistance, hence the maximum permissible regulation is reached while the line losses, which are determined chiefly by the resistance, are still very light. Thus the line is unnecessarily efficient from an economic point of view. If, however, synchronous phase modifiers are installed to maintain a steady voltage at the line termini, the line losses can be made to assume any value which is shown to be desirable from economic considerations.

Practical experience indicates that non-regulated systems cannot be operated satisfactorily when the voltage variation from no load to full lagging load is more than 15 % for the high-tension circuit. Taking into account also the pressure drop in the sending- and receiving-end transformers, this would mean a total regulation of 20 to 25 %. When, as a result of developing load, the regulation of the system has reached this value, it has been customary to install a duplicate line or alternatively to increase considerably the

operating voltage of the system. The installation of synchronous phase modifiers is usually a cheaper means of achieving the same object. By using these machines loads can be transmitted which would produce a voltage regulation of 40 to 80 °/₀ in a non-regulated line.

The influence of synchronous phase modifiers in increasing the power rating of overhead lines is best illustrated by taking a few numerical examples. Assume that power has to be transmitted a distance of 100 miles to supply a load of 80 °/₀ power factor; the receiving-end line voltage being 100 000 volts, 50 cycles. Taking a 12-foot spacing between conductors, the smallest size of stranded copper conductor it is possible to use, from consideration of corona, is 0·10 sq. in. The resistance and reactance of this, and four larger stranded conductors, are given in the second and third columns of Table 12. Under the condition that the load of a non-

TABLE 12.—*Carrying Capacities of Regulated and Non-regulated Overhead Lines.*

Regulated lines—Limiting Factor, 10 °/₀ Power Loss.
Non-regulated Lines—Limiting Factor, 15 °/₀ Voltage Variation in Transmission.
$E_r = \frac{100\,000}{\sqrt{3}}$ volts. Distance of Transmission = 100 miles.
Cos ϕ_r = 80 °/₀ lagging. Spacing = 12′. Frequency = 50 cycles.

Size of Conductor.	Resistance.	Reactance.	Non-regulated Line.		Regulated Line.	Ratio of Carrying Capacities.
			Carrying Capacity in Kilowatts.	Efficiency of Transmission.	Carrying Capacity in Kilowatts.	
0·10 sq. in.	43·5 ohms	69·6 ohms	15 700	90·6 °/₀	25 500	1·63
0·20 „	21·6 „	66·0 „	21 100	93·6 „	49 500	2·35
0·30 „	14·3 „	63·4 „	24 300	95·0 „	73 000	3·00
0·40 „	10·8 „	61·9 „	25 700	96·0 „	93 000	3·62

regulated line is limited by a regulation of 15 °/₀ due to the line impedance only, the maximum amounts of power that can be transmitted over these conductors are shown in the fourth column, and the transmission efficiencies in the fifth column. It will be seen that the line losses are unduly small, particularly with the heavier conductors. Suppose now that synchronous phase modifiers are installed at the receiving end to control the voltage, and that the maximum loads on the lines are determined by the condition that

the line losses must not exceed 10 %. The carrying capacities of the same sets of conductors with this method of control ($E_s = \frac{110\,000}{\sqrt{3}}$ volts) are given in the sixth column of the table. The last column shows the carrying capacity of the constant-voltage line expressed as a ratio of the carrying capacity of the non-regulated line, and it will be noticed that very substantial increases of load are permissible. In fact, by this method, loads can be increased until limited by the cost of the extra power losses in the line, or by the cost of the synchronous machinery needed to maintain constant voltage.

100. Effect on Power Factor.—The advantages of operating alternating-current machinery and systems on unity power factor are well known and do not need elaboration here. Briefly, the bad effects of low power factor are felt both in the initial cost of electrical equipment and in the expense of operating the same. Thus low power factor results in :—

1. Relatively large and costly electrical apparatus—including generators, line conductors, transformers and switchgear—the dimensions of which are governed by the kVA., rather than by the kW., output.

2. Reduced efficiency for the whole of the equipment owing to the copper losses for a given kW. load being inversely proportional to the square of the power factor.

Although, as previously stated, synchronous phase modifiers are primarily installed for purposes of voltage control, and not for power factor improvement, it is interesting to see what effect they have on the latter. At times of light load the synchronous phase modifiers will be working under-excited, dragging a lagging current from the line, and thus lowering the power factor to some extent. At times of moderate or heavy loads on the line the machines will be working over-excited, drawing a leading current from the line, and raising the power factor. Generally speaking, then, the operation of synchronous phase modifiers has a beneficial influence on the power factor of the system, as at such times when a high power factor is most desirable the machines play the double *rôle* of voltage regulators and power factor correctors.

101. Influence on Installation of Reactors.—In the event of a short-circuit occurring on a line, particularly near the generating end, the heavy rush of current which takes place often causes serious damage to transformers and circuit-breakers. With the

increase in size of generating stations it has become necessary to provide means for limiting these currents to a safe value, and so reactance coils are frequently installed on the low-tension side of the line transformers. Alternatively, the transformers may be designed with a high internal reactance so that they can safely withstand short-circuiting without extra help. Unfortunately, the use of reactors on a non-regulated line has the disadvantage of directly reducing the power rating, since the voltage variation which determines the power rating is thereby increased. On the other hand, with the constant-voltage system of operation a high terminal reactance can be employed, with consequent increase in the reliability of service.

102. Summary of Results.—Before summarising the results of the preceding discussion, it is necessary to consider two or three disadvantages which are introduced by the employment of synchronous phase modifiers for voltage control. First of all, there is always the possibility of synchronous machines falling out of step, and thus producing large accidental variations of voltage since the line pressure is so dependent on these machines. Interruptions to supply may also be caused which are best avoided by installing no auxiliary apparatus which it is possible to do without. Another disadvantage of synchronous machines is that they tend to increase the current flowing into a short-circuit. It should be remembered, however, that the machines are usually a long distance from the generating station, and their influence on the short-circuit current is felt chiefly at the receiving end of the line. The most dangerous short-circuits on the system are those which occur close to the generating station, and the synchronous machinery at the other end of the line cannot increase these appreciably owing to the large intervening line reactance.

It has been pointed out that the constant-voltage system allows of far more power being transmitted along an overhead line than could be dealt with satisfactorily by a non-regulated line. In other words, the number of lines for any given project must be reduced if the economies of the constant-voltage system are to be fully realised. Hence, in case of line trouble, the consequences are liable to be more serious with the constant-voltage system than with the non-regulated system. For instance, supposing that with the non-regulated system four sets of conductors are required to transmit a certain amount of power, then, with one set out of action, each of

the other three would be overloaded by 33 %. With the constant-voltage system employing only two sets of conductors to transmit the same total load, on failure of one set of conductors the load on the other set would be liable to an increase of 100 %. There is a growing tendency, however, to interconnect transmission systems in districts where this is possible, thus providing an insurance against failure of supply. A large reserve of lines is not such a *desideratum* under these circumstances.

The chief advantages and disadvantages of the constant-voltage system of transmission may thus be summarised as follows:—

Advantages.—1. Steady voltage maintained at both ends of line at all loads.

2. Increase of power that can be dealt with by one set of conductors thus permitting of considerable financial saving, particularly in long-distance heavy-power transmissions.

3. Improvement of power factor at times of moderate and heavy loads.

4. High terminal reactance can be employed, thus giving better protection.

Disadvantages.—1. Possibility of synchronous phase modifiers dropping out of step, or causing interruption to supply.

2. Increase of short-circuit current of system.

3. Lower reserve of lines in case of line trouble.

When considering whether to install synchronous phase modifiers as an integral part of a transmission system it is necessary to consider the whole problem both from an economic and technical point of view, particularly with short or moderate distances of transmission. On the longest-distance transmissions now employed in various parts of the world the constant-voltage system of operation offers such overwhelming advantages that it may now be considered standard practice.

103. Calculation of Synchronous Phase Modifier Capacity.—A formula for calculating the synchronous phase modifier capacity necessary for the maintenance of constant voltage on a long line is easily derived from the relationship

$$\dot{E}_s = \dot{E}_r A + \dot{I}_r B. \quad . \quad . \quad . \quad . \quad (76)$$

$\dot{I}_r$ is now the actual current at the receiving end of the line, and consists of the load current combined with the reactive current taken by the synchronous machinery.

Let $\dot{I}_r$ be split up into its two components, I_p the in-phase, and I_q the quadrature, component respectively, so that

$$\dot{I}_r = I_p \pm jI_q.$$

I_q is taken as positive when leading, and negative when lagging.

Substituting in equation (76), and resolving into rectangular co-ordinates, gives

$$\dot{E}_s = \dot{E}_r(A_1 + jA_2) + (I_p + jI_q)(B_1 + jB_2),$$
$$\text{or} \quad E_s^2 = (E_rA_1 + I_pB_1 - I_qB_2)^2 + (E_rA_2 + I_pB_2 + I_qB_1)^2. \quad (121)$$

Substituting $\frac{1\,000P}{E_r}$ for I_p, where P is the power delivered at the receiving end in kW. per phase (E_r being the value to neutral), and solving for I_q this becomes

$$I_q = E_r\left(\frac{A_1B_2 - A_2B_1}{B^2}\right) - \sqrt{\left(\frac{E_s}{B}\right)^2 - \left(\frac{1\,000P}{E_r} + E_r\frac{A_1B_1 + A_2B_2}{B^2}\right)^2} \quad (122)$$

The alternative solution for I_q obtained by employing the + sign in front of the radical should not be used, as it represents a condition of unstable operation.

Having found I_q, the reactive kVA. at the receiving end of the line is then given by

$$Q = \frac{E_rI_q}{1\,000} \text{ kVA. per phase.} \quad . \quad . \quad (123)$$

The kVA. required from the synchronous phase modifier is obtained by algebraically subtracting from Q the reactive kVA. of the load, the latter being

$$Q_1 = -P \tan \phi_r \text{ kVA. per phase.} \quad . \quad . \quad (124)$$

Hence the capacity of the synchronous machinery necessary for voltage regulation is

$$Q_{pm} = Q - Q_1$$
$$= \frac{E_rI_q}{1\,000} + P \tan \phi_r \text{ kVA. per phase.} \quad . \quad (125)$$

It should be noted in connection with these formulæ that Q and Q_{pm} may be either leading or lagging according to whether the sign is + or −.

The following example will be taken to illustrate the method of calculation:—

Problem 6.—The 300-mile line whose constants have been previously calculated is to be operated on the constant-voltage system, both the sending-end and receiving-end pressures being maintained constant at 154 000 volts. Determine the synchronous phase modifier capacity necessary when delivering a load of 50 000 kVA. (42 500 kW. at 85 % power factor lagging).

Referring to Art. 46 we have

$$\begin{aligned} A_1 &= 0{\cdot}869780, \\ A_2 &= 0{\cdot}035418, \\ B_1 &= 47{\cdot}937, \\ B_2 &= 180{\cdot}786, \end{aligned}$$

and from the data of the problem

$$E_r = 88\,912 \quad \text{volts to neutral,}$$

$$P = \frac{42\,500}{3}$$

$$= 14\,166{\cdot}7 \quad \text{kW. to neutral.}$$

Hence

$$E_r\left(\frac{A_1B_2 - A_2B_1}{B^2}\right) = 395{\cdot}364,$$

$$\left(\frac{E_s}{B}\right)^2 = 225\,995,$$

$$\left(\frac{1\,000P}{E_r} + E_r\frac{A_1B_1 + A_2B_2}{B^2}\right)^2 = 79\,291{\cdot}2,$$

and

$$I_q = 395{\cdot}364 - \sqrt{225\,995 - 79\,291{\cdot}2}$$

$$= + 12{\cdot}345 \quad \text{amperes.}$$

The reactive kVA. at the receiving end of the line is

$$Q = \frac{12{\cdot}345 \times 88\,912}{1\,000}$$

$$= + 1\,097{\cdot}6 \quad \text{kVA. per phase (leading),}$$

and the reactive kVA. of the load is

$$Q_1 = 14\,166{\cdot}7 \times \tan 31{\cdot}78°$$

$$= - 8\,779{\cdot}7 \quad \text{kVA. per phase (lagging).}$$

Hence the capacity of synchronous phase modifier necessary is

$$Q_{pm} = 1\,097{\cdot}6 + 8\,779{\cdot}7$$

$$= 9\,877{\cdot}3 \quad \text{kVA. per phase (leading);}$$

the total capacity for the three-phase system being

$$3Q_{pm} = 29\,632 \quad \text{kVA. (leading).}$$

The electrical conditions when no load is being supplied is readily found by substituting $P = 0$ in formulæ (122) and (125).

The reactive current at the receiving end of the line is

$$I_{qo} = E_r\left(\frac{A_1B_2 - A_2B_1}{B^2}\right) - \sqrt{\left(\frac{E_s}{B}\right)^2 - \left(E_r\frac{A_1B_1 + A_2B_2}{B^2}\right)^2}$$
$$= 395{\cdot}364 - \sqrt{225\,995 - 14\,946{\cdot}0}$$
$$= -64{\cdot}04 \quad \text{amperes,}$$

and the capacity of synchronous machinery necessary at times of zero load is therefore

$$Q_{pmo} = \frac{-64{\cdot}04 \times 88\,912}{1\,000}$$
$$= -5\,693{\cdot}9 \quad \text{kVA. per phase (lagging),}$$

or a total capacity for the three-phase system of

$$3Q_{pmo} = -17\,082 \quad \text{kVA. (lagging.)}$$

Thus a synchronous phase modifier rated at 30 000 kVA. and of standard design would be capable of dealing with both load, and no-load, conditions.

Having obtained the actual current in the line at the receiving end $\dot{I}_r = I_p + jI_q$, the remaining electrical characteristics of the line may be calculated by applying the customary formulæ.

Thus

$$\dot{E}_s = \dot{E}_rA + (I_p + jI_q)B$$
$$= 88\,912 \angle 21{\cdot}472^\circ \quad \text{volts to neutral,}$$

and

$$\dot{I}_s = (I_p + jI_q)A + \dot{E}_rC$$
$$= 193{\cdot}01 \angle 45{\cdot}066^\circ \quad \text{amperes,}$$

and other numerical results obtained therefrom are

$\phi_s = 23{\cdot}594^\circ$ leading.
$\cos \phi_s = 91{\cdot}64$ per cent.
$kW_s = 15\,726$ kW. per phase.
Loss $= 1\,559$ kW. per phase.
Efficiency $= 90{\cdot}09$ per cent.

104. Graphical Determination of Synchronous Phase Modifier Capacity.—The synchronous phase modifier capacity necessary for the voltage regulation of a long line can be conveniently determined from the voltage and current vector diagrams of the line. These diagrams have been given previously, but it is

now desirable to examine their properties in rather more detail. In the voltage diagram shown in Fig. 52, OR represents E_r, the receiving-end voltage, which is taken as reference, and OP leading OR by a small angle is E_rA, the sending-end voltage when the line is on open-circuit. When load is being supplied, the impedance consumed by the line will raise or lower the sending-end voltage according to the power factor of the load. This effect is shown by the vector PS which represents I_rB, and the resultant sending-end voltage under load is thus OS. The position of the point S, which locates the sending-end voltage vector, depends on the character of the load. With a unity power factor load at the receiving end, the point S travels along PA. At any other con-

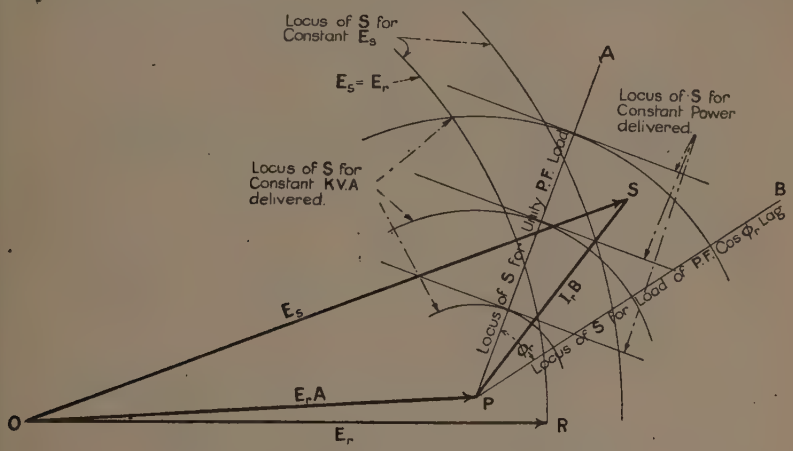

FIG. 52.—Voltage vector diagram for long line.

stant power factor load, the locus of S is a straight line drawn from P leading or lagging behind PA by the angle ϕ_r. If the amount of power delivered is kept constant while the power factor is varied, S travels along a line perpendicular to PA. With a constant kVA. load, S moves along the arc of a circle drawn with P as centre. Finally, for the condition that the sending-end voltage must be kept constant as the load on the line varies, the point S must follow the arc of a circle drawn with O as centre and having a radius equal to the given sending-end voltage E_s.

The current diagram shown in Fig. 53 is similar to the voltage diagram in that it consists of two vectors, one of which is proportional to the receiving-end voltage, and one which is proportional

to the receiving-end current. In the diagram, OR again represents E_r, the receiving-end voltage, and OM drawn at an angle of just over 90 degrees to OR is E_rC, the charging current at the sending end when the line is on open-circuit. As the line is loaded, a current MN representing I_rA is added to the charging current, thus giving the sending-end current ON. For a unity power factor load, N travels along the line MA leading OR by a small angle. At any other power factor, the locus of N is a straight line drawn

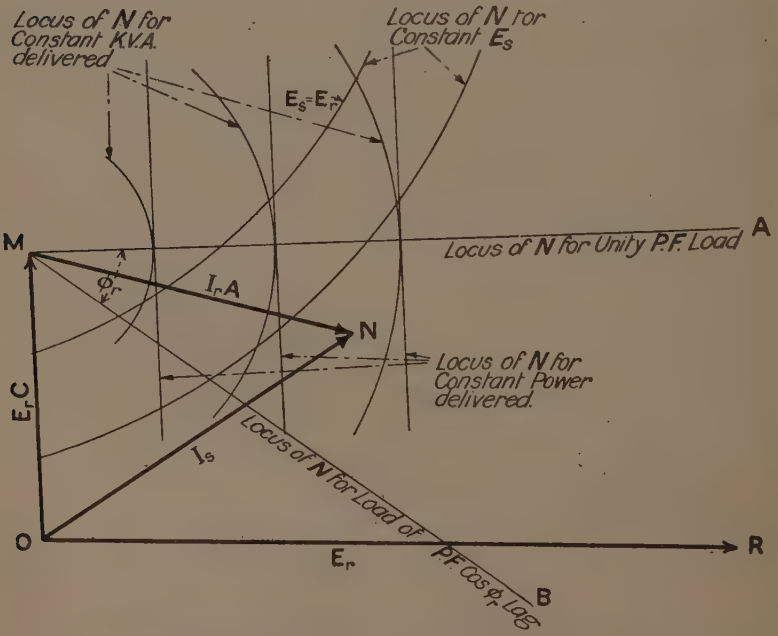

Fig. 53.—Current vector diagram for long line.

from M leading or lagging behind MA by the angle ϕ_r. With constant power delivered and varying power factor, N moves along a line perpendicular to MA. Similarly the locus of N for a constant kVA. load will be an arc of a circle drawn with M as centre.

On comparing the two diagrams it is seen that the line PA in the voltage diagram and its associated system of lines being the loci of the sending-end voltage, is exactly similar to MA in the current diagram and its associated loci for the sending-end current. Any point on the voltage diagram such as S represents a definite

amount of power delivered at the receiving end of the line, and a corresponding point N can always be found on the current diagram which represents exactly the same load conditions. OS and ON are the sending-end voltage and current respectively for this particular load. It is obvious that concentric arcs of circles may also be drawn on the current diagram as shown, giving the loci of I_s for any given constant sending-end voltage E_s.

The use of these diagrams to solve Problem 6 graphically will now be explained.

Choosing a scale of, say, 1 inch = 10 000 volts, OR and OP are drawn as previously described (Art. 47) to represent E_r and E_rA respectively (see Fig. 54). In order to get a kVA. scale take a

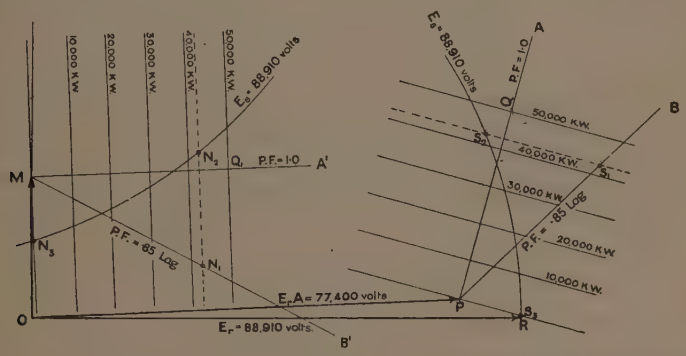

FIG. 54.—Voltage and current solutions for various receiving-end conditions.

load of 50 000 kVA. at the receiving end at unity power factor. I_r will then equal 187·5 $\angle 0°$ amperes per conductor, and

$$I_rB = 187{\cdot}5 \angle 0° \times 187{\cdot}0 \angle 75{\cdot}15°$$
$$= 35\,060 \angle 75{\cdot}15° \text{ volts.}$$

From P draw the unity power factor line at an angle of 75·15° to OR, measure up along it a distance PQ = 3·506 inches, and through Q draw a line perpendicular to PA. This is the 50 000 kW. line, and other lines can then be drawn parallel to it to show the loci of the sending-end voltage corresponding to other loads. With O as centre draw an arc of radius equal to the scale value of the sending-end voltage which it is decided to maintain constant by means of synchronous phase modifiers. In the problem under consideration $E_s = E_r$.

To find the conditions at the receiving end when a load of

50 000 kVA. (42 500 kW. at 85 % power factor lagging) is being delivered, draw on the diagram the power line corresponding to 42 500 kW., and also PB lagging behind PA by $\cos^{-1} 0{\cdot}85$ or 31·79°. The point of intersection S_1 of these two lines locates the end of the sending-end voltage vector for the non-regulated line, which vector has a value of 106 000 volts. The sending-end voltage is actually maintained at 88 910 volts, and to reduce the voltage from 106 000 to 88 910 requires an amount of phase modifier capacity represented by the distance S_1S_2 or 30 000 kVA. leading.

Under zero-load conditions, to keep E_s constant at 88 910 volts requires a phase modifier capacity represented by PS_3 or 17 000 kVA. lagging.

The voltage diagram is all that is needed to determine the phase modifier capacity for any given load, but by drawing the current diagram the remaining characteristics of the line can be found. The current diagram is most conveniently drawn on the same sheet, and to the same scale, as the voltage diagram. Both I_rB and I_rA are proportional to I_r, so if the current scale is chosen so as to make $I_rB = I_rA$, the same power scale will apply to both diagrams.

For 50 000 kVA. at unity power factor

$$\dot{I}_rA = 187{\cdot}5 \angle 0° \times 0{\cdot}8705 \angle 2{\cdot}33°$$
$$= 163{\cdot}2 \angle 2{\cdot}33° \text{ amperes,}$$

and for equality of power scale this current must be represented on the diagram by the same length which represents $\dot{I}_rB$, or 3·506 inches. Hence

$$163{\cdot}2 \text{ amperes} = 3{\cdot}506 \text{ inches,}$$

or the current scale is 1 inch = 46·55 amperes.

Now from O draw OM at an angle of 90·72 degrees to represent

$$\dot{E}_rC = 88\,910 \times 0{\cdot}001349 \angle 90{\cdot}72°$$
$$= 119{\cdot}9 \angle 90{\cdot}72° \text{ amperes,}$$

and from M draw the unity power factor line MA' at an angle of 2·33 degrees to OR. The power scale can then be readily transferred from the voltage diagram on to the current diagram by means of dividers.

With centres M and Q_1, and radii equal to PO and QO respectively, strike two arcs and the point of intersection will be the centre from which to draw the curve showing the locus of sending-end current for a constant sending-end voltage E_s. The points

N_1, N_2, and N_3 on the current diagram correspond to S_1, S_2, and S_3 on the voltage diagram. Thus ON_1 is the sending-end current when no phase modifiers are in use and is equal to 146 amperes. When the sending-end voltage is held constant at 88 910 volts, the sending end current is $ON_2 = 193$ amperes.

The sending-end phase-angle is the angle $S_2ON_2 = 23{\cdot}5°$, and the power factor 91·7 °/₀ leading.

The total kVA. input to the line can now be calculated and therefrom the line losses and efficiency of transmission.

105. The Circle Diagram.—The circle diagram may be defined as a graph showing the reactive power in the line at the receiving end for various loads transmitted, assuming definite sending- and receiving-end voltages. It thus forms a very convenient method of finding the requisite synchronous phase modifier capacity for all possible loads, and not for one particular load only. The circle diagram for a short line was described by Philip,[1] and later on was extended by Dwight [2] to take into account the effect of capacitance, that is a diagram for a long line.

The data required for drawing the diagram are obtained from equation (121)

$$E_s^2 = (E_rA + I_pB_1 - I_qB_2)^2 + (E_rA_2 + I_pB_2 + I_qB_1)^2,$$

which is the equation of a circle and can be reduced to the form

$$\left(I_p + E_r\frac{A_1B_1 + A_2B_2}{B^2}\right)^2 + \left(I_q - E_r\frac{A_1B_2 - A_2B_1}{B^2}\right)^2 - \left(\frac{E_s}{B}\right)^2 = 0. \tag{126}$$

In this equation voltages are measured to neutral, hence by multiplying throughout by $\frac{3E_r}{1\,000}$, a circle is obtained which shows the relation between the total reactive and active kVA at the receiving end of the line.

The co-ordinates of the centre of this circle are a and b, and the radius is c where

$$a = -\frac{3E_r^2(A_1B_1 + A_2B_2)}{1\,000B^2} \text{ kW.},$$

$$b = +\frac{3E_r^2(A_1B_2 - A_2B_1)}{1\,000B^2} \text{ kVA.},$$

$$c = +\frac{3E_rE_s}{1\,000B} \text{ kVA.}$$

The circle diagram for the 300-mile line, assuming that the sending- and receiving-end line voltages are both held constant at 154 000 volts, is shown in Fig. 55. In this particular case the

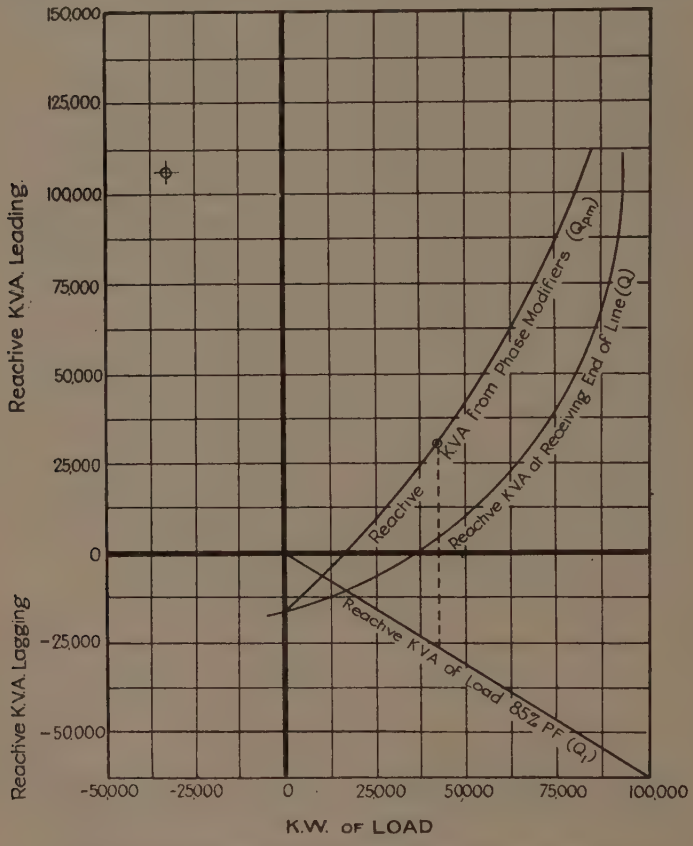

FIG. 55.—Circle diagram for constant-voltage transmission line.

numerical values of a, b, and c are

$$a = -32\,610 \text{ kW.}$$
$$b = +105\,460 \text{ kVA.}$$
$$c = +126\,800 \text{ kVA.}$$

Any point in the fourth quadrant of the diagram represents a definite load supplied at a definite power factor. To determine the kVA. required from the synchronous phase modifier at this load, the reactive kVA. of the load must be subtracted from the corre-

sponding ordinate of the circle which represents the total kVA. in the line at the receiving end. As the reactive kVA. of the load is lagging, and thus negative, this operation is equivalent to arithmetically adding the reactive kVA. of the load to the corresponding kVA. of the circle diagram.

Thus, supposing that the load although varying in magnitude has a constant power factor of 85 % lagging, a straight line can be drawn at an angle $\cos^{-1} 0{\cdot}85$ below the base line. By means of a pair of dividers the reactive kVA. of the load can then be added to the corresponding ordinate of the circle, thus plotting the curve of kVA. required from the synchronous phase modifier ; this latter curve being an ellipse.

It will be noted that there is a theoretical limit to the amount of load in kilowatts that can be transmitted, which is easily read from the diagram as it is the farthest distance to the right reached by the circle. The maximum possible load is thus

$$kW_{max} = c + a,$$

which is numerically less than c since a is negative.

The phase modifier capacity required for transmitting the load of 42 500 kW. at 85 % power factor is indicated in the diagram by the intersection of the dotted line with the phase modifier curve, and agrees with the results of previous calculations. The maximum amount of power that can be transmitted by the line at the voltages under consideration is

$$\begin{aligned} kW_{max} &= 126\,800 - 32\,610 \\ &= 94\,190 \text{ kW.} \end{aligned}$$

It will be noticed that there is a very rapid increase in the amount of phase modifier capacity required when loads in the neighbourhood of the theoretical maximum are reached.

In making the calculations for a long transmission line it is often desirable to include the effect of the raising and lowering transformers, and Dwight [3] has described a method of accomplishing this, *i.e.* a circle diagram for the complete constant-voltage system.

106. Determination of Synchronous Phase Modifier Capacity in Case of Short Line.—Hitherto the determination of synchronous phase modifier capacity has been discussed in relation to long transmission lines, as this is the more important case occurring in practice. It is interesting, however, to note the

modification of the preceding formulæ when applied to the case of short lines.

In the case of a line where capacitance effects may be ignored the fundamental equation corresponding to (121) is

$$E_s^2 = (E_r + I_pR - I_qX)^2 + (I_pX + I_qR)^2, \quad . \quad (127)$$

from which

$$I_q = \frac{E_rX}{Z^2} - \sqrt{\left(\frac{E_s}{Z}\right)^2 - \left(\frac{1\,000P}{E_r} + \frac{E_rR}{Z^2}\right)^2}, \quad . \quad (128)$$

where the current is leading when positive and lagging when negative.

The capacity of synchronous phase modifiers necessary for voltage regulation can then be obtained as before,

$$Q_{pm} = \frac{E_rI_q}{1\,000} + P \tan \phi_r \quad \text{kVA. per phase.} \quad . \quad (129)$$

Also by writing equation (127) in the form

$$\left(I_p + \frac{E_rR}{Z^2}\right)^2 + \left(I_q - \frac{E_rX}{Z^2}\right)^2 - \left(\frac{E_s}{Z}\right)^2 = 0$$

it is readily seen that, in the case of a short line, the co-ordinates of the centre of the circle are

$$a = -\frac{3E_r^2R}{1\,000Z^2},$$

$$b = +\frac{3E_r^2X}{1\,000Z^2},$$

the radius of the circle being

$$c = +\frac{3E_rE_s}{1\,000Z}.$$

107. Choice of Sending-end Voltage.—Although in the constant-voltage system of operation both the sending-end and receiving-end voltages are maintained constant, it is not essential that they should be equal. Frequently it happens that a less synchronous phase modifier capacity is needed by operating with E_s higher than E_r. This can readily be seen from the following considerations. The minimum phase modifier capacity for any given transmission scheme is obtained when the lagging current taken by the machines with the line open-circuited at the receiving end, is equal to the leading current taken when the line is carrying full inductive load. Now a lagging current in the line at zero load pro-

duces a direct loss of pressure, that is, requires a higher sending-, than receiving-end, pressure. Hence in the case of a short line, E_s should be chosen higher than E_r. As the length of line increases, the lagging current taken by the synchronous phase modifier at zero load is more and more neutralised by the leading charging current of the line. Hence for minimum phase modifier capacity, E_s approaches nearer and nearer to E_r, and in the case of very long high-voltage lines the two terminal voltages may be practically equal. The circle diagram has the valuable characteristic that, with an alteration of E_s, the only diagram constant which is affected is the radius of the circle, which varies directly with E_s. Hence concentric circles can be drawn with practically no additional trouble to show the results for different values of the sending-end voltage.

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[2] H. B. Dwight, 'The Calculation of Constant-Voltage Transmission Lines,' *Elec. Jour.*, Vol. 11, p. 487 (1914).

[3] H. B. Dwight, 'Electrical Characteristics of Transmission Systems,' *Trans. A.I.E.E.*, Vol. 41, p. 781 (1922).

CHAPTER IX.

UNDERGROUND CABLES.

108. Introductory.—Hitherto, the chief use of underground electric cables has been in connection with city distributing systems at comparatively low pressures. Recently, however, great developments have been made in the art of cable construction, and underground lines appear likely to come more and more into use for the straight transmission of power for short or moderate distances. In this work, only high-voltage cables suitable for transmitting energy in bulk from generating station to receiving station, or from receiving station to substation, will be considered.

Compared with overhead systems, cables have the following advantages:—

1. Greater freedom from interruption of service and damage due to thunderstorms, severe weather conditions, or extraneous interference. On overhead lines short-circuits and earths are always more or less prevalent, due to lightning stroke, breaking of conductors or insulators, objects falling across the wires, or other causes.

2. Removal of liability of accident to public.

3. Avoidance of unsightly appearance of poles and overhead conductors in the neighbourhood of cities.

4. Freedom from way-leave troubles, as a cable usually follows the roadways.

The above considerations often result in the installation of underground lines in populous districts, despite the fact that the first cost is much higher than that of an overhead line.

109. Insulation of Cables.—Paper is practically the only material used at present for the insulation of high-voltage cables. The advantages of paper-insulated cables compared with other types are superior durability, high dielectric strength, low capacitance and dielectric power factor, and low cost.

The paper used by British manufacturers is made largely, or

entirely, of manilla fibre. Inferior forms of fibrous-dielectric cables usually contain small proportions of manilla paper, and large proportions of straw, esparto, and wood, all chemically treated. The introduction of such substances lowers the dielectric strength and insulation resistance, and impairs the mechanical strength and elasticity of the cable. Furthermore, there is danger of the insulating material rotting in course of time due to the action of the residual chemicals therein.

Paper is a fibrous material, and therefore requires to be impregnated thoroughly with a special compound to enhance its insulating properties, to make the cable more pliable, and to render it as non-absorbent of moisture as possible. A mixture of resin and resin oil (or mineral oils), varying in composition for different makers, is frequently used for this purpose.

In the manufacture of paper-insulated cables, the copper conductor, after being stranded up in a stranding machine, is passed through an insulating machine, where the paper in the form of ribbon about five-thousandths of an inch in thickness, is wrapped on spirally to the required thickness. The reels containing the technically wet paper cable are now heated in steam vessels, first at atmospheric pressure, and later in a vacuum, till all the moisture has been extracted. The cable is then immersed in a bath of insulating oil for a period varying with the thickness of the insulation. In another method the impregnation is effected by passing the dry paper, before it is cut into ribbons, through a bath of the insulating oil. It is claimed that by this means a greater uniformity of impregnation is possible.

110. Types and Construction of Cables.—Soft or annealed copper is the metal generally used for the conductor, although it appears possible to use aluminium in some cases to advantage. In order to take full advantage of the high conductivity of the copper it is necessary that the metal should be absolutely pure. It is for this reason that all copper used for electric cables is made from electrolytically-deposited metal, this being the only practical method of ensuring purity. A minimum conductivity of 100 % can be obtained by this means. Stranded conductors are always used, to provide the necessary flexibility and eliminate the inherent mechanical weakness of solid copper to fracture when subjected to repeated bending.

All high-voltage paper-insulated cables are covered with a lead

sheath, with the object of preventing access of moisture to the insulation, and also of forming a mechanical protection against handling. The lead is applied in the plastic state under hydraulic pressure, the greatest care being taken to ensure a flawless non-hygroscopic covering. Ductility and durability are essential features in the sheathing of a cable, and to secure these characteristics, lead of at least 99·8 % purity should be employed.

There are two chief types of cable used for alternating-current power-transmission work, *viz.* single-core and three-core. The former type has the advantage in simplicity of construction, consisting essentially of a stranded copper conductor, a belt of impregnated-paper insulation, and an overall lead sheath. Plain lead-covered single-core cables can be used where the conditions are favourable, but more often a covering of jute yarn soaked in a preservative compound of a tarry nature is applied over the lead. This serves to protect the lead sheath from abrasion or damage caused by drawing in ducts or other means.

In three-core cables the three separate conductors are placed inside one lead sheath. The great advantage is thereby obtained that steel armouring may be used to protect the cable, whereas with single-core cables carrying alternating currents, armouring would usually lead to excessive voltage drop and power losses in the line.

In the construction of three-core cables, the three conductors are stranded and insulated separately, thus forming the cores of the cable. The three cores are then laid spirally together, and packed with jute or paper fillers to a truly circular shape, and round this is wrapped another layer of insulation called the belt insulation. Finally, the lead sheath is drawn over the whole, and the cable is then ready for use providing a suitable method of laying is adopted. If, however, the cable is to be laid direct in the ground, or is likely to meet with rough usage, armouring is necessary. The armouring is performed by enclosing the cable in a layer of galvanised steel wires (or steel tapes) bedded down on to a layer of jute yarn with which the lead sheath is served. After application, the armouring is finished off with an overall serving of jute yarn thoroughly impregnated, to protect the armour from chemical action in the soil. In cases where the cable is liable to meet with severe stresses, additional mechanical protection is provided by two layers of armouring, the layers being spiralled in opposite directions.

Three-core cables with sector-shaped conductors are occasionally employed as this design permits of a smaller overall diameter for a given voltage rating, and so leads to a saving in cost and weight. These special cross-sections of conductor are usually obtained by rolling or hammering a circular stranded conductor to the desired shape, and special care has to be taken to avoid sharp edges which would give rise to excessive local electrostatic stresses. The reduction in overall diameter of the cable due to employing sector-shaped conductors is a function of the size of the conductors, and so is not of much importance in connection with high-voltage cables where the ratio of conductor diameter to insulation thickness is relatively small. In this case, the additional manufacturing complications involved in producing specially-shaped conductors do not appear worth while, and it is preferable to use the ordinary circular conductors.

III. Methods of Laying.—The present-day methods of laying may be classified under three headings, *viz.* Direct, Solid, and Draw-in.

1. *Direct System.*—This method of laying cables is the simplest and cheapest, and much favoured by modern practice. The cables are laid at a depth of about three feet, on a bedding of riddled earth placed at the bottom of the trench. Where more than one cable is laid in the same trench, a spacing of 12 to 18 inches should be allowed in order to prevent mutual heating effect, and also to ensure that a fault occurring on one cable will not spread immediately to the others. In most cases, a line of concrete slabs or lengths of creosoted wooden planks are laid directly over the cable before reinstatement of the trench. This precaution gives an indication to future excavators of the presence of the cables. This method of laying gives the best conditions for dissipating the heat from the cable losses, and practically eliminates trouble due to vibration caused by traffic, etc.

Solid System.—In this system of laying the cables are laid in wood, earthenware, cast-iron, or asphalt troughs, filled up solid with bitumen or pitch, poured in when heated to a fluid state. Cables laid in this manner do not require armouring as the troughing affords a good mechanical protection.

Wood troughing has the advantage of cheapness, and can be made conveniently in greater lengths than troughing made from other materials. It should be thoroughly impregnated under

pressure with some preservative such as creosote. Glazed earthenware troughing is also largely employed and has the advantage over wood that it is not liable to rot. It cannot, however, be conveniently obtained in lengths greater than three feet, so that a large number of joints are inevitable. Cast-iron troughing can be supplied in long lengths, but is expensive. Cast-iron and earthenware troughs are usually supplied with socket ends whereby each length of troughing is jointed with the next. Asphalt troughing appears to be very suitable for a solid system as it is easily laid and jointed, has good insulating properties, and allows good dissipation of heat by conduction.

Cables laid in troughing are supported every few feet on insulating bridge pieces or saddles, so that the filling-in compound entirely surrounds the cables. Hard asphalt saddles are the best, as under proper conditions of trough filling the surfaces which come into contact with the hot filling compound are melted, and in cooling the whole forms a practically homogeneous mass. Asphalt troughing has the immense advantage of not requiring any saddles.

After the filling material, which should preferably be pure bitumen, has been run in, a covering which may be of bricks, tiles, or wood is applied to the trough. In the case of cast-iron or asphalt troughs, however, the cover is usually of the same material as the trough.

Contrary to early expectations, this system of laying does not make as permanent a job as armoured cable laid direct. Armouring, provided with a suitable taping, is probably as waterproof as the best solid system, and is more reliable since it is applied under skilled supervision in the factory, instead of by unskilled labour in the open, and possibly under adverse weather conditions. Furthermore, the necessary joints in the troughing are a source of weakness, as they are liable to result in local concentration of vibration with consequent damage to the lead sheath.

3. *Draw-in System.*—In the neighbourhood of cities where excavation is expensive and inconvenient, the draw-in system is practically always used. Ducts or pipes are laid in the ground, with manholes at suitable positions and distances, and the cables afterwards drawn therein. The great advantage is thus secured that once the conduit has been laid, repairs, alterations, or additions to the system can be made without re-opening the ground. Cables intended to be drawn into ducts should have their lead sheaths well

wrapped with impregnated jute, or preferably have a braided cover, in order to protect the sheath from damage when drawing in.

Cast-iron pipes with caulked joints form a good conduit but the high cost prevents their extensive use, and earthenware ducts are in more general use. The latter may be either in the form of separate pipes, or of multi-way ducts with round or square ways from two to eight in number. These ducts should be thoroughly glazed inside and out, and preferably constructed with spigot-and-socket joints. An alternative construction makes use of fibre conduits. These are cylindrical tubes made of wood-pulp saturated with an asphalt or bituminous compound containing a small amount of creosote, and are usually set in concrete.

The chief disadvantage of the draw-in system is that the carrying capacity of the cables is reduced, due to the close grouping of cables. So much is this the case that, in America, where this system of laying is almost universal, special methods have been proposed to facilitate the removal of heat by blowing air through the ducts, or filling them with vaseline. Furthermore, cables drawn into ducts are subject to mechanical expansion and contraction so that plenty of room should be left in the manholes, and the joints set to one side so as to provide a U-bend. Conduits also are liable to become charged with explosive gases caused, either by leakage from gas mains in the vicinity, or by leaks from the cables themselves, the insulation of which when heated gives off explosive gases. It is therefore necessary to provide efficient ventilation.

It is not intended in this work to give details of the various kinds of joints employed for high-voltage cables, but too much attention cannot be paid to this point in practice, no matter what the method of laying adopted. Joints are of the utmost importance, as they are the weak places in an otherwise homogeneously insulated conductor.

112. Current-carrying Capacity.—Whenever possible, cables should be worked at the value of current density indicated by economic laws. In many cases, however, and particularly if the load factor is low, this economic loading would result in excessive heating of the dielectric, and the current rating of a cable should then be determined solely from temperature considerations. On account of their high cost, it is most important that cables should be loaded up to their safe limit, and a great deal of research work has been done in recent years to determine safe current-carrying

capacities. In Great Britain, this work has been carried out by the British Electrical and Allied Industries Research Association.[1]

In general, when an electric cable is carrying current, there are three sources of heat, the I^2R losses in the conductors, the dielectric losses and the sheath losses. The effect of dielectric losses is practically negligible up to temperatures of 60° to 70° C. and pressures in common use. Sheath losses may be appreciable in the case of single-core cables, but can be kept to relatively low values by suitable arrangements. Both these sources of heat being of secondary importance they will be ignored in the following discussion of the subject.

The fundamental law on which the calculation of current-carrying capacity, as limited by temperature rise, is based is known as 'Ohms' law for heat.' It is obvious that in any cable which is carrying a current, when a steady temperature is attained, the amount of heat generated in the cable is exactly equal to the amount of heat dissipated by the cable. With buried cables this dissipated heat is taken up by the earth. For example, in the case of an armoured cable laid direct in the ground, the heat generated in the conductors flows along the following path in escaping: dielectric—lead sheath—protective covering (consisting of inner serving, armouring wires and outer serving)—earth in contact—to a part of the earth which is taken as the base for temperature. The thermal path may be considered as analogous to an electric circuit with resistances in series. In such an electric circuit the potential difference across each of the resistance elements would be equal to the product of its electrical resistance by the current flowing. In the corresponding thermal path the difference of temperature across each of the elements will be equal to the flow of heat multiplied by the thermal resistance of the element. The thermal flow might be expressed in calories per second but it is more convenient to express it in watts (or joules per second). The unit of thermal resistance is, then, the resistance of a path through which a difference of temperature of 1 degree C. produces a heat flow of one joule of heat per second. This unit is usually known as a thermal ohm, but Dunsheath[2] suggests that the name 'Forbe' should be given to it. The fundamental relation for evaluating the temperature rise of a buried cable may be expressed mathematically as

$$\theta = H(S + G) \quad . \quad . \quad . \quad . \quad (130)$$

where θ = temperature rise of conductor in degrees C.
H = heat developed in the conductors in watts.
S = thermal resistance between conductors and outer surface of cable in thermal ohms.
and G = thermal resistance of ground surrounding the cable.

Supposing then, that a value of temperature rise is selected corresponding to the maximum permissible operating temperature of the dielectric, and that the values of the thermal resistances are known, the current loading can be determined by an application of the above formula.

The total heat generated in the cable is

$$H = nI^2R_\theta, \quad . \quad . \quad . \quad . \quad (131)$$

where I = current per conductor,
n = number of conductors,
and R_θ = resistance of conductor in ohms at working temperature corresponding to the temperature rise θ,
and substituting this value of H in (130) gives

$$I = \sqrt{\frac{\theta}{nR_\theta(S + G)}}. \quad . \quad . \quad . \quad (132)$$

113. Maximum Permissible Temperature for Cables.—At present there is a lack of accurate information as to the temperature to which cables can be subjected for long periods without damage. It is well known, however, that at high temperatures, impregnated-paper insulation suffers injury to its mechanical and electrical properties. Furthermore, the dielectric losses which are increased at high temperatures may aggravate the trouble, and lead to rapid deterioration of the cable. So far as the properties of the dielectric are concerned, a maximum temperature of 80° to 85° C. appears to be a safe limit. It is necessary, however, to take into consideration the expansion and contraction effects and consequent mechanical strains on cables subjected to alternate heating and cooling. Taking the following coefficients of expansion:—

Copper . . .	0·0000168 per degree C.
Lead . . .	0·0000275 „

the actual expansion per 100 yards length of cable for a 50 degree C. rise would be about 3 inches for copper and 5 inches for lead. It should be borne in mind, however, that the lead sheath undergoes

less temperature variation than the conductor, the sheath temperature rise in high-voltage cables being about one-half the temperature rise of the conductor. Hence the lower temperature rise of the lead compensates to some extent for its higher coefficient of expansion, and the actual differential expansion of the two metals may be smaller than would be indicated by the above figures.

Apparently the chief mechanical trouble due to heating is the pulling-out or buckling of conductors at joints, and precautions have to be taken to prevent this occurring. Special joints such as Vernier's expansion joint are largely used, and allow for actual movement of the conductor up to about 8 inches. Buckling at joints can be guarded against by efficient support and separation of the cores, or by enclosing each core in a separate insulating tube. It is also desirable to provide means for dealing with the expansion of the lead sheath, and for this purpose special forms of joint box have been introduced to permit perfectly free movement of the lead sheath at the joint. In this connection also it should be mentioned that experiments by Beaver show that all movement of the lead sheath is not transmitted to the joints, but that due to the poor elasticity of the lead, alternation of expansion and contraction may cause a permanent distortion of the sheath.

It is readily seen that the question of operating temperature is not an easy matter to settle, but as a result of the British researches, it has been decided to adopt in this country a maximum conductor temperature of 65° C. This figure applies to the case of armoured cables laid direct in the ground. The danger of mechanical trouble due to expansion and contraction effects is greatest when the draw-in system of laying is used, as in this case the cable is practically free to move lengthwise along the duct. Hence for plain lead-sheathed cables drawn into ducts the maximum permissible conductor temperature is fixed at 50° C. In fixing these limits emphasis has been laid on the fact that buried cables should require a minimum of maintenance, and that they should never be run under conditions that would cause any shortening of their reputed life of 30 to 40 years. Taking the normal temperature of the ground as 15° C., this allows temperature rises of 50 degrees C. for armoured cables laid direct, and 35 degrees C. for lead-sheathed cables in ducts.

In America, the standardisation rules of the American Institute of Electrical Engineers allow as a maximum conductor temperature

the value (85 − V) degrees C. where V is the operating voltage in kilovolts between conductors. Thus a cable for 11 000 volts may be operated at a conductor temperature of 74° C., and a 33 000-volt cable at 52° C. It is recognised, however, that for high-pressure cables this rule gives values which are obviously too low, and as a result of research which is now going forward it will undoubtedly be modified.

114. Thermal Resistance.—The formula for the thermal resistance between conductor and sheath of a single-core cable can easily be obtained as follows :—

Considering the thermal resistance of a length of cable one centimetre long to be the sum of the thermal resistances of successive layers of insulation, there may be written

$$dS' = \frac{K dx}{2\pi x},$$

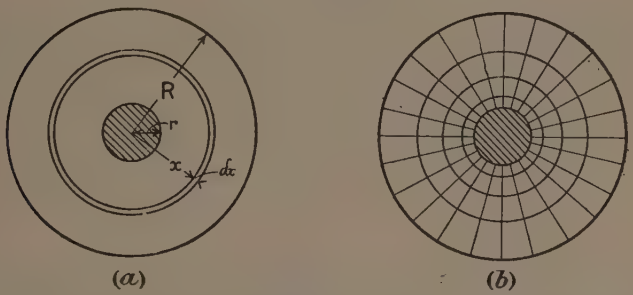

Fig. 56.—Lines of heat flow and isothermal lines in a single-core cable.

where dx is the thickness, and $2\pi x$ the area, of a cylindrical element of radius x (Fig. 56 (a)). K is the thermal resistivity in thermal ohms, *i.e.* the temperature difference in degrees C. between opposite faces of a centimetre cube to cause transference of one watt of heat. From the above

$$S' = \frac{K}{2\pi}\int_r^R \frac{dx}{x},$$

$$= \frac{K}{2\pi} \log_e \frac{R}{r} \quad \text{thermal ohms per cm.} \quad . \quad . \quad (133)$$

Fig. 56 (b) shows the lines of heat flow and isothermal lines of the cable.

The similarity between the above formula and the formulæ for the capacitance and insulation resistance of a single-core cable

(Art. 123) is readily noticed. As a matter of fact there is a close relationship between the theories of heat, electrostatics, and current flow which is established clearly by the following table :—

TABLE 13.—*Analogy between Thermal, Dielectric, and Electric Circuits.*

Thermal Circuit.	Dielectric Circuit.	Electric Circuit.
Thermal flux	Dielectric flux	Current
Thermal resistance	Elastance	Resistance
Temperature difference	P.D.	P.D.

This analogy is found exceedingly useful when dealing with multi-core cables. Thus, supposing that the conductors of a multi-core cable are bunched together and an alternating pressure applied between the conductors and sheath, then the path taken by the dielectric flux is also the path taken by the leakage current, and the heat generated in the conductors. Also the equipotential lines coincide with the isothermal lines.

If, then, in a three-core cable,

R' = insulation resistance per centimetre length of cable,

S' = thermal resistance „ „ „

and C' = capacitance „ „ „

all these values being for the arrangement of the three conductors against the sheath, it is easy to show that

$$R'C' = \frac{\rho\epsilon}{4\pi}; \quad S'C' = \frac{K\epsilon}{4\pi}; \quad \frac{S'}{R'} = \frac{K}{\rho}.$$

where ρ is the volume resistivity, K is the thermal resistivity, and ϵ the permittivity of the dielectric. If these physical constants are known, the value of any two of the three quantities R', S', and C' can be determined, provided that the third is known either by calculation or experiment.

The calculation of these quantities for a three-core cable is a matter of some difficulty. Russell[3] has attacked the electrostatic problem, however, and his value for the capacitance of the three conductors of a three-core cable against the sheath is

$$C' = \frac{3\epsilon}{2\log_e\left(\frac{R^6 - a^6}{3R^3a^2r}\right)} \text{ electrostatic units per cm.,} \quad (134)$$

where R = inside radius of lead sheath
a = radius of circle on which lie the centres of the conductors,
and r = radius of conductors.

Hence making use of the relationships given above, the corresponding value for the thermal resistance is

$$S' = \frac{K}{6\pi}\log_e\left(\frac{R^6 - a^6}{3R^3a^2r}\right) \text{ thermal ohms per cm.} \quad . \quad (135)$$

Mie,[4] also, has gone into the matter, and gives the following formula for the thermal resistance of a three-core cable :—

$$S' = \frac{K}{6\pi}\log_e\frac{1 - a\beta + \sqrt{(1 - a^2)(1 - \beta^2)}}{a - \beta}$$

thermal ohms per cm., (136)

where $a = \left(\frac{a + r}{R}\right)^3$

and $\beta = \left(\frac{a - 3r}{a + 3r}\right)a.$

For purposes of comparison the values of thermal resistance between conductors and sheath for typical three-core 0·20 sq. in. cables have been worked out, and are given in Table 14. In each case the value of the thermal resistivity has been taken as 550.

TABLE 14.—*Thermal Resistance of Three-core 0·20 sq. in. Cables as given by Russell's and by Mie's Formulæ.*

Working Pressure in kV.	a.	r.	R.	Calculated Value of Thermal Resistance.	
				Russell.	Mie.
11	1·29 cm.	0·735 cm.	2·79 cm.	51·6	40·8
33	1·58 „	0·735 „	3·58 „	61·6	53·8
66	1·92 „	0·735 „	4·18 „	63·8	57·6

It is seen that there is a considerable difference between the results obtained by the two formulæ, and this is due to various approximations introduced in the course of obtaining the mathematical solutions. With both formulæ, however, the error is less when small conductors and thick insulation is being considered, which corresponds to the case of cables for the higher pressures.

It is interesting to note that Russell's formula gives values of thermal resistance which are too high, and Mie's formula gives results rather too low.

It appears that neither of these theoretical formulæ are completely satisfactory, and Wedmore and Simon have shown that more exact values for the thermal resistance can be obtained by a geometrical method. In brief, this method consists in drawing a large-scale section of the conductors and sheath, and plotting therein the flow lines either from first principles, or else by modification of the flow lines as calculated by one of the theoretical formulæ. Wedmore [1] gives curve showing the ratio of thermal resistance of an eccentric conductor to sheath as a ratio of the value for a similar conductor placed centrally. Simon [5] has expressed his results in the form of an empirical formula, and states that the thermal resistance of a three-core cable, to a close approximation, is

$$S' = \frac{K}{6\pi}\left(0{\cdot}85 + \frac{0{\cdot}2t}{T}\right)\log_e\left[\left(8{\cdot}3 - \frac{2{\cdot}2t}{T}\right)\frac{T+t}{d} + 1\right]$$

thermal ohms per cm., (137)

where t = belt-insulation thickness,

T = conductor-insulation thickness,

and d = conductor diameter.

Fortunately, a check on any suggested formula can be obtained by experimental methods of solution. These methods are based on the analogy existing between the electrostatic, electric, and thermal fields of the three conductors against the sheath. Thus Atkinson [6] and Sachetto [7] have measured the ohmic resistance between the bunched conductors and the sheath of three-core cable models, composed of three tubes uniformly spaced, and surrounded by a fourth tube, all located in a tank of electrolyte. Melsom and Fawssett [1] have used an electrostatic model of the cable and measured the capacitance between three inner tubes representing the conductors, and an outer tube corresponding to the sheath.

In this connection, Fig. 57, showing the lines of heat flow and isothermal lines in a three-core cable, and reproduced by courtesy of Mr. W. A. Del Mar of the Habirshaw Electric Cable Co., is of interest. This diagram, also, was obtained experimentally by exploring the potential distribution in a tank of electrolyte containing electrodes to represent the cable.

From a comparison between Atkinson's experimental data and

values obtained by calculation from Simon's formula it appears that the latter is capable of giving results correct to within about 2 %. Furthermore, it is simpler in use, and S' may be readily calculated on a slide rule which it would be dangerous to do with formulæ (135) and (136) seeing they are 6th degree expressions.

Applying Simon's formula to the case of the three-core 0·20 sq. in. cables previously taken, the values of the thermal resistance between conductors and sheath are as follows :—

11 000 V. cable	. .	43·6	thermal ohms per cm.
33 000 V. „	. .	56·2	„ „ „
66 000 V. „	. .	59·8	„ „ „

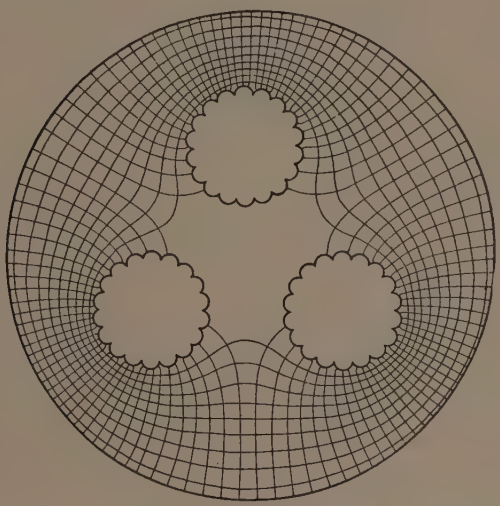

Fig. 57.—Lines of heat flow and isothermal lines in a three-core cable.

K being taken as 550. These values check very closely with those derived from the curves given by Wedmore.

The thermal resistance S of an armoured cable may be conveniently regarded as made up of two portions, S' and S'', such that $S' + S'' = S$, and where

S' = thermal resistance between conductors and sheath

S'' = thermal resistance between sheath and outer surface of cable (protective covering).

The thermal resistivity of lead is only about 2·9 and so the influence of the sheath itself can be neglected. Thus, with a

temperature gradient of 20 degrees C. between conductors and outside of sheath, the gradient in the sheath itself would be of the order of 0·1 degree C. only. As regards the effect of the protective covering, it is sufficiently accurate to assume it as being thermally homogeneous, with a mean value of thermal resistivity K_1. An average value of K_1 for the protective covering (wire armouring, bedding, braiding and serving) is 300.

The thermal resistance of the covering is then

$$S'' = \frac{K_1}{2\pi} \log_e \frac{R_2}{R_1} \quad . \quad . \quad . \quad . \quad (138)$$

where R_2 = external radius of outer covering of cable,
and R_1 = outer radius of lead sheath.

Taking the case of the 33 000-volt, 0·20 sq. in., three-core cable, the values of R_2 and R_1 will be about 5·1 cm. and 4·0 cm. respectively.

Hence

$$S'' = \frac{300}{2\pi} \log_e \frac{5{\cdot}1}{4{\cdot}0}$$
$$= 11{\cdot}6 \quad \text{thermal ohms per cm.},$$

and the thermal resistance of the complete cable is

$$S = 56{\cdot}2 + 11{\cdot}9$$
$$= 68{\cdot}1 \quad \text{thermal ohms per cm.}$$

115. Thermal Resistance of the Soil.—Analogously to the case of heat transmission through the cable dielectric and protective covering, there are two factors to be taken into consideration in determining the thermal resistance of the soil:—

1. The thermal resistivity of the soil.
2. The lines of heat flow from the cable.

The thermal resistivity of the soil depends largely upon the amount of moisture present, and partly upon the type of soil. In this connection Griffiths [1] has conducted a series of tests both with sandy loam and heavy clay in order to determine the effect of varying moisture content, and curves showing some of the results obtained are shown in Fig. 58.

As a result of a large amount of data collected from various parts of the country, it appears that a fair average would be 10 °/₀ of moisture for sandy loam, and 20 °/₀ for clay soil, corresponding to thermal resistivities of about 120 and 160 respectively. How-

ever, large differences of moisture content occur in practice, and it would appear advisable to make actual observation of the condition of the soil in all cases where it is possible This is more important where cables are worked on high load factor, and thus run for long periods at or near their maximum current loading. In order to

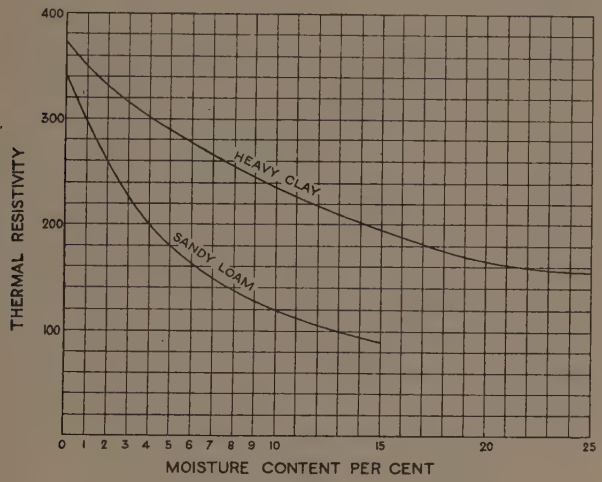

FIG. 58.—Thermal resistivity of soil.

give an idea of the influence of various soil resistivities on the current-carrying capacities of cables, Table 15 shows the current loading of an 11 000-volt, 0·10 sq. in., three-core cable laid direct, at a depth of 3 feet. The effect, however, varies with different sizes and types of cable.

TABLE 15.—*Effect of Soil Resistivity on Current-carrying Capacity of an* 11 000*-volt,* 0·10 *sq. in., Three-core Cable.*

Thermal Resistivity of Soil, *g.*	Maximum Permissible Current.
90	225 amperes
120	213 „
180	193 „
340	160 „

The exact nature of the heat flow through the soil has been the subject of much discussion, but a universally accepted conclusion

has not yet been reached. Kennelly[8] assumes that the surface of the ground above the cable is a plane isothermal, implying that all the heat is ultimately transmitted to the surface of the ground, which, being in contact with free air, remains at a uniform temperature. On this assumption it can be shown that the thermal resistance of the soil

$$G = \frac{g}{2\pi} \log_e \frac{2l}{R_2} \quad \text{thermal ohms per cm.} \qquad . \quad (139)$$

where g = thermal resistivity of the soil.
l = depth of cable axis below surface of ground.
and R_2 = overall radius of cable (supposed small in comparison with l).

According to Apt[9] the effect of the earth surrounding the cable can be considered to be equivalent to that of a cylinder of material equal in resistivity to the soil, and having a radius equal to the depth of the cable axis below the surface of the ground. On this assumption the cable is surrounded by a series of concentric isothermals and

$$G = \frac{g}{2\pi} \log_e \frac{l}{R_2} \quad \text{thermal ohms per cm.} \qquad . \quad (140)$$

Neither of the fundamental assumptions on which these formulæ are based appear to correspond to the actual heat flow obtaining in practice. For instance, the formula of Kennelly which has been very widely used assumes that the surface of the ground is at uniform temperature, but many cases are known of the route of a heavily-loaded cable being clearly traced by the drying of the soil immediately above it, which suggests a higher temperature for the soil immediately over the cable. An attempt has been made therefore, in connection with the British research, to determine experimentally the shape of the isothermals of heat flow under ideal conditions. Furthermore, a large series of tests have been made on cables of various sizes laid direct in the ground, and from these results, considered in conjunction with the information obtained from the heat flow experiments, it is suggested that the following modification of Kennelly's formula should be used :—

$$G = \frac{g}{3\pi} \log_e \frac{2l}{R_2} \quad \text{thermal ohms per cm.} \qquad . \quad (141)$$

Taking as a numerical example, the 33 000-volt, 0·20 sq. in., three-core cable whose thermal resistance has been previously

calculated, and supposing it buried at a depth of 3 feet in soil of thermal resistivity 120, then according to formula (141)

$$G = \frac{120}{3\pi} \log_e \frac{182{\cdot}8}{5{\cdot}1}$$
$$= 45{\cdot}4.$$

The current loading of this cable *in situ* can now be calculated by means of formula (132)—

$$I = \sqrt{\frac{\theta}{nR_\theta(S + G)}}.$$

Let the air temperature be taken as 15° C., and the maximum permissible temperature rise of the cable (θ) as 50 degrees C.

The resistance per kilometre of conductor at 15° C. from tables is 0·133 ohms. Hence the resistance at 65° C. is

$$R_\theta = 1{\cdot}33 \times 10^{-6} \times (1 + 0{\cdot}004 \times 50)$$
$$= 1{\cdot}60 \times 10^{-6} \text{ ohms per cm.}$$

Also

$$n = 3$$

and

$$S + G = 67{\cdot}8 + 45{\cdot}4$$
$$= 113{\cdot}2 \text{ thermal ohms per cm.,}$$

therefore

$$I = \sqrt{\frac{50}{3 \times 1{\cdot}60 \times 10^{-6} \times 113{\cdot}2}}$$
$$= 303 \text{ amperes.}$$

116. Grouped Cables Laid Direct in the Ground.—Hitherto, only cables laid singly have been dealt with. In practice, however, cables are often laid near each other in the same trench, and in such cases the mutual heating effect must be taken into consideration. If the permissible temperature rise of the cable when running alone is θ_1, and θ_x is that due to diffusion from another cable, then the current loading must be reduced in the ratio

$$\sqrt{\frac{\theta_1 - \theta_x}{\theta_1}}, \quad . \quad . \quad . \quad . \quad (142)$$

if this permissible temperature rise is not to be exceeded.

For instance, consider the case of two similar cables lying in the same horizontal plane, with their axes at a depth l centimetres below the ground surface.

Then the temperature rise of one cable due to its own losses is

$$\theta_1 = H\left(S + \frac{g}{3\pi}\log_e\frac{2l}{R_2}\right).$$

If the interaxial spacing is x centimetres, the temperature of the isothermal passing through one cable, due to the heat generated in the other, is

$$\theta_x = H\left(\frac{g}{3\pi}\log_e\frac{2l}{x}\right),$$

the latter term being derived on the assumption that the cables are not spaced too far apart in comparison with the depth of laying.

Hence substituting in (142), the ratio of current reduction necessary is

$$\sqrt{\frac{H\left(S + \frac{g}{3\pi}\log_e\frac{2l}{R_2} - \frac{g}{3\pi}\log_e\frac{2l}{x}\right)}{H\left(S + \frac{g}{3\pi}\log_e\frac{2l}{R_2}\right)}}.$$

$$= \sqrt{\frac{\left(S + \frac{g}{3\pi}\log_e\frac{x}{R_2}\right)}{\left(S + \frac{g}{3\pi}\log_e\frac{2l}{R_2}\right)}}. \quad . \quad . \quad . \quad . \quad (143)$$

For example, take the case of two 33 000-volt, 0·20 sq. in., three-core cables buried at a depth of 3 feet, and spaced at an interaxial distance of 12 inches: $g = 120$.

$$\frac{g}{3\pi}\log_e\frac{2l}{R_2} = 45{\cdot}4,$$

$$\frac{g}{3\pi}\log_e\frac{x}{R_2} = 22{\cdot}7,$$

and from previous calculation

$$S = 67{\cdot}8,$$

so that due to mutual heating effect the current loading of the cables are reduced in the ratio

$$\sqrt{\frac{67{\cdot}8 + 22{\cdot}7}{67{\cdot}8 + 45{\cdot}4}} \quad . \quad . \quad . \quad . \quad (143)$$

$$= 0{\cdot}89.$$

117. Cables Laid on the Solid System.—It does not appear likely that this method will be much employed with high-voltage transmission cables, and there is not much published data on current-carrying capacities of cables laid under these conditions. Where this

method of laying is adopted, plain lead-sheathed cables are employed and there are clearly four factors to be taken into consideration, *viz.* the thermal resistance of (*a*) the cable dielectric, (*b*) the trough-filling compound, (*c*) the trough itself, and (*d*) the surrounding earth. One reason for the lack of current-loading tables is the uncertainty regarding the thermal resistivity of the trough-filling compound. Pure bitumen itself has a thermal resistivity of about 500, but other materials are in use as filling compounds with resistivity values varying from 200 to 500. More information is also required on the thermal resistivities of non-metallic troughing. Provided, however, that the thermal resistivities of the various elements of the heat path are known or can be obtained, the current loading of a cable laid on the solid system can be calculated in an analogous manner to that employed for cables buried direct in the soil.

Generally speaking, the solid system of laying does not ensure as good facilities for heat dissipation as is obtained with cables laid direct in the earth, and thus the current loading must usually be decreased. The British Insulated Cables, Ltd., for instance, give reduction factors for multiplying the current loading of armoured cables laid direct, to obtain safe loadings for plain lead-sheathed cables laid solid in stoneware troughs. The value of the reduction factor varies from 0·94 for a three-core 0·10 sq. in. cable, to 0·97 for a 0·30 sq. in. cable, but these figures must only be taken as approximate as they depend to some extent on the voltage class of the cables under consideration.

118. Current-carrying Capacity of Cables in Air.—It is convenient to discuss the current loading of cables suspended in still air before considering the case of cables drawn into ducts. The general method of calculation is the same as that used for buried cables, except that the term G in formula (132) is replaced by A, the resistance to the flow of heat from the outside surface of the cable to the surrounding air. If h is the emissivity of the cable surface expressed in watts per square centimetre per degree C. temperature rise of surface above the surrounding air, then the equivalent thermal resistance per centimetre length of a plain lead-sheathed cable is

$$A = \frac{1}{2\pi h R_1}, \quad . \quad . \quad . \quad . \quad (144)$$

where R_1 = outside radius of sheath.

The emissivity constant h has values ranging from 0·0008 to 0·0011, depending on the nature of the cable surface. When the sheath of a plain lead-sheathed cable is new and bright, the value is distinctly lower, but when the surface of the lead has become dull, the emissivity constant is substantially the same as for a black surface. There appears to be no great and constant difference between the emissivity constant for the black surface of an armoured cable, and that for a plain lead-sheathed cable, and for purposes of calculation a mean value of $h = 0{\cdot}0009$ may be taken.

119. Cables Drawn into Ducts.—The determination of the current-carrying capacity of cables drawn into ducts is considerably simplified by the fact that the temperature rise due to any given current loading is practically the same as the temperature rise that would occur in air with the same value of current. As a result of experiment, it is found that the factor connecting the values of current, for an equivalent rise of temperature in air, varies from 0·935 to 0·98. Apparently the greater part of the resistance to the flow of heat from the external surface of the cable is in the layer of air immediately around the latter, and the relative sizes of cable and duct, and the material of the latter, make little difference to the final result. Taking as an average value of the reduction factor 0·95, the current-carrying capacity of a cable drawn into a duct can be found by calculating the permissible current loading in air, and multiplying by this factor.

For example, take a 33 000-volt, 0·20 sq. in., three-core, plain lead-sheathed cable, the thermal resistance between conductors and sheath being 56·2.

The equivalent thermal resistance outside the sheath when surrounded by air is

$$A = \frac{1}{2\pi h R_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (144)$$

$$= \frac{1}{2\pi \times {\cdot}0009 \times 4}$$

$$= 44{\cdot}2 \quad \text{thermal ohms per cm.},$$

the total thermal resistance of the heat path being thus 100·4.

The current-carrying capacity in air for a permissible temperature rise of 35 degrees C. is

$$I = \sqrt{\frac{35}{3 \times 100{\cdot}4 \times 10^{-6} \times 1{\cdot}52}} \quad . \quad . \quad (132)$$

$$= 276 \quad \text{amperes},$$

and the current loading when drawn into a duct is

$$= 276 \times 0{\cdot}95$$
$$= 262 \text{ amperes.}$$

When cables are run in close proximity in multi-way ducts, a considerable decrease in current-carrying capacity has to be made, but not much experimental work has yet been done on this subject. The figures shown in Table 16 may be taken as approximate reduction factor for cables laid in this way :—

TABLE 16.—*Approximate Reduction Factors for Current Loading of Cables Grouped in Ducts.*

2 cables . . .	0·92	5 cables . . .	0·75
3 „ . . .	0·85	6 „ . . .	0·70
4 „ . . .	0·79	8 „ . . .	0·63

120. Comparative Current-carrying Capacities of Cables Laid under Various Conditions.—A rough idea of the effect of different laying conditions on the current loading of a typical high-voltage cable is given in Table 17 ; the carrying capacity of the cable when armoured and laid direct in the soil being taken as 100.

TABLE 17.—*Comparative Current Loadings of a Typical High-voltage Three-core Cable Laid under Various Conditions.*

Temperature rise 50 degrees C. in each case.

Method of Laying.	Comparative Current Loading.
Lead-covered cable in water	142
Armoured cable in water	129
Lead-covered cable in air	106
Armoured cable in air	105
Lead-covered cable in duct	101
Armoured cable laid direct	100
Lead-covered cable laid solid	96

It will be noticed that submarine laying permits of an increase of rating of about 40 %. This is due to the fact that the lead then assumes the water temperature, particularly if the water is circulating, and the only rise of temperature of the conductor above that of the water is that across the dielectric. As regards the three common methods of laying underground cables, there does not appear to be very much difference between the current ratings, but

it should be noted that in each case the same temperature rise has been assumed for purposes of comparison. In the case of cables drawn into ducts, however, a lower operating temperature is advised in order to avoid trouble due to expansion effects. Taking the permissible temperature rise for the latter method of laying as 35 degrees C. instead of 50 degrees C., the current rating of the cable would be reduced from 101 to 84.

121. Temperature Rise as a Function of Time.—So far in dealing with the heating of underground cables, only the ultimate temperatures attained have been considered. The period

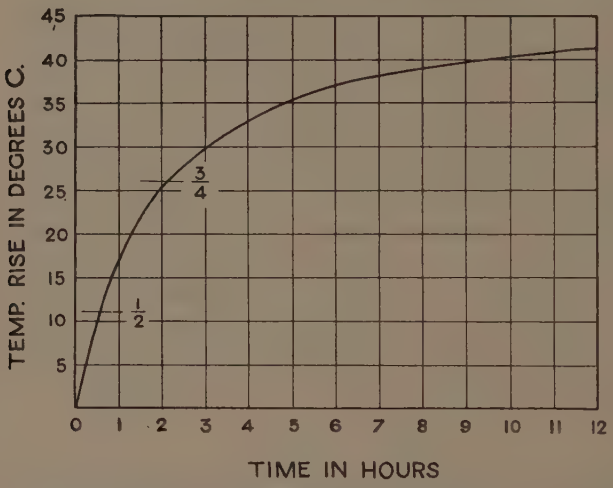

Fig. 59.—Temperature rise of 20 000-volt, 0·35 sq. in., three-core armoured cable laid direct in the ground; ultimate temperature rise = 50 degrees C.

often necessary for the attainment of an absolutely steady state is measured in days, rather than in hours, but varies with different classes of cable and different methods of laying. The rate at which the temperature rises is also considerably influenced by the specific heat of the soil, as it requires time to heat the layers of soil until they become steady isothermal surfaces. Table 18 may be taken as a rough comparison of different methods of laying as regards their effect on the rate of increase of temperature, and Fig. 59 shows an experimentally-obtained time / temperature rise curve for a 20 000-volt, 0·35 sq. in., three-core cable laid direct in the ground, the final temperature rise being 50 degrees C.

TABLE 18.—*Approximate Time Required for Temperature Rise to attain 90 °/₀ of its Final Value, assuming a Steady Load.*

Method of Laying.	Time.
Suspended in air	3 hours
Laid direct in ground	15 ,,
Laid in ducts	10 ,,
Laid solid in bitumen (stoneware troughs)	10 ,,

122. Current-carrying Capacity of Cables for Emergency Loading.—A higher current density can be allowed, without exceeding the specified temperature rise, in cases where the load is only of short duration. The permissible overload depends on the previous temperature of the cable, and may be very considerable if the cable has been operating on one-half or less of the normal maximum load. To determine this permissible emergency load a time / temperature rise curve for the cable *in situ* is required for the specified temperature rise (say 50 degrees C.).

To illustrate the method employed, take the simplest case where the cable has not previously been carrying any load. The permissible current is deduced by setting off the emergency time on the time / temperature rise curve starting from zero; then the square root of the ratio that 50 bears to the temperature rise reached in the short time selected, gives the multiplier for that time, to apply to the continuous permissible current. Thus, referring to Fig. 59, the temperature rise after 2 hours is approximately 25 degrees C., so the multiplying factor is 1·41, which means that the particular cable under consideration will carry for 2 hours starting from cold 41 °/₀ more current than the maximum continuous rating, without exceeding the specified temperature rise of 50 degrees C.

If the cable has previously been carrying a continuous load the relative temperature rise for this load can be calculated from formula (132), and this rise is taken as the starting-point on the time / temperature rise curve. The emergency time is then set off from this point, and the total temperature rise from zero read off and treated as before, thus giving the multiplying factor for the specified conditions.

In the curve of Fig. 59 has been marked the final temperature

points 11 degrees C. for one-half normal, and 26 degrees C. for three-quarters normal, load respectively. Supposing, then, that the 0·35 sq. in. 20 000-volt cable has been previously carrying a steady load equal to one-half its rating, and it is required to find what load can be carried for an emergency period of two hours. The temperature rise corresponding to the initial load on the cable is 11 degrees C., and marking off from this point on the curve the scale value of 2 hours horizontally, a temperature rise of 28 degrees C. is obtained. Then the multiplying factor is $\sqrt{\frac{50}{28}} = 1{\cdot}33$, or a permissible overload of 33 % can be carried for the two hours, without exceeding the limiting temperature rise of 50 degrees C.

Considering cables of the same voltage class, it is found that the greater the size of conductor the greater is the permissible overload. The values given in Table 19 apply to two cables similar in every way except size, and show widely different emergency factors. High-voltage cables also exhibit different characteristics from low-voltage ones, and hence a time / temperature rise curve should be taken for each case on which accurate information is sought.

TABLE 19.—*Multiplying Factors to obtain Emergency Current from Maximum Permissible Continuous Loading.*

20 000-volt three-core cable laid direct in ground.

Size of Cable.	0·15 Sq. In.		0·35 Sq. In.	
Previous Proportion of Full Load.	Half.	Three-quarter.	Half.	Three-quarter.
Emergency Time in Hours.				
1	1·20	1·10	1·48	1·28
2	1·12	1·06	1·32	1·22
3	1·09	1·04	1·25	1·19
4	1·06	1·03	1·20	1·16

123. Capacitance and Insulation Resistance of Single-core Cables.—In the transmission of power by single-core, lead-sheathed cables the three cables forming a three-phase system are laid close together in a concrete conduit or in multi-way ducts, the lead sheaths being generally bonded together and earthed at

several points. The three sheaths are thus maintained at zero potential, and in determining the capacitance of a conductor it is only necessary to consider the dielectric field existing between the conductor and sheath. Let Fig. 60 represent a section of a single-core cable having a conductor radius of r centimetres, and an internal sheath radius of R centimetres. If a charge of q units per centimetre length is given to the conductor, lines of dielectric flux pass out radially, and produce at a distance x centimetres from the axis of the cable a flux density of

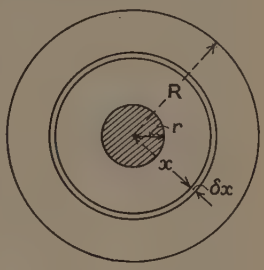

FIG. 60. — Capacitance and insulation resistance of a single-core cable.

$$D = \frac{4\pi q}{2\pi x}$$

$$= \frac{2q}{x} \text{ lines per square centimetre.}$$

If the permittivity of the insulating material is ϵ, the electrostatic force at the point is

$$F = \frac{D}{\epsilon} = \frac{2q}{\epsilon x} \text{ dynes,} \quad . \quad . \quad . \quad (145)$$

and the potential difference between the conductor and sheath is

$$E = \int_r^R \frac{2q}{\epsilon} \frac{dx}{x}$$

$$= \frac{2q}{\epsilon} \log_e \frac{R}{r} \text{ ergs.} \quad . \quad . \quad . \quad (146)$$

The capacitance per centimetre length of the cable is

$$\frac{q}{E} = \frac{q}{\frac{2q}{\epsilon} \log_e \frac{R}{r}}$$

$$= \frac{\epsilon}{2 \log_e \frac{R}{r}} \text{ electrostatic units,}$$

or expressed in terms of practical units,

$$C_0 = \frac{\epsilon \times 2{\cdot}54 \times 12 \times 5\,280}{2{\cdot}303 \log_{10} \frac{R}{r} \times 9 \times 10^{11}}$$

$$= \frac{0{\cdot}0388\epsilon}{\log_{10} \frac{R}{r}} \text{ microfarads per mile.} \quad . \quad . \quad (147)$$

Considering again a 1 centimetre length of cable, the insulation resistance of a thin layer at a radial distance x centimetres from the cable axis is

$$dR' = \frac{\rho dx}{2\pi x},$$

where dx is the thickness of the layer, and ρ its volume resistivity.

Hence the insulation resistance per centimetre length of cable is

$$R' = \frac{\rho}{2\pi}\int_r^R \frac{dx}{x}$$
$$= \frac{\rho}{2\pi}\log_e \frac{R}{r},$$

or expressed in mile units,

$$R' = 2{\cdot}28\rho \log_{10} \frac{R}{r} \times 10^{-12} \text{ megohms per mile.} \quad (148)$$

Average values of ϵ and ρ for impregnated paper insulation are

$$\epsilon = 3{\cdot}3,$$
$$\rho = 5 - 8 \times 10^{14} \text{ ohms},$$

these figures being taken at about 15° C.

The permittivity increases slightly with rise of temperature, the increase being of the order of 10 % for a temperature rise of 60 degrees C.

Temperature, however, has a very marked effect on the resistivity of impregnated paper, which decreases rapidly with increase of temperature. This variation in resistivity may be represented closely by the expression

$$\rho_t = \rho_0 e^{-Kt},$$

where the coefficient K has a value of about 0·04 or 0·05.

124. Inductance and Resistance of Single-core Cables.—In the case of single-core cables, the fact of there being a separate lead sheath around each conductor often has a considerable influence on the electromagnetic properties of the circuit, and it is the effective values of reactance and resistance which must be determined and used for purposes of calculation. The lead sheath is, in effect, a conductor parallel to the copper conductor of the cable, and therefore when a current flows along the cable it will induce a voltage along the sheath tending to cause the flow of current

lengthwise of the sheath, as well as numerous secondary reactions in other directions. The actual current flowing in the sheath depends on the manner of laying.

Following the classification of Cramp and Calderwood,[10] the sheath currents may be divided into two main groups:—

1. Eddy currents whose outward and return paths lie entirely in the sheath of one cable, called 'sheath eddies.'

2. Eddy currents whose outward path is along the sheath of one cable and return path along the sheath of another, called 'sheath-circuit eddies.'

Supposing that the lead sheaths of the cables are open-circuited, either by being entirely insulated from each other or by being electrically interconnected and earthed only at one end of the run, the sheath-circuit eddies cannot exist. The sheath eddies, however, are able to circulate, and there will also be a certain voltage induced between lead sheath and earth at the far end. If the sheaths are short-circuited by bonding together and earthing at each end of the run, these bonds are, in effect, a short-circuit of the voltage induced along the sheath. There will then be a flow of sheath-circuit eddies along the sheaths.

Lead Sheaths Open-circuited (Unearthed or Earthed at One End of Run only).—In this case only sheath eddies can exist, and a clear account of the distribution of these currents has been given by Cramp and Calderwood. The power loss due to the currents is greatest when the sheaths are closest together, and diminishes as the distance between neighbouring cables is increased. Under any circumstances the magnitude of the currents are small, and the power loss due to them can be ignored in practice.

For example, take the case of two 0·30 sq. in., 37-strand, single-core cables to be used on a 66 000-volt, 50-frequency, single-phase transmission: the dimensions of the cables being as follows:—

Radius of conductor (r)	= 0·915	centimetres.
Internal radius of sheath (R)	= 2·5	,,
External radius of sheath (R_1)	= 2·8	,,

Then, under the worst possible conditions, *i.e.* with the sheaths practically in contact, the ratio of the sheath loss to that of the copper loss in the conductor is less than 2 %. Hence, when laid

with open-circuited sheaths, the losses per conductor due to the current transmitted can be taken as those due to the I^2R losses in the conductor only, and the effective resistance is the ordinary conductor resistance as obtained from tables. Similarly, the effect of the open-circuited sheath on the conductor inductance is negligible.

Lead Sheaths Short-circuited (Earthed at Each End of Run).—When the cable sheaths are bonded together and earthed at each end, they form a closed circuit enclosing an alternating flux produced by the currents in the conductors. Under these conditions, in addition to sheath eddies, there will exist sheath-circuit eddies in the lead. The power loss due to the latter is small in magnitude when the cables are touching, but increases rapidly as the distance between the cables is increased.

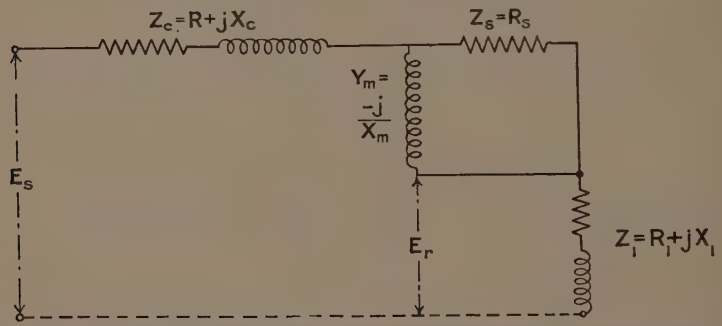

FIG. 61.—Single-core lead-covered cable represented as an inductive circuit.

The phenomenon may be investigated quantitatively by a method due to Berg.[11] In this method, the lead sheath is treated as a short-circuited secondary to the e.m.f. generated by the inductive reactance of the conductor. It consumes some of the e.m.f. and reduces the inductive drop, but on the other hand increases the ohmic power losses of the circuit. Hence, for purposes of calculation, there will be substituted for the conductor the primary of a transformer of the same resistance R as the conductor, and having a magnetising current the same as the conductor current at the voltage corresponding to the reactance e.m.f. The reactance X_c of the primary in this equivalent transformer is due to the flux between the lead sheath and the conductor, and is therefore quite small. The secondary resistance R_s corresponds to the resistance of the lead sheath, while the reactance

is zero. The primary admittance Y_m of the equivalent transformer is $\frac{-j}{X_m}$, where X_m is the mutual reactance between the conductor and sheath. The circuit can thus be represented diagrammatically as in Fig. 61.

Let E_s = impressed voltage from conductor to neutral,
$Z_1 = R_1 + jX_1$ = impedance of load.

The joint admittance of the two parallel circuits of impedance X_m and R_s respectively is $\frac{-j}{X_m} + \frac{1}{R_s}$,

and the corresponding impedance

$$Z_2 = \frac{1}{\frac{-j}{X_m} + \frac{1}{R_s}} = \frac{X_m R_s}{-jR_s + X_m} = \frac{X_m R_s (X_m + jR_s)}{R_s^2 + X_m^2}.$$

The total impedance is

$$Z_c + Z_2 + Z_1,$$

the current is

$$\frac{E_s}{Z_c + Z_2 + Z_1},$$

the voltage across the load is

$$\frac{E_s Z_1}{Z_c + Z_2 + Z_1},$$

and the voltage drop in the line is

$$E_s - \frac{E_s Z_1}{Z_c + Z_2 + Z_1} = \frac{E_s(Z_c + Z_2)}{Z_c + Z_2 + Z_1}. \quad . \quad . \quad . \quad . \quad (149)$$

Now in an ordinary transmission line, if Z_{eff} is the impedance of each line conductor, the current is

$$\frac{E_s}{Z_{\text{eff}} + Z_1},$$

and the voltage drop in the line is

$$\frac{E_s Z_{\text{eff}}}{Z_{\text{eff}} + Z_1}. \quad . \quad . \quad . \quad . \quad (150)$$

Comparing (149) and (150) it is evident that the ordinary line equation can always be used for purposes of calculation if

$$\begin{aligned} Z_{\text{eff}} &= Z_c + Z_2 \\ &= R + jX_c + \frac{X_m R_s (X_m + jR_s)}{R_s^2 + X_m^2} \\ &= R + \frac{X_m^2 R_s}{R_s^2 + X_m^2} + j\left(X_c + \frac{X_m R_s^2}{R_s^2 + X_m^2}\right). \quad . \quad (151) \end{aligned}$$

Hence, with short-circuited sheath, the resistance of the conductor is increased from R to the effective value

$$R_{\text{eff}} = R + \frac{X_m^2 R_s}{R_s^2 + X_m^2}. \quad . \quad . \quad . \quad (152)$$

That is to say, this value of resistance, when multiplied by the square of the current flowing in the conductor, gives the total power loss per phase in both the conductor and the sheath.

Also, due to the mutual inductive effect of the sheath, the conductor reactance is somewhat decreased. For with open-circuited sheath $R_s = \infty$ and the reactance has its normal value

$$X = X_c + X_m, \quad . \quad . \quad . \quad . \quad (153)$$

whereas with short-circuited sheath the effective value is

$$X_{\text{eff}} = X_c + \frac{X_m R_s^2}{R_s^2 + X_m^2}, \quad . \quad . \quad . \quad (154)$$

and substituting for X_c from (153) gives

$$X_{\text{eff}} = X - \frac{X_m^3}{R_s^2 + X_m^2}. \quad . \quad . \quad . \quad (155)$$

To illustrate the influence of the sheath, the effective resistance and inductance of the 0·30 sq. in. cable has been calculated for different spacings, both with open-circuited and short-circuited sheath, and the results are shown in Table 20. The column, showing the percentage increase in power losses due to the latter method of installation, is specially interesting.

TABLE 20.—*Effective Resistance and Reactance per Mile of 0·30 sq. in. Single-core Cable.*

$r = 0{\cdot}915$ cm. Temp. of conductor = 50° C.
$R = 2{\cdot}5$,, Temp. of sheath = 30° C.
$R_1 = 2{\cdot}8$,, $\rho_s = 23{\cdot}2 \times 10^{-6}$ ohms.
Frequency = 50 cycles.

Interaxial Spacing.	Sheath Open-circuited.			Sheath Short-circuited.		
	Resistance, R.	Reactance, $X = X_c + X_m$.	Voltage Induced per Mile, IX_m ($I = 400$ amperes).	Resistance, $R + \frac{X_m^2 R_s}{R_s^2 + X_m^2}$.	Reactance, $X - \frac{X_m^3}{R_s^2 + X_m^2}$.	Increase in Power Losses Due to Sheath.
2·4 ins.	0·159 ohms	0·218 ohms	33·6 volts	0·168 ohms	0·217 ohms	5·9 %
6 ,,	0·159 ,,	0·311 ,,	70·8 ,,	0·199 ,,	0·302 ,,	25·1 ,,
12 ,,	0·159 ,,	0·381 ,,	98·8 ,,	0·233 ,,	0·357 ,,	46·3 ,,
24 ,,	0·159 ,,	0·452 ,,	127 ,,	0·297 ,,	0·406 ,,	86·9 ,,
60 ,,	0·159 ,,	0·544 ,,	164 ,,	0·332 ,,	0·449 ,,	109 ,,

If the sheaths are left open-circuited the voltage induced therein is given by

$$e_{sh} = IX_m, \quad . \quad . \quad . \quad . \quad (156)$$

where I is the current in the conductor. The values of this voltage corresponding to a current of 400 amperes per conductor have been worked out, and are given in column 4 of the table. It will be seen that, in this case, the voltages existing between sheaths in a long run of cable may be really dangerous to those handling the cables. Furthermore, the magnitude of the effect is proportional to the current in the conductors, so that in cases of short-circuit occurring on some part of the system, when the current flowing may be fifty times its normal value, the voltage between sheath and earth may reach a high value. This condition may cause arcing between the sheath and earth, and result in serious damage to the former.

Special methods of earthing to minimise the effect of high sheath voltages without actually closing the sheath circuit have been discussed by Clark and Shanklin,[12] but these methods involve a considerable amount of complication and expense. It is usually considered the best practice to bond the sheaths together and earth directly, accepting the additional power losses incurred thereby. These additional losses can easily be kept to a reasonable magnitude by laying the cables as close to one another as possible. Probably the best method of installation is to cover the lead with a good serving of jute tape or braid, and lay the cables in troughing with the outer surfaces in contact. Cramp and Calderwood[10] have pointed out that this method is preferable to laying the cables with the lead sheaths in direct contact, as in the latter case it is possible for pitting to occur at the points where lead actually touches lead, ultimately leading to break-down of the sheath. Allowing for the thickness of the serving, the minimum interaxial spacing of the particular cable under consideration would be about 2·4 inches, and the sheath losses for short-circuited sheath at this spacing would be about 6 % of the conductor I^2R losses. This figure corresponds to a supply frequency of 50 cycles per second, but for any other frequency the losses may be easily determined as they are proportional to f^2.

The actual current flowing in the sheaths when these are short-circuited is

$$I_{sh} = \frac{IX_m}{\sqrt{X_m^2 + R_s^2}}, \quad . \quad . \quad . \quad . \quad (157)$$

and for minimum spacing ($I = 400$ amperes)

$$I_{sh} = \frac{33{\cdot}6}{\sqrt{(0{\cdot}084)^2 + (0{\cdot}747)^2}}$$
$$= 44{\cdot}7 \text{ amperes.}$$

So far, the cables have been assumed to be arranged symmetrically, but in practice the three cables forming a three-phase system are often arranged in one plane. In the latter case there is a great difference between the sheath losses of the middle cable, and the outside cables. The average losses, however, are not very far different from those obtained by calculating the line as if the three cables were equidistant at the geometric mean spacing. A further point of interest is that the two outside cables have the higher losses, and since the outer cables have usually the best facilities for heat dissipation this tends to equalise the temperature rise of the three.

It should be mentioned that throughout the present discussion on sheath losses the proximity effect of the sheath currents has been neglected. In other words, it has been assumed that the current flowing in the sheath is uniformly distributed around its circumference, although if the cables are close together it is well known that a greater proportion of the current flows in the outer parts of the sheaths, and less in the parts where the sheaths are nearest together. Dwight[13] has given an accurate, but rather tedious, method of calculating the losses in earthed cable sheaths taking this proximity effect into consideration. It is satisfactory to note, however, that the simpler formulæ developed in this article give results quite accurate enough for all ordinary line calculations.

125. Electrostatic Stresses in Single-core Cables.—A single-core cable can be represented as in Fig. 56 (a) by concentric cylinders, the radius r of the inner cylinder corresponding to the conductor radius, and the internal radius R of the outer cylinder corresponding to the internal radius of the lead sheath. Assume that the potential of the inner cylinder is E, and that of the outer one is zero. Then the electrostatic force or potential gradient at radius x is

$$g = \frac{2q}{\epsilon x}, \quad . \quad . \quad . \quad . \quad (145)$$

and from (146)

$$q = \frac{E\epsilon}{2 \log_e \frac{R}{r}}.$$

Hence the potential gradient or electrostatic stress at radius x is

$$g = \frac{E}{x \log_e \frac{R}{r}} \quad . \quad . \quad . \quad (158)$$

It has its maximum value at the surface of the inner cylinder

$$g_{max} = \frac{E}{r \log_e \frac{R}{r}} \quad . \quad . \quad . \quad (159)$$

and its minimum value at the surface of the outer cylinder

$$g_{min} = \frac{E}{R \log_e \frac{R}{r}} \quad . \quad . \quad . \quad (160)$$

so that

$$\frac{g_{max}}{g_{min}} = \frac{R}{r}. \quad . \quad . \quad . \quad . \quad (161)$$

The stresses are radial, and Fig. 56 (*b*) may be taken to represent their distribution and the equipotential lines.

As regards the permissible value of the stress in a cable dielectric, this depends on the margin of safety desired, and on the degree of dielectric heating which can be allowed. In high-voltage cables considerations of dimensions demand that dielectric stresses should be pressed to the highest practicable limit. It should be borne in mind, however, that the dielectric losses increase rapidly, not only with regard to voltage, but also (above a critical point) with temperature, so that if the stresses are fixed too high the dielectric heating, superimposed on that due to the I^2R losses in the conductors, is liable to become cumulative, with destructive effects on the cable. In practice, the dielectric strength of impregnated-paper insulation varies between 200 and 300 kilovolts per centimetre (R.M.S. values), and working stresses of the order of 40 to 50 kilovolts per centimetre (R.M.S.) may be taken as a reasonable maximum, thus giving a safety factor of about 6.

126. Economic Conductor Radius of Single-core Cable.—For a given voltage and overall diameter of single-core cable there is one definite radius of conductor which will cause the stress at the conductor surface to be a minimum. A small conductor will allow of a greater thickness of insulation, but on the other hand the smaller radius of curvature tends to increase the stress; while the effect of too large a radius of conductor is to cause an increase

of the stress through reduction of the total thickness of insulation. The stress at the conductor surface is

$$g_{max} = \frac{E}{r \log_e \frac{R}{r}} \quad . \quad . \quad . \quad (159)$$

and if E and R are constant this stress has a minimum value when

$$\log_e \frac{R}{r} = 1, \quad . \quad . \quad . \quad . \quad (162)$$

or

$$R = 2{\cdot}718r.$$

In this case the actual stress at the conductor surface

$$g_{max} = \frac{E}{r}. \quad . \quad . \quad . \quad . \quad (163)$$

For high-voltage cable the value of r determined by the above method would, in general, give a conductor of cross-section much larger than would be required from the point of view of current-carrying capacity. In such a case aluminium might be used instead of copper as a conductor material. If this does not bring the conductor radius up to the required limit, the conductor may be built up of copper wires stranded round a hemp centre. A better method still is to strand the conductor wires round an inner lead tube, and encase them with a second thin lead tube as shown in Fig. 62. This construction has the obvious advantage of not only increasing r to the required dimensions, but also of presenting a smooth surface to the dielectric instead of the corrugated surface of a stranded conductor. The previously derived formula for the electrostatic stress at the conductor surface was obtained on the assumption of a smooth cylindrical conductor, and only supplies an ideal value for this stress. With the ordinary stranded conductors in common use, the stresses near the conductor are increased, due to the greater curvature of the surface of the individual wires. Figures given by Jona [14] and Atkinson [15] show that the effect of stranding is generally to increase the maximum stress on the dielectric by about 20 %. It is, accordingly, worth while to prevent these high local stresses by encasing the conductors in a smooth lead tube.

FIG. 62. — Special core construction for high-voltage single-core cable.

127. Grading of Cables for Alternating Pressures.—It has been shown above that the electrostatic stress in a single-core cable

decreases towards the sheath, its value at a distance x from the axis being

$$g = \frac{2q}{\epsilon x}. \quad . \quad . \quad . \quad . \quad (145)$$

In a cable of homogeneous insulation, the voltage which may be safely applied is limited by the stress at the inner conductor, and if the dielectric is strong enough at this point it is too strong and needlessly expensive further away. If, however, by some means the stress can be so distributed that its value in the outer layers of the dielectric is increased without increasing the stress near the conductor, a less thickness of insulation will be required for any given operating voltage, and an important economy effected. This result may be achieved in practice by grading the cable in one of two ways :—

1. By forming the dielectric in layers of insulating material having different values of permittivity.

2. By using the same material for all layers, but inserting one or more metallic intersheaths in the dielectric concentric with the conductor. These intersheaths are not intended to carry any part of the working current but only to fix the potentials at different points in the insulation.

These methods are known as 'capacitance' grading, and 'intersheath' grading respectively.

Capacitance Grading.—It will be seen from an examination of formula (145), that if it were possible to vary the insulation of a cable so that its permittivity at every point was inversely proportional to the distance from the axis, the stress in the dielectric would be absolutely constant. This is the principle of the graded cable, first suggested by O'Gorman.[16] Such perfect grading is practically impossible, since the permittivities of materials which are suitable for the insulation of conductors do not vary to a great extent. However, very good results can be obtained by using two or three layers of dielectric.

As an illustration, take first of all the design of a 0·30 sq. in. non-graded cable for a pressure of 60 000 volts between conductor and lead. If the stress in the dielectric must not exceed 40 000 volts per centimetre (R.M.S.), the radius of the conductor for a cable of economic proportions would be

$$r = \frac{E}{g_{max}} = \frac{60\,000}{40\,000}$$
$$= 1{\cdot}5 \quad \text{centimetres.}$$

A solid stranded conductor of this radius would have a sectional area nearly three times as large as desired, and a hollow conductor constructed on the lines of Fig. 62 would probably be adopted. The internal radius of the sheath would be

$$R = 2{\cdot}718 \times 1{\cdot}5$$
$$= 4{\cdot}08 \quad \text{centimetres.}$$

Consider now the case of a cable having the same overall dimensions, but graded by means of two layers of material having permittivities ϵ_1 and ϵ_2 respectively. In order to secure the same value of maximum stress in each layer

$$\frac{2q}{\epsilon_1 r} = \frac{2q}{\epsilon_2 r_1}, \quad . \quad . \quad . \quad . \quad (145)$$

or

$$\epsilon_1 r = \epsilon_2 r_1,$$

where r_1 is the internal radius of the second layer. A 0·30 sq. in. conductor of the solid stranded type has an overall radius of 0·915 centimetres, and assuming $\epsilon_1 = 5$ and $\epsilon_2 = 3$ it follows that

$$r_1 = \frac{0{\cdot}915 \times 5}{3}$$
$$= 1{\cdot}525 \quad \text{centimetres.}$$

The voltage across the inner layer of dielectric is

$$E_1 = g_{\max} r \log_e \frac{r_1}{r},$$

and the voltage across the outer layer is

$$E_2 = g_{\max} r_1 \log_e \frac{R}{r_1}.$$

Hence the total voltage which can be safely applied to the cable is

$$E = E_1 + E_2$$
$$= g_{\max}\left(r \log_e \frac{r_1}{r} + r_1 \log_e \frac{R}{r_1}\right). \quad . \quad . \quad (164)$$

Taking as before a maximum stress of 40 000 volts per centimetre

$$E = 40\,000\left(0{\cdot}915 \log_e \frac{1{\cdot}525}{0{\cdot}915} + 1{\cdot}525 \log_e \frac{4{\cdot}08}{1{\cdot}525}\right)$$
$$= 40\,000(0{\cdot}4674 + 1{\cdot}5008)$$
$$= 78\,730 \quad \text{volts.}$$

Hence by grading the dielectric in two layers, and without increasing the overall dimensions of the cable, the safe operating voltage has been raised from 60 000 volts to 78 730 volts, being an increase of about 31 °/₀. Figs. 63 (*a*) and (*b*) show the two cables diagrammatically and the stresses in each case.

In the above discussion, the insulating materials have been assumed to have the same dielectric strength. If this is not so, the maximum stress in each layer should be made proportional to the dielectric strength of that layer, thus leading to a uniform factor of safety for the complete cable. Hence, in general, if ν_1, ν_2, etc., are the dielectric strengths of the different layers, the radii of the layers can be calculated from the relationship

$$\nu_1\epsilon_1 r = \nu_2\epsilon_2 r_1 = \nu_3\epsilon_3 r_2. \quad . \quad . \quad . \quad (165)$$

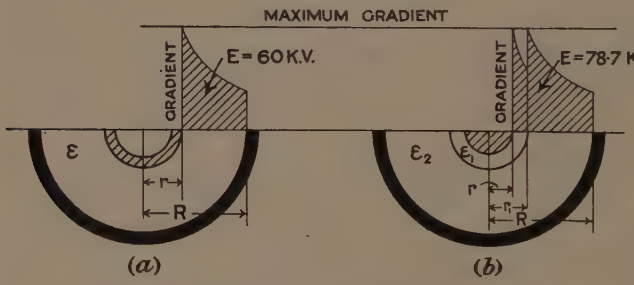

FIG. 63.—Ungraded and capacitance-graded cable.

Intersheath Grading.—In this method of grading, suggested by Morris,[17] the insulation is the same throughout the total thickness of the cable, but is divided into two or more sections by means of metallic cylinders. Each section can be made to take its proper share of the total voltage by applying a definite potential from an outside source to the intersheaths. The intersheaths can be made of lead tubes as this provides a smooth surface in contact with the dielectric, and the sole current carried by any intersheath is the difference between the charging currents taken by the sections on each side of it. The most obvious way of anchoring the potentials of the various intersheaths is by connecting them to tappings taken from the secondary winding of the power tranformers.

As an illustration of the method, suppose that the 0·30 sq. in. cable (r = 0·915 centimetres, R = 4·08 centimetres) is to be graded by means of one intersheath. If the lead intersheath is 0·125

centimetres in thickness, and is situated half-way between conductor and sheath, its internal and external radii will be $r_1 = 2{\cdot}435$ centimetres, and $r_2 = 2{\cdot}560$ centimetres respectively. The voltage across the first section is

$$E_1 = g_{\max}\left(r \log_e \frac{r_1}{r}\right),$$

and the voltage across the second section is

$$E_2 = g_{\max}\left(r_2 \log_e \frac{R}{r_2}\right).$$

Assuming as before a maximum stress of 40 000 volts per centimetre (R.M.S.), and substituting numerical values

$$\begin{aligned} E &= E_1 + E_2 \\ &= 40\,000\left(0{\cdot}915 \log_e \frac{2{\cdot}435}{0{\cdot}915} + 2{\cdot}560 \log_e \frac{4{\cdot}08}{2{\cdot}560}\right) \\ &= 83\,550 \quad \text{volts.} \end{aligned}$$

Hence by means of one intersheath, without increasing the overall dimensions of the cable, its voltage rating has been increased by about 39 % (see Fig. 64).

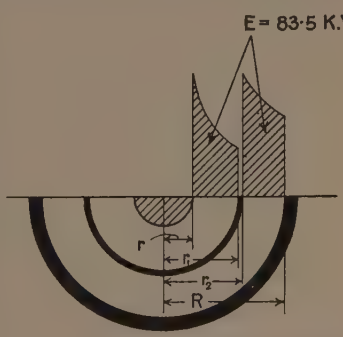

FIG. 64.—Intersheath-graded cable.

Where long lengths of cable have to be dealt with, the cross-sectional area of the intersheaths must be ample to carry the charging current of the system. Let the capacitance of the condenser formed by the conductor and the intersheath be c_1 farads, and the capacitance between the intersheath and the outer sheath of the cable be c_2 farads. Then, considering the inner condenser, a capacitance current of $\omega c_1 E_1$ amperes flows out along the conductor and returns by the intersheath. Similarly for the outer condenser, a capacitance current $\omega c_2 E_2$ flows out along the intersheath and flows back along the outer sheath. Hence the actual current carried by the sheath is

$$\omega(c_2 E_2 - c_1 E_1); \qquad . \quad . \quad . \quad (166)$$

this is of course the sending-end value. At the receiving end the capacitance current is zero, and at a point distant x from the

sending end of a cable of length l the intersheath current is approximately

$$\omega(c_2E_2 - c_1E_1)\left(1 - \frac{x}{l}\right).$$

Referring to the 0·30 sq. in. graded cable

$$c_1 = \frac{0{\cdot}\,0388 \times 3{\cdot}2}{\log_{10}\dfrac{2{\cdot}\,435}{0{\cdot}\,915}} \times 10^{-6}$$

$$= 0{\cdot}\,2921 \times 10^{-6} \quad \text{farads per mile,}$$

$$c_2 = \frac{0{\cdot}\,0388 \times 3{\cdot}2}{\log_{10}\dfrac{4{\cdot}08}{2{\cdot}\,560}} \times 10^{-6}$$

$$= 0{\cdot}\,6134 \times 10^{-6} \quad \text{farads per mile.}$$

Hence for a 20-mile 50-frequency transmission, the capacitance current in the intersheath at the sending end is

$$2\pi \times 50(47\,730 \times 0{\cdot}\,6134 - 35\,820 \times 0{\cdot}\,2921)10^{-6} \times 20$$
$$= 117{\cdot}8 \quad \text{amperes.}$$

128. Practical Difficulties Involved in Grading.—Although from a theoretical standpoint there are considerable advantages to be gained by grading, it has not yet met with any success in practice. Developments along this line, however, may possibly occur when the higher voltages for underground transmission become more common. If an arbitrary limit of 3 inches is fixed on account of difficulties in handling, and the dielectric is worked at a maximum stress of 50 kV. per cm. (R.M.S.), grading would be necessary for solid-conductor, single-core cables designed for pressures of over 70 kV. between conductor and sheath.

The chief handicap of the capacitance method of grading is due to the fact that there are such few high-grade insulating materials of reasonable cost, whose permittivities can be made to vary over the necessary range. Impregnated paper has values of permittivity ranging from 2·8 to 4·0. Higher values than this could be obtained by loading rubber with various chemicals as, according to Jona,[14] its permittivity can thereby be varied from 4·2 to 6·1. The cost of such a rubber dielectric, however, would be high, and probably prohibitive for this reason. Furthermore, in a composite dielectric of rubber and paper, beside the risk of chemical action taking place between the two layers, there is the possibility that the individual materials would not retain their permittivities unchanged from

year to year. If such a progressive increase or diminution in the values of these constants took place, the grading of the cable would be spoiled, and break-down might occur at a lower pressure than with the ungraded cable. For these reasons it seems preferable to have the insulation a homogeneous compact mass of impregnated paper.

The chief disadvantage of the intersheath method of grading is the relatively flimsy character of the intersheath, and the danger of damaging or interfering with its continuity during transportation and installation. High local potentials might then concentrate at the defective place, resulting in failure. Another point is that the length of transmission would be unduly limited, as the intersheath could not carry the heavy charging current in long lines.

129. Effect of Temperature Gradient on Distribution of Electrostatic Stresses in Single-core Cables.—In practice, when a cable is carrying load, a considerable difference of temperature exists between the conductor and sheath. Since the permittivity of insulating materials increases with rise of temperature, this internal temperature drop has a modifying effect on the distribution of the electrostatic stresses. In the case of a single-core cable having a homogeneous dielectric, the stress at a point x centimetres from the cable axis is

$$g = \frac{2q}{\epsilon x} \quad . \quad . \quad . \quad . \quad . \quad (145)$$

Now ϵ increases with increase of temperature, and therefore it increases as x decreases. Hence the influence of the internal temperature drop in the cable is to make the electrostatic stresses in the dielectric more uniform. With impregnated-paper insulation the magnitude of this automatic grading effect is not large, but a reduction of about 5 % in the value of the maximum stress may be expected.

130. Representation of Three-core Cable Capacitances by Condensers.—In overhead systems the capacitance to earth is generally negligible, but in an underground transmission by multi-core cable the condensers formed by the comparatively small thickness of insulation between conductors and earth must be reckoned with. Russell [18] has shown that in a three-core cable eight different capacitances can be obtained, corresponding to various combinations

of the three conductors and the sheath. It is sufficient here, however, to imagine that the three conductors and sheath form an arrangement of condensers as shown in Fig. 65. The capacitance of each of the condensers connected between the conductors and sheath is c_s, and the capacitance of the condensers connecting the conductors to one another is c_c. It is apparent that the capacitance c_s is to earth or neutral. Also the mesh-connected capacitances c_c may be converted to equivalent star-connected capacitances by multiplying by three. Therefore the effective capacitance of each conductor to neutral is

$$C = c_s + 3c_c. \quad . \quad . \quad . \quad (167)$$

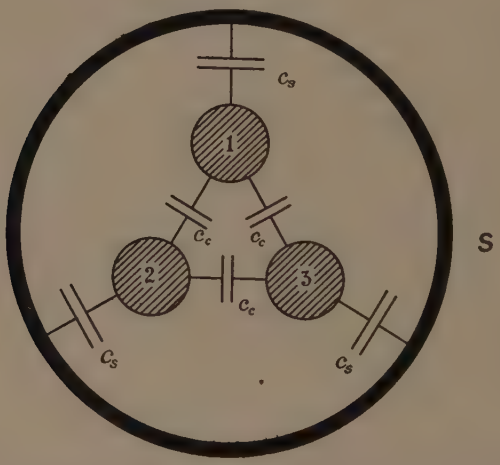

FIG. 65.—Representation of three-core cable capacitances by condensers.

Mathematical formulæ for calculating C are rather complicated and depend on certain assumptions which are not always satisfied in a practical cable. Tables of cable capacitances are therefore usually based on test data obtained from the finished cables.

For example, supposing that the following measurements of capacitance are made:—

(a) between any two conductors as (1) and (2).

(b) between the three conductors connected together (1, 2, 3) and the sheath (s).

In terms of the imaginary capacitances c_s and c_c of Fig. 65

$$\text{Capacitance } (a) = \tfrac{1}{2}(c_s + 3c_c).$$

In the measurement of (a) it is obvious that conductor (3) is at the same potential as the sheath, so that it is immaterial whether (3) is left free or connected to the sheath.

Also

$$\text{Capacitance } (b) = 3c_s.$$

From these two measurements both c_s and c_c can be obtained.

For the purpose of transmission line calculations, however, the quantity required is the capacitance of conductor to neutral, and its value can be found as shown above by simply taking twice the value of the measured capacitance (a).

131. Capacitance of Three-core Cable.—If test figures are not available, the capacitance to neutral of a three-core cable can be calculated from first principles and with reasonable accuracy if the following assumptions are made:—

1. That the diameter of the conductors is small in comparison with the diameter of the sheath.
2. That the cable dielectric is homogeneous.
3. That the conductors are cylindrical, and symmetrically spaced with respect to the cable axis.

Russell's formula for this case is [19]

$$C_0 = \frac{0{\cdot}0388\epsilon}{\log_{10}\left(\dfrac{\sqrt{3}a(R^2 - a^2)}{r(R^4 + R^2a^2 + a^4)^{\frac{1}{2}}}\right)} \text{ microfarads per mile,} \quad (168)$$

where ϵ = permittivity of the dielectric,
R = internal radius of lead sheath,
a = radius of circle passing through centres of conductors,
and r = radius of conductor.

This formula checks fairly closely with experimental results. At the lower voltages with large sizes of conductor, however, a certain amount of error is introduced, and it seems preferable to use the following empirical expression derived by Simon [5] from Atkinson's experimental data, and which is stated to be correct within 2 to 3 % for the complete range of sizes.

$$C_0 = \frac{0{\cdot}108\epsilon}{\log_e\left[\left\{0{\cdot}52\left(\dfrac{t}{T}\right)^2 - 1{\cdot}70\dfrac{t}{T} + 3{\cdot}84\right\}\dfrac{T + t}{d} + 1\right]}$$

microfarads per mile, (169)

where t = belt-insulation thickness,
T = conductor-insulation thickness,
and d = conductor diameter.

This formula is also far more convenient to use than (168).

In an actual three-core cable, the spaces between the cores may be filled with a different type of insulating material to that used on the conductors, or even if the same material is employed for the fillers, differences in tightness of packing may occur. Furthermore, when the cable is carrying current, a considerable temperature gradient exists in the dielectric. The resulting variation in permittivity throughout the cable section due to these causes modifies, to some extent, the theoretical capacitance, and formula (169) may be considered amply accurate enough for all practical requirements.

An analogous formula to (169) can be derived for the insulation resistance of a three-core cable but is not of much practical use, as the power loss due to leakage, with three-phase voltage, is negligible compared with that caused by dielectric hysteresis.

132. Electrostatic Stresses in Three-core Cables.—The electrostatic stress, at any point in the dielectric of a single-core cable, can be calculated in terms of the applied voltage and the cable dimensions by a simple mathematical formula. In the case of a three-core cable, the magnitude of the dielectric stresses cannot, in general, be determined in this manner, and it is necessary to fall back on experimental data. Fortunately, chiefly owing to the investigations of Atkinson,[20] a great deal of information on the subject is available, and the problem may be considered as satisfactorily solved from an experimental point of view. The method employed has been to construct cable models consisting of three symmetrically-spaced tubes representing the three conductors, surrounded by a fourth tube representing the sheath, all immersed in electrolyte. When the three inner tubes are connected to a three-phase supply, the lines of current flow correspond exactly to the distribution of dielectric flux in an actual cable having a geometrically similar cross-section. Hence the electrostatic stresses at any point in the cable can be found by measurements of the potential gradient existing at the corresponding point in the electrolyte. More recently other models have been employed, consisting of sheets of tinfoil on which were soldered three copper rings to represent the conductors, surrounded by a fourth ring to represent

the sheath. Emanueli [21] has also attacked the problem, using cable models made of electrodes in a tank of conducting liquid to determine stress values.

Of course, in a three-core cable the electrostatic field is polyphase, and rotates at uniform angular velocity round about the geometric centre of the cable, in a manner analogous to the rotation of the magnetic field in an induction motor. Fig. 66 (supplied by the Habirshaw Electric Cable Co.) shows the stress distribution and equipotential lines in the cable at one particular instant.

In general, the stresses in a multi-core cable depend upon the absolute dimensions of the cable and upon the voltage applied, but

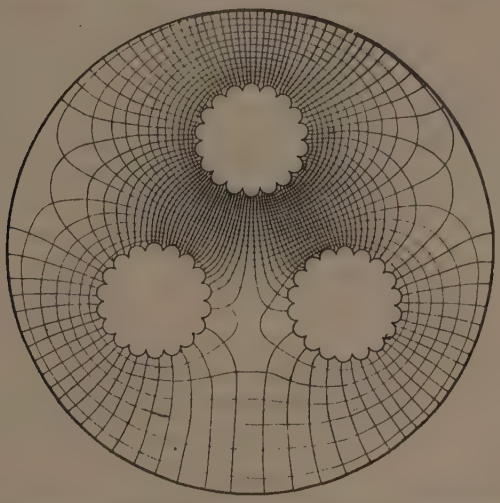

Fig. 66.—Electrostatic field of three-core cable at a particular instant.

it is evident that the stresses will be the same in cables which are geometrically similar, if the applied voltages are proportional to the sizes of the cables. In other words, the ratio of maximum to average stress is a constant for cables which are geometrically similar. The highest stresses occur in the triangular section of the cable formed by joining the centres of the conductors, and it has been found that their magnitude is independent of the belt-insulation thickness. These stresses vary according to the ratio of conductor-insulation thickness to conductor diameter, and may be completely determined if the ratio of actual stress to average is plotted as a function of the ratio of conductor-insulation thickness to conductor diameter.

The stress of greatest interest is the maximum stress in the cable dielectric, and this occurs at the conductor surface at the point nearest to the centre of the cable. So far, there has not been developed any formula to calculate the value of this stress, hence the need of an experimental determination.

Another stress of interest is that at the conductor surface on the line joining conductor centres. With the usual thicknesses of

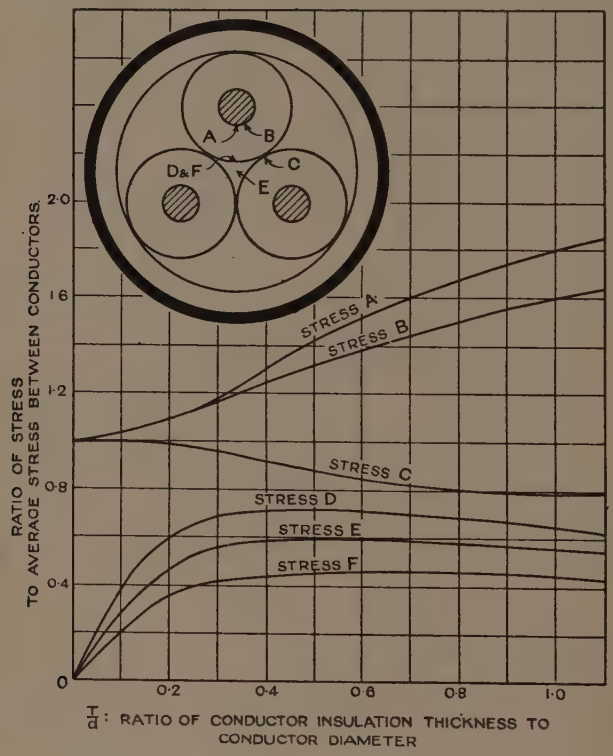

FIG. 67.—Electrostatic stresses in three-core cables.

cable insulation this stress is very nearly as great as the maximum stress, and furthermore can be calculated by means of a formula due to Russell.[22]

These two stresses and various others at representative points in the dielectric are best expressed in terms of the average stress between conductors, meaning by this latter quantity the numerical average obtained by dividing the voltage between conductors by the total intermediate thickness of insulation. The ratio of actual stress

to average can then be plotted as a function of the ratio of conductor-insulation thickness to conductor diameter, and the results obtained are reproduced in Fig. 67. In this diagram stress B has been calculated by Russell's formula, and the remaining curves are the experimental results obtained by Atkinson. By the use of these curves the determination of the stresses in three-core cables is made as simple as in the corresponding single-core case. It should be noted that all the stresses shown in Fig. 67 are radial to the conductor surface with the exception of stress F, which is a tangential stress measured along the circular boundary of the conductor insulation.

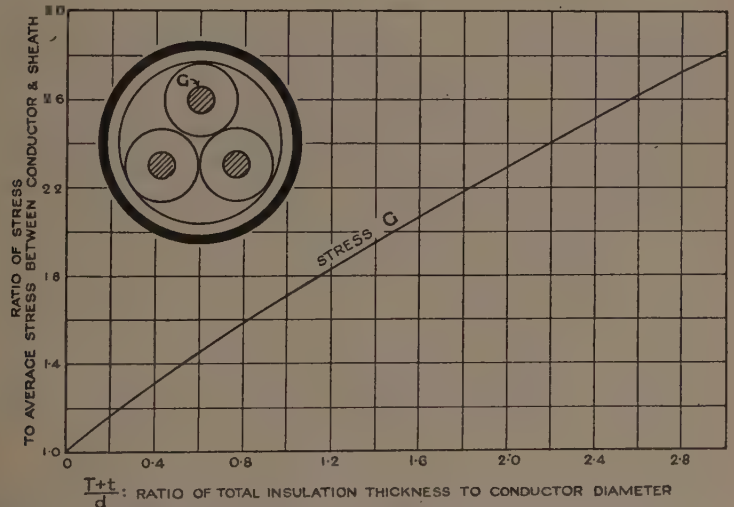

FIG. 68.—Electrostatic stress in three-core cables at conductor surface and towards sheath.

In addition to the maximum stress and various other stresses in the centre triangle of the cable, the stress at the conductor surface and toward the sheath is also of interest. Its magnitude depends very largely upon the thickness of the belt insulation. In the case of a cable having a thin belt or no belt, the stress towards the sheath can be made the same as the maximum stress, and in such a case indeed the stresses around the entire circumference of a conductor are nearly uniform. If the cable, as is the usual practice with high-voltage cable, has a belt-insulation thickness of the same order as that of the conductor insulation, the stress towards the

sheath will be considerably less than the maximum. Russell also gives a method of calculating the stress in this case, but there are several variables in the formula, and an exact solution for all conditions would involve an extensive series of curves. It is possible, however, by taking somewhat similar co-ordinates to those previously employed, to plot one curve shown in Fig. 68 which is correct within about 1 °/₀ for any point. It should be noted in connection with Fig. 68 that the stress is a function, not only of the conductor insulation, but also of the belt insulation, and it must therefore be expressed not in terms of the average stress between conductors, but of the average stress between conductor and sheath. Similarly as abscissæ must be taken the ratio of total-insulation thickness to conductor diameter.

To illustrate the use of the preceding data, the electrostatic stresses will be calculated for the case of a 33 000-volt, three-core, 0·20 sq. in. cable. The conductor diameter is 1·47 centimetres, the thickness of the conductor insulation is 0·635 centimetres, and the belt-insulation thickness is also 0·635 centimetres.

Hence for the purpose of determining the stresses in the inner triangle of the cable the average stress is

$$\frac{33}{1{\cdot}27} = 26{\cdot}0 \quad \text{kV. per centimetre,}$$

and the ratio of conductor-insulation thickness to conductor diameter is

$$\frac{0{\cdot}635}{1{\cdot}47} = 0{\cdot}43.$$

With 0·43 as abscissæ, a series of factors can be read off from the curves of Fig. 67, which factors, when multiplied by the average stress 26·0, give the actual stresses at the various points in the cable.

The maximum stress in the cable is

$$\text{Stress } A = 1{\cdot}34 \times 26{\cdot}0 = 34{\cdot}8 \quad \text{kV. per centimetre,}$$

and the other values are

$$\begin{aligned}
\text{Stress } B &= 1{\cdot}27 \times 26{\cdot}0 = 33{\cdot}0 \quad \text{kV. per centimetre.} \\
C &= 0{\cdot}91 \times 26{\cdot}0 = 23{\cdot}7 \quad \text{kV. ,, ,,} \\
D &= 0{\cdot}69 \times 26{\cdot}0 = 17{\cdot}9 \quad \text{kV. ,, ,,} \\
E &= 0{\cdot}58 \times 26{\cdot}0 = 15{\cdot}1 \quad \text{kV. ,, ,,} \\
F &= 0{\cdot}43 \times 26{\cdot}0 = 11{\cdot}2 \quad \text{kV. ,, ,,}
\end{aligned}$$

The stress at the surface of the conductor in the direction of the sheath is similarly derived from Fig. 68.

The average stress in the total thickness of insulation between conductor and sheath is

$$\frac{33}{\sqrt{3} \times 1{\cdot}27} = 14{\cdot}9 \quad \text{kV. per centimetre,}$$

and the ratio of total-insulation thickness to conductor diameter is

$$\frac{1{\cdot}27}{1{\cdot}47} = 0{\cdot}86.$$

Then with 0·86 as abscissa, the corresponding ordinate is found as 1·62. Hence the stress at that point on the conductor surface which is nearest to the sheath is

$$\text{Stress } G = 1{\cdot}62 \times 14{\cdot}9 = 24{\cdot}1 \quad \text{kV. per centimetre.}$$

It must be borne in mind that the stress data given above are for the condition of homogeneous dielectric. If, in an actual cable, the permittivity of the filler material differs very much from that relating to the main insulation, the stresses may be somewhat modified thereby. Also no account has been taken of the effect of stranding in increasing the stress at the surface of the individual wires in the conductor. This, however, does not detract from the value of the method in furnishing an accurate means of comparing the electrostatic stresses in various designs of cable, or in various parts of the same cable.

133. Special Designs of Three-core Cable.—The limiting value of voltage rating for three-core cables may be determined to a great extent by the heating of the triangular section of insulation formed by the lines joining the centres of the conductors. In the first place, the temperature and electrostatic stresses are highest in this central triangle, resulting in heavy local dielectric losses. Also, as first pointed out by Höchstädter,[23] high stresses occur in this part of the cable in a direction tangential to the surfaces of the conductor insulation. The potential difference between points of contact of the three insulated cores is equal to 87 % of the line voltage, and the tangential stresses produced give rise to equalising leakage currents along the surfaces of the dielectric, these currents being accentuated if the space between the cores is filled with a relatively imperfect insulating material such as impregnated jute. In many cases charring of the insulation with consequent failure has thus been caused.

In the improved method of construction suggested by Höchstädter, the belt insulation of the cable is omitted, and each of the

three cores covered with a conducting film of metallised paper. H-type cables have also been made with a covering over the individual cores consisting of a thin copper tape applied with a narrow gap between turns. In either case the effect is to make the cable electrically into a mechanical combination of three single-core cables. The insulation of the conductors can then be kept absolutely homogeneous, and the metal coatings at earth potential. The tangential stresses are eliminated, and better facilities are obtained for the dissipation of heat from the central portions of the cable. Furthermore, the presence of the metal sheath prevents direct core-to-core faults, break-down taking place to earth in case of failure, which is an advantage from the operating point of view. The advantages of this type of cable are thus higher current-carrying capacity, increased voltage rating for a given overall diameter, and greater mechanical flexibility.

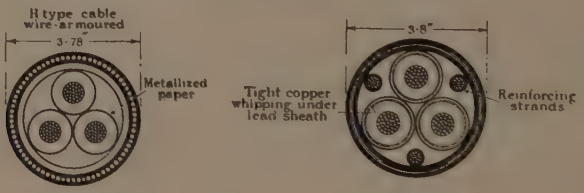

Fig. 69.—H-type and S.L. type of three-core cable.

H-type cables are made with a common lead sheath over the assembled cores as shown in Fig. 69, and the cores are laid up before the impregnation process is carried out. In an improved method of construction introduced by W. T. Henley's Telegraph Works Co., and known as the S.L. type,[24] the individual cores are dried, impregnated and lead covered before being laid up, and there is no enclosing lead sheath over the assembled cores. Fig. 69 also illustrates a particular design of S.L. cable, and it should be noted that a tight whipping of copper tape is employed over the core papers immediately under the lead in order to provide better mechanical support than would be obtained by depending on the lead sheath itself. The three lead-covered cores are also covered with a coat of paper and compounded tape before being laid up with jute wormings into a cable. In order to give the cable longitudinal strength while retaining the maximum flexibility, three steel reinforcing strands are introduced into the

spaces between the cores as shown, and the armouring then consists of two layers of steel tape.

The S.L. type of construction results in a rather greater current-carrying capacity than that of the H-type, owing to the greater mass of metal surrounding the individual cores and the greater cooling effect thereby produced. Furthermore, better impregnation of the insulation can be secured by dealing with each core separately, instead of when laid together. Another advantage is that large-diameter lead tubes are avoided, resulting in a better mechanical construction. In other words, for a given overall diameter of cable, a given amount of bending results in less distortion of the lead sheaths in the S.L. type, than in other forms of metal-sheathed cables.

Of course induced voltages or currents will exist in the lead sheaths depending on whether the sheaths are left open-circuited or bonded together, exactly as in the case of single-core cables previously discussed, but it does not appear that these are of a dangerous magnitude. Experimental results, for instance, obtained on a 33 000-volt, 0·20 sq. in., three-core, S.L. cable show that at the full working current of the cable the induced pressure in the open-circuited sheaths is of the order of 25 volts per mile. When the sheaths are bonded together at each end as would usually be the case in practice, the additional power loss when the cable is carrying full load is only 3 to 4 % of the copper losses. The thermal resistance of the S.L. type is about one-half of that of a plain three-core cable, so that notwithstanding the additional heating of the sheaths an increase in current rating can be obtained for the same temperature rise. The permissible increase in current-carrying capacity depends on the size of the cable, depth of laying, and thermal resistivity of the soil, etc., but improvements of the order of 10 % appear to be possible.

134. Dielectric Losses.—The energy losses occurring in the dielectric of cables are due to leakage and so-called dielectric hysteresis. The former loss is the I^2R loss of the current passing by conduction through the resistance of the dielectric, and is independent of frequency, so that it occurs both with direct and alternating voltages. The leakage current is proportional to the impressed voltage, hence the loss is proportional to the square of the impressed voltage.

With alternating pressures there is a further loss of energy

usually styled hysteresis loss, and which under normal operating conditions is very much larger than the leakage loss. Apparently some energy is consumed in reversing the stresses in solid dielectrics, and this appears as heat, causing a rise in temperature of the dielectric. At normal working stresses the dielectric hysteresis loss is proportional to the square of the electrostatic stress, that is to the square of the impressed voltage.

In order to take into account these energy losses, the charging current of a cable may be considered as made up of two components, one being the true capacitance current of which the phase is exactly 90 degrees in advance of the impressed voltage, the other being the 'energy' component in phase with the voltage. The dielectric

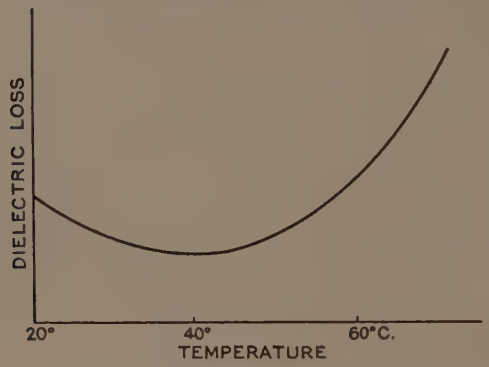

FIG. 70.—'V' curve of dielectric loss.

losses are equal to the product of the voltage and the in-phase component of the charging current. Hence if in any cable

W_l = energy losses in watts per phase due to leakage,
W_h = energy losses in watts per phase due to dielectric hysteresis,
E = impressed voltage to neutral,
I_c = charging current per conductor,
and ϕ_d = angle of advance of charging current,

the dielectric losses per phase are

$$W_d = W_l + W_h = EI_c \cos \phi_d, \quad . \quad . \quad (170)$$

where $\cos \phi_d$ is known as the dielectric power factor.

Ordinarily, the dielectric losses of paper-insulated cables are small, but in the case of cables for the highest voltages, where the

stresses are necessarily pushed to their extreme limit, they may have to be taken into consideration in fixing the current rating of the cable.

A very important and interesting relationship in the study of cable dielectrics is the variation of dielectric loss with temperature. It has been shown by a number of investigators during the past few years that, over the working temperature range, the dielectric loss in most cables falls to a minimum at about 40° C. and then rises again. The variation when plotted gives rise to what is known as the 'V' curve of dielectric loss, and the minimum temperature point corresponds very closely to the temperature of liquefaction of the resinous impregnating compound. A typical curve for paper-insulated cable is exhibited in Fig. 70. Since the variation of capacitance current with temperature is small, the dielectric power factor / temperature curve is of the same general shape as the dielectric loss / temperature curve. In this connection, it seems preferable to exhibit dielectric-loss data in terms of dielectric power factor rather than in power loss per mile of cable in order to facilitate comparison between different types and sizes of cables.

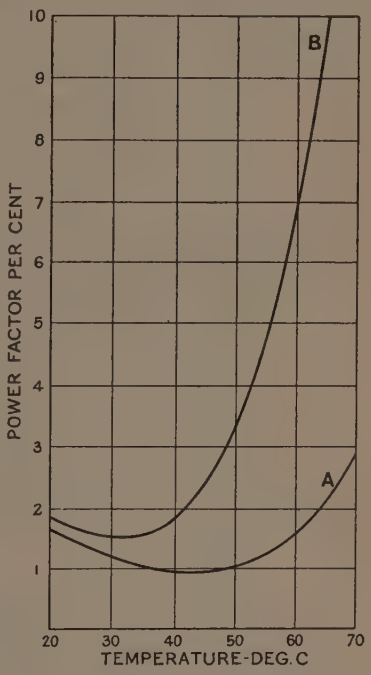

FIG. 71.—Effect of type of paper on power factor / temperature curve.

In order to explain the shape of the 'V' curve Höchstädter [25] assumes that two types of loss are going on in the dielectric, one a hysteresis loss which definitely falls with rise of temperature, and the other a conduction loss which has a positive temperature coefficient. The combination of these two factors produces first a fall, and subsequently a rise, of the losses, as the temperature is increased. It must be confessed, however, that this explanation and others which have been proposed are not entirely satisfactory,

and the real nature of the change indicated by the curve has not yet been discovered.

The actual shape of the 'V' curve depends on several factors and may be deliberately changed by the use of different papers, impregnating compounds, and methods of manufacture. For example, the two curves of Fig. 71, due to Dunsheath,[26] were obtained on two single-core cables of exactly similar dimensions, impregnated with the same compound, and manufactured under identical conditions. The only difference between them was that the papers employed for the two lengths were composed of different fibres. Fig. 72, taken from a paper by Del Mar and Hanson,[27]

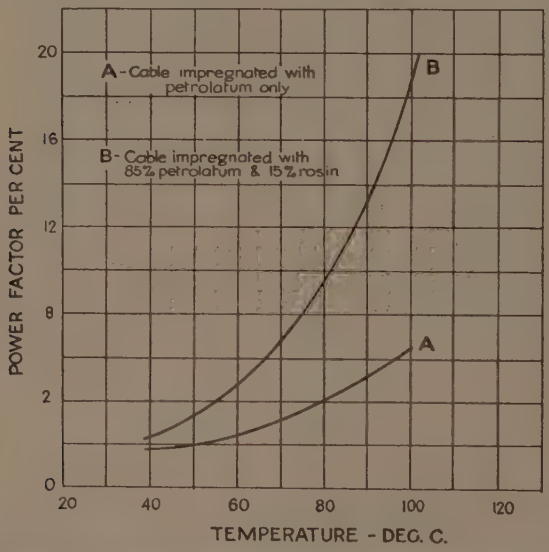

FIG. 72.—Effect of different impregnating compounds on power-factor / temperature curve.

shows the results obtained on another pair of similar cables in which both had the same insulating paper and treatment, but in which different impregnating compounds were employed. A trace of moisture in the paper will also alter the shape of the curve very considerably.

In the development of high-voltage cable many of the earlier constructed cables were found to possess heavy dielectric losses, and in some cases these losses have led to cumulative heating and consequent failure in service. These excessive losses have been

found to be due to various causes such as inadequate drying of the paper, the use of impregnating compounds of low resistivity, and to ionisation phenomena.

There appears to be no doubt that moisture has been to blame for the high dielectric losses of many of the earlier cables. A very slight amount of moisture in the insulation is sufficient to produce quite heavy losses. The resistivity of the impregnating compound also has a considerable influence, the losses increasing as the resistivity of the compound decreases. In fact, according to Del Mar and Hanson, in thoroughly dried and well-made cables it is possible to predict the dielectric power factor of the cable by merely making resistivity tests on the impregnating mixture. As regards ionisation losses these may be serious in certain cases, and will be considered in more detail in the following article.

Apart from the influence of heavy dielectric losses in causing cable break-downs, the money value of such losses is also of considerable importance as they are continuous while the cable is under pressure, no matter what the load being supplied. In recent years a considerable amount of research work has been done in connection with the reduction of these dielectric losses. Good results have been obtained and 44 000-volt, 50-cycle, three-core cable can now be supplied which will have a dielectric power factor not exceeding 1 % up to a working temperature of 55° or 60° C. This figure corresponds to about 2 % of the I^2R losses in the cable when carrying full load as determined from thermal considerations. Further evidence of the low dielectric losses of modern cables is afforded by data given by Emanueli [25] and relating to a 60 000-volt single-core cable. The dielectric power factor of this cable at 42 cycles on test was 1·2 % at 20° C., 0·6 % at 40° C., 1·3 % at 60° C., and 4·0 % at 80° C.

Contrary to what might be thought, the dielectric power factor is not independent of frequency. As the frequency of the impressed voltage is increased, the dielectric losses of the cable also increase, but at a slower rate than the frequency, so that the losses, say, at 50 cycles, are not twice the value of those at 25 cycles. In other words, as the charging current is proportional to the frequency, the dielectric power factor diminishes when the frequency increases. These remarks apply to cables running at the usual operating temperatures of 50° to 60° C. or over. At ordinary atmospheric temperatures the dielectric power factor *increases* with increase of

frequency. These phenomena are clearly illustrated by the curves of Fig. 73 given by Emanueli,[29] and it will be seen that the minimum points of the 'V' curves appear to lie on a line parallel to the abscissae.

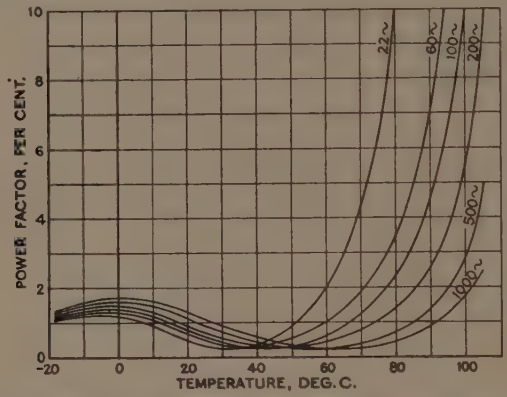

Fig. 73.—Power factor / temperature curves at various frequencies.

135. Ionisation Phenomena.—In many cables additional energy losses in the dielectric are caused by the presence of occluded gas or air in the insulation. This gas may occur in the form of small pockets or bubbles between the wrappings of the paper, or larger pockets may be formed during the process of manufacture by loosely-wrapped paper or improper filling which allows creases to form. The use of viscous impregnating compounds also leads to formation of these vapour pockets as stiff compounds will not follow the movements of the paper tapes when the cables are bent. Large voids are consequently caused at the bends which the compound cannot flow into and fill up. According to the ionisation theory, when the stress across these gas pockets exceeds about 21 or 22 kilovolts per centimetre (R.M.S.), ionisation sets in and increases the normal energy losses due to leakage and dielectric hysteresis. Hence above the critical voltage of ionisation the dielectric losses in cables increase more rapidly than the square of the impressed voltage. This effect is best shown by plotting the effective conductance of the dielectric against the impressed voltage. If W_d is the total dielectric loss per phase, and E the impressed voltage to neutral, the conductance of the cable dielectric is

$$G = \frac{W_d}{E^2} \quad . \quad . \quad . \quad . \quad (171)$$

Typical conductance / voltage curves are given in Fig. 74.[30] Up to a certain value of impressed voltage at which the ionisation stress is reached, G is constant; beyond that point, G increases due to the occluded gases being ionised; and at yet higher voltages it becomes nearly constant again, when the gases are nearly totally ionised. Special precautions are now taken in the manufacture

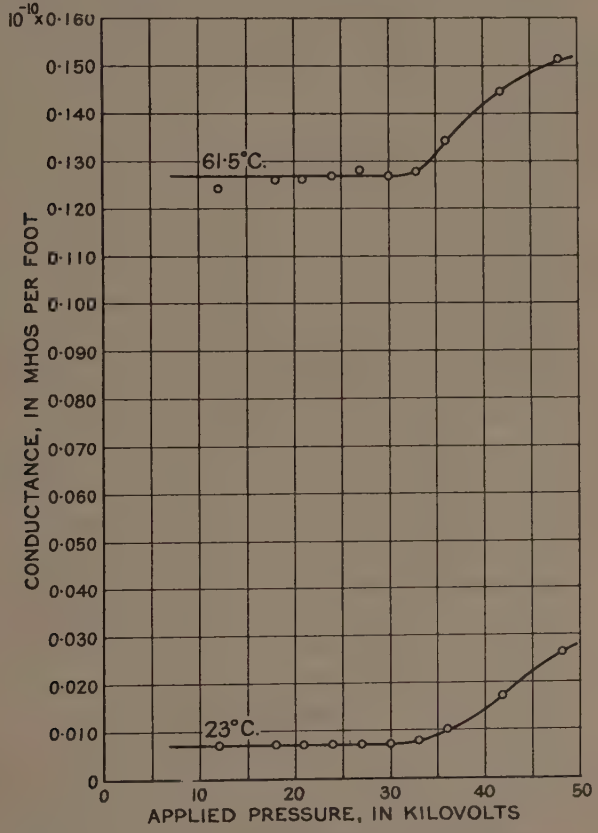

Fig. 74.—Variation of cable conductance with voltage.

and in the materials used for high voltage cable, and by this means the ionisation losses can be neglected even at stresses of 40 kilovolts per centimetre or more. One method of minimising the trouble due to Beaver is to impregnate the paper in the strip form before applying to the cable. After impregnation, the paper is wrapped on to the conductors by special machinery under the

surface of the hot impregnating compound, so that the occlusion of air in the lappings is almost impossible.

If, however, the ionisation curve of a three-core cable is taken after it has been loaded up to its maximum temperature and allowed to cool again, it is found that the losses are considerably increased. The coefficient of expansion of impregnating oil is many times greater than that of the lead sheath, so that as the cable heats up the lead is stretched beyond its elastic limit and permanently enlarged. When the cable cools, due to a reduction in load, the insulating medium contracts, but the lead cannot follow up this movement, and spaces are formed which give rise to the increased ionisation losses. These spaces may also be created by deformation of the sheath set up by bending or drawing the cable into ducts. The trouble is more pronounced with three-core cables, presumably because the lay of the cores and conductor expansion produce a tendency for spaces to form between the cores where the stress is high. It is no doubt the subjection of these spaces to the tangential stresses which leads to electrical discharge over the surface of the paper, slow charring of the paper fibres, and ultimate failure. H-type and S.L.-type cables have an advantage over the plain three-core type in that separation of the cores, by mechanical distortion of the cable or by thermal expansion, cannot introduce voids into the dielectric under stress. Should the cores become separated from one another by a small fraction of an inch, from any cause whatever, the conducting sheath moves with the core, and the space that is formed is outside the electrostatic field and out of contact with the dielectric.

136. Influence of Dielectric Losses on Current-carrying Capacity of Cables.—In cables having a high dielectric loss the maximum permissible operating temperature, and hence the maximum permissible current, may be determined by the temperature at which the heating of the cable due to dielectric losses becomes cumulative. In modern low-loss cables, however, this feature is not usually the determining factor in limiting the temperature of operation. The limiting temperature of these cables is that temperature which will cause permanent deterioration of the impregnated-paper insulation, or damage to the sheath or cable fittings.

Nevertheless, it appears desirable to have some method of determining the carrying capacity of a cable which will take into account the dielectric losses as well as the copper losses. A simple

means of accomplishing this has been given by Bang and Louis.[31] First of all, knowing the thermal resistance of the path followed by the heat generated in the cable, a line is drawn showing for different conductor temperatures the number of watts per foot that can be dissipated by the cable. In order that the temperature of the cable may be constant the heat generated in the cable must equal the heat dissipated. If, then, the curve showing the variation of dielectric losses with temperature is plotted on the same diagram, the maximum permissible copper losses at any particular

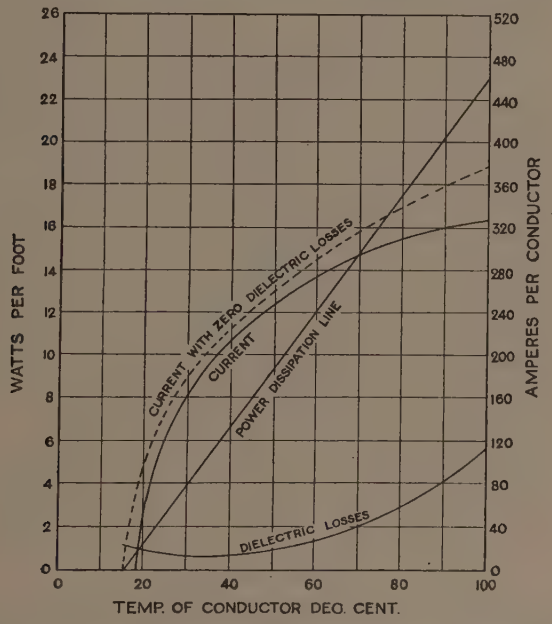

FIG. 75.—Effect of dielectric loss on current-carrying capacity of cable.

temperature can be ascertained by subtracting the dielectric losses from the total power losses. From the latter figure the current in the cable can be readily calculated.

As an example, take the case of the 33 000-volt, 0·20 sq. in., three-core cable, buried direct in earth under the conditions previously assumed. The thermal resistance of the heat path per foot of cable is

$$\frac{113{\cdot}2}{12 \times 2{\cdot}54} = 3{\cdot}72 \quad \text{thermal ohms,}$$

and taking the earth temperature as 15° C. the power-dissipation line is a straight line drawn at an angle from the point corresponding to this temperature, as shown in Fig. 75. The dielectric losses are then plotted, and the curve showing the maximum permissible current can then be calculated and drawn in.

For purposes of comparison, the permissible current with zero dielectric losses is also shown in the diagram. At 65° C. the reduction in current due to dielectric losses is from 303 amperes to 285 amperes—a decrease of about 6 °/₀ in the current rating of the cable. It should be noted, however, that for the sake of clearness in the diagram, the dielectric losses in this example have been assumed to be much greater than would occur in modern cables.

137. Determination of Line Constants for Underground Line.—In order to show clearly the derivation of the line constants for an underground transmission, two numerical examples will be taken :—.

Problem 7.—Find the constants of a 30-mile line of three-core cable, 0·20 sq. in. conductor insulated with 0·25 in. conductor insulation, and 0·25 in. belt insulation, 0·12 in. lead sheath, for operation at 33 000 volts, 50 cycles.

From wire tables [32] the resistance per mile of conductor at 15·6° C. is 0·215 ohms. Assuming an operating temperature of 65° C., and allowing 2 °/₀ extra for laying up the cores,

$$R = 0{\cdot}215(1 + 0{\cdot}004 \times 49{\cdot}4) \times 1{\cdot}02 \times 30$$
$$= \underline{7{\cdot}89 \quad \text{ohms.}}$$

The inductance calculated by the usual formula, with $d = 1{\cdot}081$ in., and $r = 0{\cdot}29$ in., is

$$L_0 = \left(0{\cdot}85 + 0{\cdot}741 \log_{10} \frac{1{\cdot}081}{0{\cdot}29}\right)10^{-3} \qquad (101)$$
$$= 0{\cdot}508 \times 10^{-3} \text{ henries per mile,}$$

and the reactance is consequently

$$X = 0{\cdot}508 \times 10^{-3} \times 2\pi \times 50 \times 30$$
$$= \underline{4{\cdot}80 \quad \text{ohms.}}$$

The capacitance is determined from Simon's formula

$$C_0 = \frac{0{\cdot}108\epsilon}{\log_e \left[\left\{0{\cdot}52\left(\frac{l}{T}\right)^2 - 1{\cdot}70\frac{t}{T} + 3{\cdot}84\right\}\frac{T + t}{d} + 1\right]}. \qquad (169)$$

In this case

$$\frac{t}{T} = \frac{0{\cdot}25}{0{\cdot}25} = 1,$$

$$\frac{T + t}{d} = \frac{0{\cdot}5}{0{\cdot}58} = 0{\cdot}86,$$

and taking ϵ as 3·3 gives

$$C_0 = \frac{0{\cdot}\,108 \times 3{\cdot}3}{\log_e [\{0{\cdot}52 - 1{\cdot}70 + 3{\cdot}84\}0{\cdot}86 + 1]}$$
$$= 0{\cdot}\,299 \times 10^{-6} \text{ farads per mile.}$$

Hence the susceptance is

$$B = 0{\cdot}\,299 \times 10^{-6} \times 2\pi \times 50 \times 30$$
$$= \underline{0{\cdot}\,00282 \text{ mhos.}}$$

Assuming that the dielectric power factor at the operating temperature is 1 °/₀, the conductance

$$G = 0{\cdot}\,00282 \times 0{\cdot}01$$
$$= \underline{0{\cdot}\,0000282 \text{ mhos.}}$$

In the above calculation for R no account has been taken of the eddy-current losses in the lead sheath. Undoubtedly these losses do occur, and various formulæ have been proposed for calculating their effect, but none apply to the case of greatest practical importance, *i.e.* a wire-armoured cable. Experimental results, however, seem to show that the losses are quite small and without any substantial modifying effect on the value of the conductor resistance.

Problem 8.—Find the constants of a 40-mile line of three single-core cables, 0·30 sq. in. conductor. Internal radius of lead sheath (R) 2·5 cm., external radius (R_1) 2·8 cm., for operation at 66 000 volts, three-phase. The cables laid closely together in triangular formation, the sheaths being bonded together at several points and earthed.

From wire tables [32] the resistance of the conductor at 15·6° C. is 0·\,140 ohms per mile, and taking the operating temperature as 50° C., the conductor resistance

$$R = 0{\cdot}\,140(1 + 0{\cdot}\,004 \times 34{\cdot}4) \times 40$$
$$= 6{\cdot}36 \text{ ohms.}$$

In order to determine the effective conductor resistance when the sheaths are short-circuited, the values of R_s and X_m must first be obtained.

Assuming the temperature of the lead sheath is 30° C., its resistivity ρ_s may be taken as $23{\cdot}2 \times 10^{-6}$ ohms, and the resistance of the sheath

$$R_s = \frac{23{\cdot}2 \times 10^{-6} \times 5\,280 \times 12 \times 2{\cdot}54 \times 40}{\pi\{(2{\cdot}8)^2 - (2{\cdot}5)^2\}}$$
$$= 29{\cdot}9 \quad \text{ohms.}$$

The interaxial spacing (d) will be about 2·4 in. or 6·1 cm., so that

$$X_m = \omega M$$
$$= 0{\cdot}741\omega \log_{10} \frac{2d}{R + R_1} 10^{-3} \times 40$$
$$= 0{\cdot}741 \times 2\pi \times 50 \times \log_{10} \frac{12{\cdot}2}{2{\cdot}5 + 2{\cdot}8} \times 10^{-3} \times 40$$
$$= 3{\cdot}37 \quad \text{ohms.}$$

Then the effective value of the conductor resistance is

$$R_{\text{eff}} = R + \frac{X_m^2 R_s}{R_s^2 + X_m^2} \quad . \quad . \quad . \quad . \quad (152)$$
$$= 6{\cdot}36 + \frac{3{\cdot}37 \times 3{\cdot}37 \times 29{\cdot}9}{(29{\cdot}9)^2 + (3{\cdot}37)^2}$$
$$= \underline{6{\cdot}73 \quad \text{ohms.}}$$

The reactance with sheaths open-circuited would be

$$X = \left(0{\cdot}85 + 0{\cdot}741 \log_{10} \frac{6{\cdot}1}{0{\cdot}915}\right) 10^{-3} \times 2\pi \times 50 \times 40$$
$$= 8{\cdot}72 \quad \text{ohms,}$$

but with short-circuited sheaths this is reduced to

$$X_{\text{eff}} = X - \frac{X_m^3}{R_s^2 + X_m^2} \quad . \quad . \quad . \quad . \quad (155)$$
$$= 8{\cdot}72 - \frac{(3{\cdot}37)^3}{(29{\cdot}9)^2 + (3{\cdot}37)^2}$$
$$= \underline{8{\cdot}68 \quad \text{ohms.}}$$

The capacitance is obtained from the usual formula, and taking ϵ as 3·3,

$$C_0 = \frac{0{\cdot}0388 \times 3{\cdot}3}{\log_{10} \frac{2{\cdot}5}{0{\cdot}915}} \quad . \quad . \quad . \quad . \quad (147)$$
$$= 0{\cdot}293 \times 10^{-6} \quad \text{farads per mile,}$$

from which the susceptance

$$B = 0{\cdot}293 \times 10^{-6} \times 2\pi \times 50 \times 40$$
$$= \underline{0{\cdot}00369 \text{ mhos.}}$$

Assuming that the dielectric power factor has a value of 1 °/₀ at the operating temperature,

$$G = 0{\cdot}00369 \times 0{\cdot}01$$
$$= \underline{0{\cdot}0000369 \text{ mhos.}}$$

After determining the values of resistance, reactance, susceptance, and conductance as above, the auxiliary line constants are obtained, and the calculation of an underground line is a straightforward transmission problem no different from that of an overhead line.

138. Characteristics of Special Types of Cables.—The inductance and capacitance of three-core cables having sector-shaped conductors cannot be stated accurately, as there is no standardisation of conductor cross-section. Speaking generally, the inductance is from 5 to 10 °/₀ less than for round conductors of the same area and separated by the same thickness of insulation, and the capacitance is increased by the same percentage. The thermal resistance of the sector-type of cable is rather lower than that of a cable having equivalent circular conductors, but the reduction is not large enough to make any definite improvement in the carrying capacity of the cable when buried in the ground. In all cases the characteristics of the higher-voltage cables are less affected by the shape of the conductors, owing to the increased interaxial spacing of the latter.

Three-core cables having separate metal-sheathed cores, such as the H-type and S.L.-type, can be regarded as a mechanical combination of three single-core cables, and their electrical characteristics can be calculated accordingly. The thermal characteristics of the two types of cables, however, vary to some extent. In the S.L. cable, the metal surrounding the cores is so thick that the difference in temperature round its periphery must be very small and the metal may be considered as an isothermal, the thermal resistance of the cable being determined as in the case of an assembly of three single-core cables. In the H-type cable, however, where the metal covering of the cores is metallised paper or thin copper tape, there is a definite temperature gradient round

the covering, and though an equipotential, it is not an isothermal, surface. The thermal resistance of such a cable appears to be intermediate in value between that of an S.L.-type cable, and that of a plain three-core cable.[33]

139. Developments in Power Transmission by Single-core Cables.—The use of three-core cables for the highest transmission voltages appears to be precluded, owing to the difficulty of providing the necessary insulation between core and core and between core and earth within the limiting diameter of the lead envelope, which should not exceed about 4 or 4½ inches. For instance, a 66 000-volt, three-core, armoured cable has an overall diameter of about 4½ inches, and cables above this size would not only be difficult to handle, but would be liable to mechanical injury whilst being laid in position unless extraordinary precautions were taken.

The recognition of this difficulty has caused many minds to turn to the possibility of using three single-core cables for high-voltage three-phase transmission. Apart from the greater ease of laying and jointing the cables, such a system can be laid out so as to considerably reduce the service demanded from the circuit-breakers. By using single-phase transformers, and taking suitable precautions in the arrangement of the switchgear, it is possible to ensure that all faults are to earth and none between phases. This feature should prove of great advantage, as the general experience of those operating high-voltage systems is that faults to earth do not produce such severe shocks as faults between phases, and are therefore not so difficult to deal with. This is due largely to the use of neutral resistances which prevent the fault current reaching such a destructive value as occurs in the case of faults between phases.

One of the most recent developments is the production of single-core cables for installation on 132 000-volt three-phase systems. An experimental line of this character has been in operation for the last two years in Italy with promising results, and considerable quantities of cable for the same pressure have been recently installed in America. In both cases the cables are of the same general design, the conductor being made up of copper wires stranded around a copper spiral, thus forming a hollow core. In the process of manufacture, the paper insulation is wrapped on, and covered with lead, after which a protecting layer of paper and tarred jute is applied. Impregnation is carried out after the lead has been applied, the

moisture being first removed by passing a current of dry air through the hollow core, and the impregnating oil being then allowed to flow in. During operation, the core is kept full of impregnating oil under pressure and this supplies the means for filling up any voids or spaces which may be formed when the cable goes through a temperature cycle. The American cables have a conductor area of 0·60 sq. in., and the overall diameter is about 3·1 inch.

Another possibility in connection with single-core cables is the employment of steel-wire armouring, which would enable the cables to be laid direct in the ground, instead of in concrete troughs or ducts as at present. Hitherto, lead-covered single-core cables have been of the unarmoured type, it having been assumed that a magnetic covering would introduce excessive power losses and reactance drop in the line. As a result, however, of researches by Harvey and Busby,[34] it has been found that the magnetic phenomena, whilst rendering the use of low-voltage armoured cable impracticable, assume less importance in cables designed for the higher voltages. For instance, according to Cramp,[35] in the specific case of a 132 000-volt single-phase transmission with lead-covered armoured cables, the losses in the armour at full load would only be of the order of 17 % of the I^2R losses in the conductors. The losses in the lead would be negligibly small in comparison with those in the armour, and it appears that the employment of high-tension, single-core, lead-covered and armoured cables is quite a feasible proposition.

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CHAPTER X.

ECONOMIC PRINCIPLES AND CALCULATIONS.

140. Introductory.—It is proposed to deal in this chapter with one or two of those fundamental economic principles which are of vital importance, and closely influence the electrical design of a transmission line. It should not be forgotten that the justification of any scheme of power transmission must be commercial, as well as technical success. As Still has said: 'It is an easy matter to design a bridge of ample strength for the load it has to carry, or a transmission line with conductors of so large a size, insulators with so large a factor of safety, and supports so closely spaced and so strong, that the electrical losses will be small, and the risk of mechanical failure almost nil; but neither the bridge nor the transmission line will reflect credit on the designing engineer unless he has had before him constantly the commercial aspect of the work entrusted to him, and has so chosen or designed the various parts, and combined these in the completed whole, that all economic requirements are as nearly as possible fulfilled.'

The application of economic factors to the design and layout of a complete power-transmission scheme is outside the scope of the present work, and it is only possible to consider here the method of determining the best size of line conductor, and the choice of line voltage.

141. Economic Size of Line Conductor.—The cost of conductor material is generally a very considerable part of the total cost of a transmission line, and hence the determination of the size of conductor to be employed is a very important problem. No matter what particular scheme of transmission is adopted, a little consideration will show that the first cost of the conductors, and the annual expenditure represented by the energy wasted in heating the conductors, follow opposite laws. To reduce the power losses in the line it is necessary to employ conductors of low resistance and therefore large cross-sectional area. On the

other hand, to reduce the first cost, conductors of small cross-section must be installed. Thus, the initial cost and subsequent working expenses are both governed by the size of conductor chosen, but while the former increases with the area the latter decreases as the area increases. It is evident that in any transmission scheme there must be at least one particular size of conductor for which the sum of the annual charges (interest and depreciation on initial cost of conductors plus annual cost of energy dissipated therein) is a minimum.

Taking into consideration first of all merely the initial capital outlay on conductor material, the annual charges on this initial cost for a given line are

$$P_1 = k_1 A a, \quad . \quad . \quad . \quad . \quad (172)$$

where A is the cross-sectional area of the conductor, a is the percentage to be taken to cover the annual interest and depreciation, and k_1 is a constant.

The cost of the energy dissipated in the line per annum is

$$P_2 = \frac{k_2}{A}, \quad . \quad . \quad . \quad . \quad (173)$$

k_2 again being a constant. The total expenditure will be a minimum when

$$\frac{dP_1}{dA} + \frac{dP_2}{dA} = 0.$$

This gives

$$k_1 a = \frac{k_2}{A^2},$$

and

$$A = \sqrt{\frac{k_2}{a k_1}}. \quad . \quad . \quad . \quad . \quad (174)$$

Inserting this value of A in (172) and (173) gives

$$P_1 = \sqrt{k_1 a k_2},$$

and

$$P_2 = \sqrt{k_1 a k_2},$$

hence

$$P_1 = P_2 \quad . \quad . \quad . \quad . \quad (175)$$

or the most economical area of conductor is that which makes the annual cost of the power losses equal to the annual interest and depreciation on the capital cost of the conductor material.

142. Kelvin's Law and its Modifications.—The above statement, known as Kelvin's law, was first enunciated by Lord Kelvin (then Sir William Thomson) at a meeting of the British Association in 1881.[1] To quote his exact words: 'The most economical size of the copper conductor for the electric transmission of energy . . . will be found by comparing the annual interest of the money value of the copper with the money value of the energy lost annually in the heat generated in it by the electric current. . . . the gauge to be chosen for the conductor does not depend on the length of it through which the energy is to be transmitted but solely on the strength of the current to be used, supposing the cost of the metal and of a unit of energy to be determined.'

Kelvin's law is based on the assumption that the cost of poles or towers, insulators, and the labour of erection and stringing of wires is either negligible or independent of the actual size of the conductor. In practice, neither of these assumptions is correct. This is owing to the fact that line supports and insulators tend to become more expensive as heavier conductors are used in consequence of the increased mechanical stresses occasioned thereby. Also the cost of stringing the wires will increase somewhat as the size of wire increases. As shown by Kapp,[2] a closer approximation to the truth is obtained by assuming that the initial capital expenditure on the complete line installation can be split up into two parts, one independent of the area of the conductor, and the other directly proportional thereto. It is therefore more correct to write

$$P_1 = (k_0 + k_1 A)a, \quad . \quad . \quad . \quad (176)$$

where k_0 represents that part of the capital outlay which is constant, and independent of the size of conductors.

This modification in the formula for P_1 makes no difference in the differential equation, as

$$\frac{dk_0}{dA} = 0,$$

hence, as before,

$$A = \sqrt{\frac{k_2}{ak_1}},$$

and substituting

$$P_1 = ak_0 + \sqrt{k_1 a k_2}$$

and

$$P_2 = \sqrt{k_1 a k_2},$$

or the interest and depreciation on the capital cost of the installation and the annual cost of the dissipated energy are now in the relation

$$P_1 = P_2 + ak_0. \quad . \quad . \quad . \quad (177)$$

Thus the interest and depreciation on the initial cost of the line must in this case be greater than the annual cost of the energy losses.

Substituting for P_1 in (177) and rewriting there follows that

$$P_2 = k_1 A a, \quad . \quad . \quad . \quad . \quad (178)$$

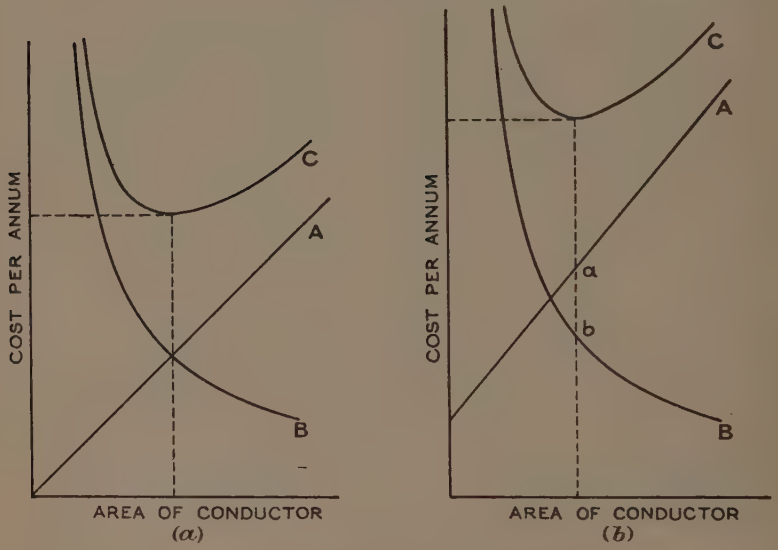

FIG. 76.—Graphical illustration of Kelvin's law.

or the most economical area of conductor is that which makes the annual cost of the dissipated energy equal to the annual interest and depreciation on that portion of the capital outlay on the line which can be considered to be proportional to the weight of conductor material employed.

Fig. 76 (*a*) exhibits the above results very clearly. In the diagram the straight line *A* shows the relation between the annual interest and depreciation of the capital spent on the line conductors, and the size of the conductor. Similarly, the rectangular hyperbola *B* gives the cost per annum of the energy wasted in the conductors. By adding the ordinates of the two

curves A and B, the curve C is obtained, of which the minimum value corresponds with that conductor cross-section which it will be most economical to use. This minimum occurs where the two curves A and B cross, *i.e.* where the values of their ordinates are equal. In Fig. 76 (b) the first diagram is modified so as to deal with the complete line installation by (1) adding to the capital cost of the conductors an amount independent of the conductor area, (2) slightly increasing the slope of A so as to take into account the higher cost of line supports, stringing of wires, etc., with the heavier conductors. In this case, at the points on curves A and B corresponding to the minimum value of C, the ordinate of curve A is greater than that of B by the distance ab, *i.e.* the amount of the independent portion of the total capital expenditure.

A simpler and more general statement which includes both the above results may be called the law of maximum economy and is expressed as follows: *For maximum economy the annual cost of the wasted energy in the transmission line added to the annual charges on the initial capital outlay shall be a minimum.*

143. Kelvin's Law in Practice.—Although the principles previously enunciated are well known, there is often considerable difficulty in applying them to a proposed scheme of power transmission. This is chiefly owing to the difficulty of estimating the power losses, and of fixing a correct money value on them when estimated.

The difficulty of estimating even approximately the probable amount of energy wasted during a year's working is a very real one. Even if the load to be delivered can be accurately forecasted, the determination of the current value to be used in conductor calculations is uncertain, and as Beard [3] has shown, even a knowledge of the load factor by itself is not sufficient. The losses over a given period of time are proportional to the square of the current flowing, so that the R.M.S. current flowing during the given period must be used in calculating them. Hence it is necessary to take into account the actual shape of the load curve. This is readily seen by considering the two extreme forms of a 50 % load-factor curve given in Fig. 77. In the full line curve the R.M.S. current is equal to the average current, while in the dotted curve it is $\sqrt{2}$ times the average, for this particular load factor. Load curves met with in practice lie between these extremes, and for a typical 50 %

load-factor curve the R.M.S. current would be about 1·20 times the average current. In every projected transmission scheme, therefore, probable load curves for different parts of the year should be drawn out, and the mean value of I^2 calculated therefrom. In the lack of other data the curve shown in Fig. 78 can be used to obtain values of the ratio of R.M.S. to average current for typical load curves.

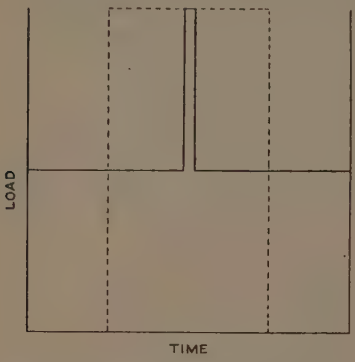

FIG. 77.—Extreme forms of load curve for a given number of units at 50 % load factor.

The cost per kilowatt-hour of the wasted energy is also somewhat difficult to decide with any accuracy. Generally speaking, the cost of the wasted energy is more than the generating cost because the load factor of the losses is less than that of the load, and in addition to this ought to be added the annual

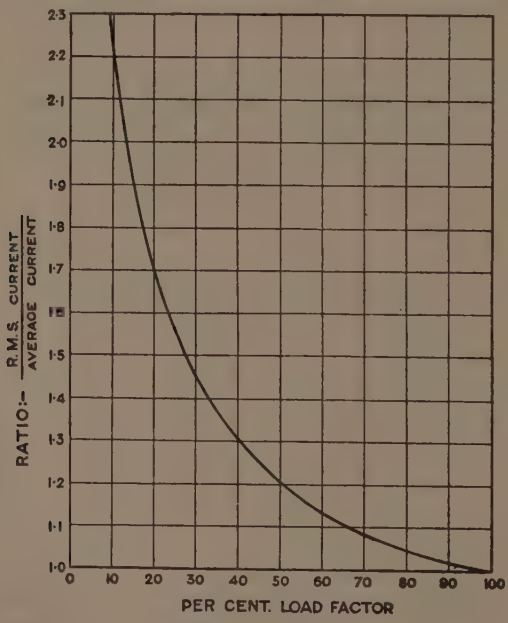

FIG. 78.—Ratio of R.M.S. to average current for typical load factor curves.

charges on the increased size of conductor necessary to transmit the losses. The works cost per unit might be increased by as much as 30 % to cover the lower load factor of the losses and these extra annual charges. Furthermore, in cases where the demand for power exceeds the supply, that is if all the power saved in the line could be promptly sold, the price to allow for the dissipated energy is actually the selling price. Every case should be taken on its own merits.

With regard to a reasonable percentage to allow for interest and depreciation on the cost of conductors, this will depend on commercial considerations and the estimated life of the conductors, also on the probable scrap value of the metal when it is replaced. So far as interest is concerned, if cash is to be paid for the conductors the figure to be taken for interest on capital should be on a par with the expected percentage profit on the complete undertaking. In fixing the depreciation it should be remembered that the scrap value of the conductors will probably be small. This is owing to the fact that the transmission line will very likely be a long way from the place where there is a demand for the copper, and a considerable deduction from the price will be necessary to meet the transportation charges and the labour cost of removing the wires from the poles.

144. Economic Current Density in Overhead Conductors.—As previously stated, the annual charges on the initial cost of a transmission line installation can be represented fairly closely by a linear equation

$$P_1 = (k_0 + k_1 A)a. \quad . \quad . \quad . \quad (176)$$

It should be remembered that the second term in the above expression varies at a rather higher rate than it would do if the cost of the line conductors only were being examined. With overhead lines, however, it is generally found sufficient to take into account the variation of cost of conductor material, and ignore other secondary factors. This simplification, in practice, gives results to the degree of accuracy warranted by the state of knowledge of the other data of the problem.

A convenient formula for the economic current density in overhead conductors can now be derived, bearing in mind that the annual charges are: (1) the annual charge for the interest and depreciation on the capital outlay on the conductors, (2) the

annual cost of the energy wasted in the conductors, and that the assumed equality of these two items of cost determines the size of conductor and consequent economic current density.

Let p_1 = cost of conductor in pence per pound.

a = percentage to be taken to cover annual interest and depreciation on conductors.

R_m = resistance of conductor 1·0 sq. in. cross-section, in ohms per mile.

m = weight of conductor 1·0 sq. in. cross-section, in lbs. per mile.

A = section of the conductor in square inches.

p_2 = cost of one kilowatt-hour of wasted energy in pence.

I = R.M.S. current in conductor during the year.

Then the energy dissipated annually (8 760 hours) in one mile of the conductor is

$$\frac{I^2 R_m \, 8\,760}{1\,000\,A} \quad \text{kilowatt-hours,}$$

and the cost of this energy is

$$\frac{p_2 I^2 R_m \, 8\,760}{1\,000\,A} \quad \text{pence.} \qquad . \quad . \quad . \quad (179)$$

The cost of one mile of the conductor is

$$mAp_1 \quad \text{lb.,}$$

and the annual interest and depreciation on this cost is

$$\frac{mAp_1 a}{100} \quad \text{pence.} \qquad . \quad . \quad . \quad . \quad (180)$$

For equality between (179) and (180),

$$\frac{p_2 I^2 R_m \, 8\,760}{1\,000\,A} = \frac{mAp_1 a}{100},$$

or

$$\frac{I}{A} = \sqrt{\frac{mp_1 a \times 1\,000}{R_m p_2 \times 100 \times 8\,760}} \qquad . \quad . \quad (181)$$

For hard-drawn, stranded copper conductors at 20° C.

$$m = 21\,100 \quad \text{lb.,}$$
$$R_m = 0{\cdot}0445 \quad \text{ohms,}$$

and substituting, the economic current density is found as

$$\frac{I}{A} = 23{\cdot}3\sqrt{\frac{ap_1}{p_2}} \qquad . \quad . \quad . \quad . \quad (182)$$

For hard-drawn, stranded aluminium conductors (m = 6 230 and R_m = 0·0755) the economic current density is

$$\frac{I}{A} = 9{\cdot}7\sqrt{\frac{ap_1}{p_2}} \quad . \quad . \quad . \quad . \quad (183)$$

145. Influence of Line Voltage on Cost of Transmission System.—It is often not fully realised that the application of Kelvin's law by itself is insufficient to enable the size of the line conductors to be calculated. Kelvin's law merely gives the current density at which the conductors should be worked for maximum economy, but the actual conductor cross-section cannot be determined unless the value of the line current is known, and this depends upon the pressure at which the energy is transmitted.

If the cost of the conductors forming the transmission line and the I^2R losses themselves were the only consideration, the highest obtainable line voltage would be the best, on account of the corresponding reduction of current for a given amount of power to be transmitted. But the transmission system must be looked at as a whole, and it will be found that it does not pay to increase the voltage above a certain point. Above this limit the saving in conductor material is more than counterbalanced by the increased cost of the line insulation and terminal apparatus. Furthermore, at extremely high pressures, other sources of losses such as corona and leakage over insulators begin to make their appearance. The choice of line voltage for any particular transmission scheme is thus mainly an economic question, and must be determined by taking into account the cost of the various parts of the complete system which are influenced by alterations in the line voltage.

146. Determination of Economic Voltage and Size of Conductors.—In order to illustrate the application of the above principles, a concrete scheme will be taken. It is assumed that a maximum load of 20 000 kVA. (16 000 kW. at 80 °/₀ power factor, 50 cycles), is to be supplied over a distance of 50 miles, the load factor being 60 °/₀. Conductors of hard-drawn, stranded copper are to be used, and the average line span is 600 feet. To guard against interruption to service two sets of conductors are employed carried on one set of steel towers.

Fig. 79 shows diagrammatically the layout of the system, which is designed to allow for the greatest flexibility of operation. Two three-phase transformers (or banks of single-phase transformers) each of 10 000 kVA. capacity are provided, both at the

sending and receiving end of the line. A saving in the high-tension switchgear could be effected by treating each transformer group and set of conductors as one unit, in which case isolating switches would only be required on the high-tension side to disconnect the transformers from the line when necessary. While

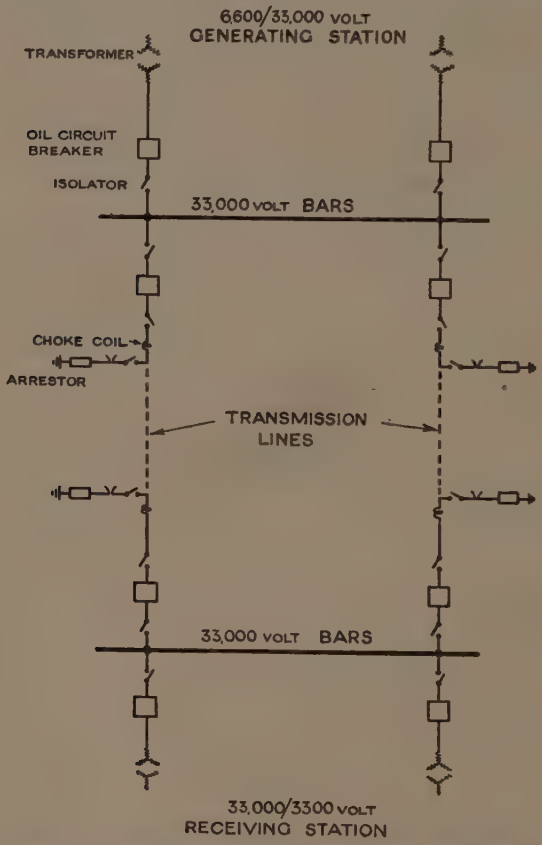

FIG. 79.—Layout of transmission system to illustrate example on economic design.

this arrangement simplifies the switching system it does so at the expense of the flexibility of the system. In the proposed layout the line circuit-breakers are of larger current-carrying capacity than the transformer circuit-breakers, and in case of breakdown to one line the other could be permitted to carry the full load of

20 000 kVA. This is a great advantage, and the additional cost of the line circuit-breakers is comparatively small.

The first step in the solution is to determine the economic current density for the line conductors. Let the cost of hard-drawn, stranded copper conductors on site p_1 be taken as 9d. per lb., the cost of the wasted energy p_2 as 0·5 pence per kilowatt-hour, and the interest and depreciation on the cost of conductors a as 10 °/₀ per annum.

Then from formula (182) the economic current density is

$$\frac{I}{A} = 23{\cdot}3\sqrt{\frac{9 \times 10}{0{\cdot}5}}$$
$$= 313 \quad \text{amperes per sq. in.}$$

Considering for the moment that the line pressure is 33 000 volts, the current required when the maximum load of 10 000 kVA. per set of conductors is being transmitted is

$$\frac{10\,000 \times 1\,000}{\sqrt{3} \times 33\,000} = 175 \quad \text{amperes.}$$

For a typical 60 °/₀ load-factor curve the ratio of R.M.S. to average current is 1·13, hence the R.M.S. current flowing in each conductor can be taken as

$$I = 175 \times \frac{60}{100} \times 1{\cdot}13$$
$$= 119 \quad \text{amperes,}$$

and the economic area of conductor is

$$A = \frac{119}{313}$$
$$= 0{\cdot}381 \quad \text{sq. in.}$$

As the weight of a copper conductor 1 mile long, and 1·0 sq. in. cross-section, is approximately 21 100 lb., the total weight of copper required in the line is

$$0{\cdot}381 \times 21\,100 \times 6 \times 50$$
$$= 2{\cdot}41 \times 10^6 \quad \text{lb.,}$$

and thus the total cost of the line conductors on a 33 000-volt system is

$$\frac{2{\cdot}41 \times 10^6 \times 9}{240}$$
$$= £90\,400.$$

The cost of the copper required for other assumed pressures can then be readily calculated, taking advantage of the fact that the cost is proportional to the current and hence inversely proportional to the pressure.

The cost of the terminal apparatus and insulators for operation at 33 000 volts would be approximately as follows:—

Transformers.

Two 10 000-kVA. 3-phase 6 600 / 33 000-volt 50-cycle transformers.
Two 10 000-kVA. 3-phase 33 000 / 3 300-volt 50-cycle transformers.
Price £15 000

Switchgear.

Four 33 000-volt transformer and control equipments complete with oil circuit-breakers, isolators and control panels
Price £2 900
Four 33 000-volt line control equipments similar to above but of larger current-carrying capacity
Price £3 300
Four 33 000-volt electrolytic impulse-gap lightning arrestors complete with isolators and choke coils
Price £2 100

Line Insulators.

Three thousand 33 000-volt pin-type insulators
Price £1 500

The total cost of terminal apparatus and insulators for a 33 000-volt system would thus be about £24 800.

In order to obtain the cost of this apparatus at other pressures use will be made of Table 21, which gives the relative cost of apparatus designed for various standard voltages expressed as a percentage of 6 600-volt costs. These figures are given by Nesbitt,[4] and for the sake of argument will be assumed to be correct for the particular problem under consideration. It need hardly be said, however, that these relative costs are liable to fluctuate, and in practice this kind of information would be obtained direct from the manufacturers as and when required.

TABLE 21.—*Approximate Relative Costs of High-tension Apparatus.*

(6 600-volt costs taken as 100 %.)

	Voltage.							
	6 600	11 000	22 000	33 000	44 000	66 000	88 000	110 000
Transformers . .	100	102	108	115	125	150	175	200
Switchgear . .	100	100	100	110	115	155	255	420
Electrolytic lightning arrestors . .	100	151	205	320	490	640	1 600	1 900
Insulators . .	100	135	430	650	1 250	3 500	5 500	6 500

On the basis of Nesbitt's figures the result of a comparison between installation costs at the various standard pressures is exhibited in Table 22, and it will be seen that the pressure of 66 000 volts is the most economical for the conditions laid down. In some cases, however, the difference between the economical and the next higher pressure may be quite small, and in such cases it is very probable that the higher voltage would be adopted. One important consideration influencing the choice of the higher pressure is that more trouble is experienced with heavy currents than with high voltages, owing to the more serious effects of transient disturbances due to switching or short-circuits. Hence greater security, and sometimes lower maintenance and operating costs, may be obtained by designing the system for a high pressure. It should be noted also that many mistakes have been made in the past by under-estimating the ultimate demand for electric power, and consequently adopting too low a pressure. When in doubt, the higher voltage should be selected, and will probably justify its adoption by reason of future growth of the system.

Adopting, therefore, the pressure of 66 000 volts for the line voltage of the system, the corresponding theoretical area of conductor required is 0·190 sq. in., and the nearest standard conductor, 7 / 0·193 stranded copper, having an area of 0·20 sq. in. would be chosen.

The expenditure on line supports, buildings, and wayleaves, is not included in the above comparison. The two latter items are practically constant irrespective of the line voltage, but the

TABLE 22.—*Comparison of Installation Costs at Various Voltages.*
(See Problem—Art. 146.)

	Voltage.							
	6 600	11 000	22 000	33 000	44 000	66 000	88 000	110 000
Theoretical conductor area	Sq. In. 1·905	Sq. In. 1·143	Sq. In. 0·571	Sq. In. 0·381	Sq. In. 0·286	Sq. In. 0·190	Sq. In. 0·143	Sq. In. 0·114
	£	£	£	£	£	£	£	£
Conductors	452 000	271 200	135 600	90 400	67 800	45 200	33 900	27 100
Transformers	—	—	14 100	15 000	16 300	19 600	22 800	26 100
Switchgear	3 100	3 100	5 600	6 200	6 500	8 700	14 400	23 700
Lightning arrestors	700	1 000	1 300	2 100	2 800	4 200	10 500	12 500
Insulators	200	300	1 000	1 500	2 900	8 100	12 700	15 000
Total cost	456 000	275 600	157 600	115 200	96 300	85 800	94 300	104 400

cost of the line supports does increase slightly at the higher pressures owing to the greater spacing required for the conductors, and this difference should be taken into consideration if sufficient information is available. In a rigorous comparison it would also be necessary to take into account the probable variation of operating and maintenance costs as influenced by line voltage.

In getting out preliminary estimates for a transmission scheme the following empirical formula due to Still,[5] and based on modern American practice, may be taken as a guide:—

$$V = 5{\cdot}5\sqrt{l + \frac{3P}{100}} \quad . \quad . \quad . \quad . \quad (184)$$

where V is the line pressure (between conductors), l is the distance of transmission in miles, and P is the estimated maximum number of kilowatts per phase to be delivered over one pole- or tower-line.

That both the distance and amount of power to be transmitted do influence the economic voltage of a transmission line can readily be seen. For as the distance of transmission is increased the influence of the terminal-apparatus costs upon the line pressure becomes of less and less importance, leading to a higher economic pressure. Also if a large amount of power is to be supplied, larger generating and transforming units can be employed, with

consequent reduction in the cost per kilowatt of terminal-station equipment.

147. Other Factors Influencing Electrical Design of System.—After working out the size of conductor required from considerations of economy, the matter should always be further investigated to ascertain if the conductor is also technically suitable. Occasionally the economic size of conductor cannot be adopted in practice for one or more of the following reasons:—

1. The regulation of the line may be greater than can be conveniently dealt with.

2. The section of the conductor may be too small, or too large, from a mechanical point of view.

3. The diameter of the conductor may be so small as to lead to excessive corona losses.

4. The current density may be so high as to raise the temperature of the conductor above the safe limit.

1. In the case of excessive line regulation there are two remedies. The first method is to adopt a higher line pressure, and in consequence a smaller current and smaller conductor. The reactance pressure drop ωLI, which chiefly determines the regulation of the system, is thereby decreased, not exactly in proportion to I (as the reactance is increased slightly by the larger value of $\frac{d}{r}$), but nearly to the same extent. Thus the decrease in the percentage regulation is brought about both by the decrease in the actual pressure drop along the line, and the increase in the line voltage itself. The other method of improving the regulation is to split a single circuit into two or more parallel circuits, having the same total area of conductor per phase as the economic conductor. Thus, assuming two sets of conductors in parallel each set taking one-half of the total load, the reduction in the reactive pressure drop ωLI due to the variation in I, is of far more importance than the slight increase of L (due to the higher value of $\frac{d}{r}$), and results in the regulation being reduced very considerably. For instance, assuming the transmission of 40 000 kVA. $\left(\cos\phi_r = 80\ \%\ \text{lagging}, E_r = \frac{66}{\sqrt{3}}\text{kV.}\right)$ over 19 / 0·166 strand, 0·40 sq. in. conductors spaced at 90 inches, the full-load regulation would be 19·8 %. Taking into consideration

the voltage drops in the terminal transformers also, this value would be too great to deal with, but by employing two sets of conductors (each 7 / 0·193 strand, 0·20 sq. in., same spacing) the regulation is brought down to the manageable figure of 12·4 %. It is true that the duplicate set of conductors entails an increase in cost of line supports and insulators, but this is more than compensated for by the valuable insurance provided against total stoppage of supply.

2. In the case of the conductor cross-section coming out so small that the line would be mechanically weak, the obvious remedy is to increase the sectional area somewhat, without regard to the question of economy, or it may be found that aluminium conductors can be adopted with advantage. Generally speaking, it is not advisable to employ line conductors having less than the following sectional areas:—

No. 8 S.W.G. (0·02 sq. in.) for solid copper conductors.

0·025 sq. in. for stranded copper conductors.

0·04 sq. in. for stranded aluminium conductors.

If, on the other hand, the conductor diameter works out too large, there may be difficulty in handling or taking the strain at the individual insulators. The circuit should therefore be subdivided into two sets of conductors in parallel.

3. Excessive corona losses on account of the small diameter of conductor leads to the employment of specially made-up conductors (*see* Art. 93), or here again, aluminium or steel-cored aluminium may be found more advantageous as a conductor material.

4. It is very rare that trouble is experienced through overheating of conductors on an overhead line owing to the fairly low current densities in use and the good facilities for cooling. In short-distance heavy-current transmissions, however, the current may be sufficient to injure the conductor as copper starts to anneal when the temperature is above 100° C. Trouble may also be caused by excessive sag, and current densities should be such that this figure is not exceeded. As a result of carefully made tests upon various types and sizes of conductors in still air, Luke[6] gives the following formula for determining the temperature rise:—

$$I = \sqrt{\frac{\pi d l K_d \theta}{R_\theta}}, \quad . \quad . \quad . \quad . \quad (185)$$

where I = continuous current-carrying capacity.
d = diameter of conductor in inches.
l = length of conductor in inches.
K_d = dissipation constant obtained from Fig. 80.
θ = temp. rise permissible in deg. C.
R_θ = resistance of conductor at final temperature corresponding to θ.

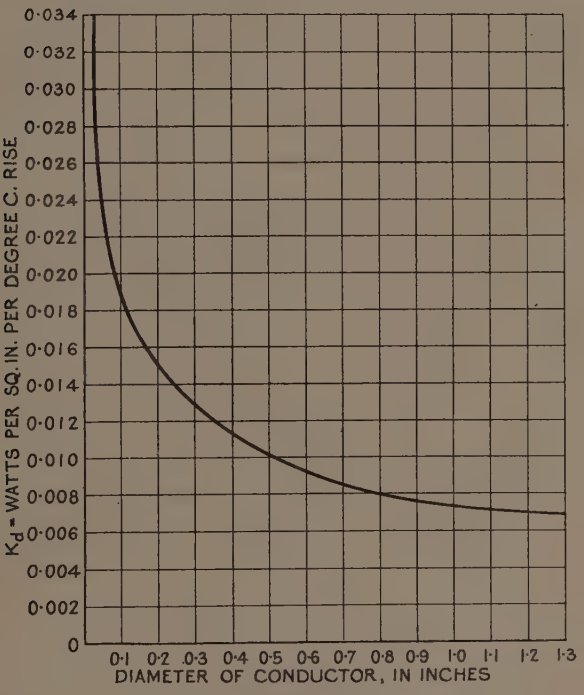

Fig. 80.—Luke's dissipation constants.

This formula gives very safe results as in practice the outside air is always in movement, and this greatly reduces the temperature rise of a conductor suspended therein. It should be noted that a stranded aluminium conductor will carry about 14 % more current than a stranded copper conductor of equal conductivity.

Before finally deciding upon the transmission voltage careful consideration should be given to the voltage of adjacent systems. There is an increasing tendency to combine neighbouring generating and transmission systems, and should such an

amalgamation take place it would be advantageous if the equipment—transformers, switchgear, insulators, etc.—of the two systems were interchangeable for service on either system.

At the present time a large number of different pressures are in use for transmission schemes, and the design and construction of transformers, circuit-breakers, lightning apparatus, etc., for all these various pressures is expensive. An endeavour is now being made to standardise transmission voltages so as to allow manufacturers to concentrate on fewer voltage ratings with benefit both to themselves and the consumers. In America the following standard line voltages are recommended:—

33 000	110 000
44 000	132 000
66 000	154 000
88 000	220 000

and in Great Britain the pressures adopted by the British Engineering Standards Association are [7]:—

33 000	110 000
66 000	132 000

148. Application of Economic Principles to Underground Lines.—It is found that the variation in cost of high-voltage cable with respect to the size of conductor follows very closely a linear law. In other words, the cost per mile of cable is

$$P_1 = k_0 + k_1 A,$$

the actual value of the constants k_0 and k_1 depending on the voltage class of the cable. Unfortunately, the Kapp-Kelvin law cannot be applied direct to the calculation of the economic current density, as beside the I^2R losses there are also the dielectric and sheath losses to be considered. Furthermore, owing to the greater capacitance of an underground line, some method is required of taking into account the effect of the charging current on the I^2R losses due to the load current.

The best method of dealing with the problem is to tabulate the cost of the line losses and the capital charges corresponding to the standard sizes of cable for various assumed transmission voltages. The final choice can then be made by bringing into the reckoning the cost of the terminal apparatus. A numerical example will serve best to illustrate the method.

Consider the following transmission scheme, *viz.* the transmission of 15 000 kVA. (12 000 kW. at 80 °/₀ power factor lagging, 50 cycles, load factor 50 °/₀), a distance of 10 miles, by means of a three-core cable.

At a pressure of 22 000 volts the line current corresponding to maximum load is

$$\frac{15\,000 \times 1\,000}{\sqrt{3} \times 22\,000}$$
$$= 394 \quad \text{amperes,}$$

and for a typical 50 °/₀ load curve the R.M.S. current flowing in each conductor is

$$I = 394 \times \frac{50}{100} \times 1{\cdot}20$$
$$= 236 \quad \text{amperes.}$$

This is the value at the receiving end of the line and at the sending end it will usually be smaller, owing to the effect of the charging current. A close approximation to the actual losses in the conductor is obtained by taking the mean value of the current as

$$\sqrt{(I \cos \phi_r)^2 + (I \sin \phi_r - I_{cr})^2},$$

where I_{cr} is the current taken by a condenser having a capacitance one-half that of the conductor, shunted between conductor and neutral at the receiving end of the line (Art. 37). In other words, the line I^2R losses may be determined by multiplying the losses which would be produced by the load current, by the reduction factor

$$k = \frac{(I \cos \phi_r)^2 + (I \sin \phi_r - I_{cr})^2}{I^2} \quad . \quad . \quad (58)$$

As the voltage drop along an underground line is comparatively small, a simplification can be introduced by calculating I_{cr} as though the pressure across it was the sending-end voltage to neutral.

In the case of a 22 000-volt, 0·20 sq. in. cable (capacitance say 0·32 microfarads, resistance 0· 244 ohms per mile at 50° C.),

$$I_{cr} = 2\pi \times 50 \times 0{\cdot}16 \times 10^{-6} \times \frac{22\,000}{\sqrt{3}} \times 10$$
$$= 6{\cdot}5 \quad \text{amperes,}$$

and
$$k = \frac{(236 \times 0{\cdot}8)^2 + (236 \times 0{\cdot}6 - 6{\cdot}5)^2}{(236)^2}$$
$$= 0{\cdot}97.$$

Hence the total I^2R losses in the cable per annum are

$$\frac{236 \times 236 \times 0{\cdot}244 \times 10 \times 3 \times 8\,760 \times 0{\cdot}97}{1\,000}$$

$$= 3{\cdot}46 \times 10^6 \text{ kilowatt-hours.}$$

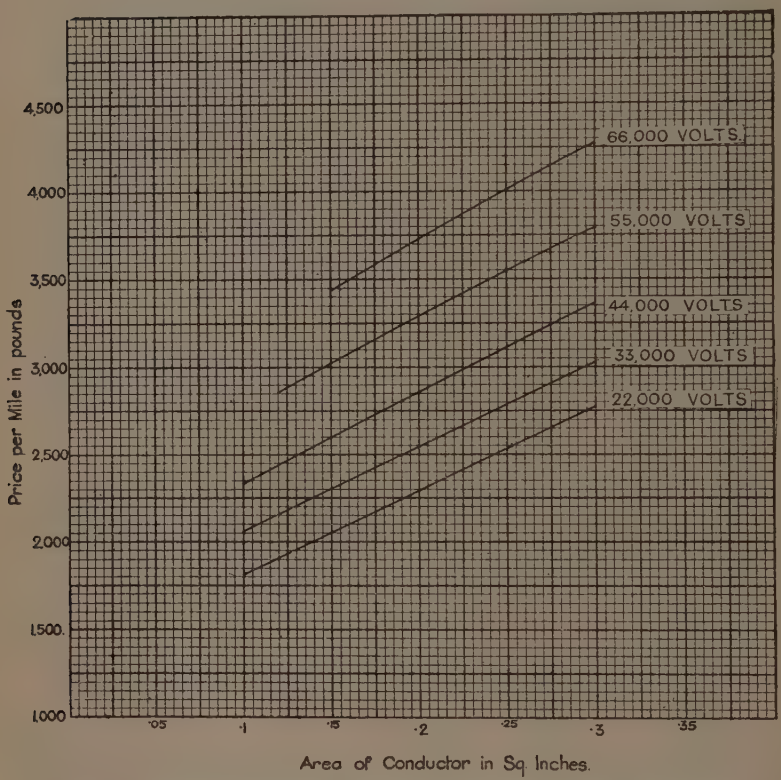

FIG. 81.—Approximate cost of three-core cables laid direct in the ground.

Assuming that the average value of the dielectric power factor is 1 %, the total annual dielectric losses in the cable are

$$\frac{2\pi \times 50 \times 0{\cdot}32 \times 10^{-6} \times 22\,000 \times 22\,000 \times 10 \times 3 \times 8\,760}{1\,000 \times 100 \times \sqrt{3} \times \sqrt{3}}$$

$$= 0{\cdot}04 \times 10^6 \text{ kilowatt-hours.}$$

In a similar manner the losses and therefrom the cost of the losses can be determined for other cable sizes and voltages.

In connection with the calculated results given in Table 23 it should be noted that:—

1. The cost figures for the cables have been obtained from the curves of Fig. 81 which give approximate prices per mile for three-core cables, these being single-wire armoured, served and compounded, and suitable for laying in the ground. The prices include the supply and laying of the cables and also the jointing, but are exclusive of the cost of any trenchwork or warning boards. The cost of these latter two items will of course be the same whatever size or class of cable is employed so that this does not enter into the comparison.

2. The percentage for annual interest and depreciation on the capital outlay has been taken as 15 °/₀ for terminal apparatus and 10 °/₀ for cables, and the cost of the wasted energy as 0·5 pence per kilowatt-hour.

3. The conductor temperature has been assumed to be 50° C., and the dielectric power factor 1 °/₀ in all cases. Also in calculating the effect of the charging current on the losses an average value of k has been used for each voltage class of cable, this value corresponding to the charging current of the 0·20 sq. in. conductor.

TABLE 23.—*Form of Tabulation for Determining Voltages and Size of Conductors.* (See Problem—Art. 148).

Voltage = 22 kV. R.M.S. current = 236 amperes. k = 0·97.

	Conductor Area in Sq. In.				
	0·10.	0·15.	0·20.	0·25.	0·30.
Annual I^2R losses, kWH. × 10^6 . . .	6·92	4·61	3·46	2·77	2·31
Annual dielectric losses, kWH. × 10^6 .	0·04	0·04	0·04	0·05	0·05
Total losses, kWH. × 10^6	6·96	4·65	3·50	2·82	2·36
	£	£	£	£	£
Cost of losses per annum . . .	14 500	9 700	7 300	5 880	4 920
Interest and depreciation on cost of cable per annum	1 810	2 050	2 290	2 530	2 770
Total cost per annum	16 310	11 750	9 590	8 410	7 690

TABLE 23.—*Continued.*

Voltage = 33 kV. R.M.S. current = 157 amperes. $k = 0{\cdot}94$.

	Conductor Area in Sq. In.				
	0·10.	0·15.	0·20.	0·25.	0·30.
Annual I^2R losses, kWH. $\times 10^6$	2·96	1·97	1·48	1·18	0·99
Annual dielectric losses, kWH. $\times 10^6$	0·08	0·08	0·09	0·09	0·10
Total losses, kWH. $\times 10^6$	3·04	2·05	1·57	1·27	1·09
	£	£	£	£	£
Cost of losses per annum	6 340	4 280	3 270	2 650	2 270
Interest and depreciation on cost of cable per annum	2 060	2 300	2 540	2 780	3 030
Total cost per annum	8 400	6 580	5 810	5 430	5 300

Voltage = 44 kV. R.M.S. current = 118 amperes. $k = 0{\cdot}89$.

	Conductor Area in Sq. In.				
	0·10.	0·15.	0·20.	0·25.	0·30.
Annual I^2R losses, kWH. $\times 10^6$	1·59	1·06	0·80	0·64	0·53
Annual dielectric losses, kWH. $\times 10^6$	0·13	0·14	0·15	0·16	0·17
Total losses, kWH. $\times 10^6$	1·72	1·20	0·95	0·80	0·70
	£	£	£	£	£
Cost of losses per annum	3 580	2 520	1 970	1 660	1 450
Interest and depreciation on cost of cable per annum	2 340	2 590	2 850	3 100	3 360
Total cost per annum	5 920	5 110	4 820	4 760	4 810

TABLE 23.—*Continued.*

Voltage = 55 kV. R.M.S. current = 94 amperes. $k = 0{\cdot}85$.

	Conductor Area in Sq. In.				
	0·10.	0·15.	0·20.	0·25.	0·30.
Annual I^2R losses, kWH. × 10^6	0·97	0·65	0·49	0·39	0·33
Annual dielectric losses, kWH. × 10^6	0·19	0·20	0·22	0·23	0·24
Total losses, kWH. × 10^6	1·16	0·85	0·71	0·62	0·57
	£	£	£	£	£
Cost of losses per annum	2 410	1 780	1 470	1 300	1 180
Interest and depreciation on cost of cable per annum	2 760	3 010	3 270	3 530	3 790
Total cost per annum	5 170	4 790	4 740	4 830	4 970

Voltage = 66 kV. R.M.S. current = 79 amperes. $k = 0{\cdot}81$.

	Conductor Area in Sq. In.				
	0·10.	0·15.	0·20.	0·25.	0·30.
Annual I^2R losses, kWH. × 10^6	0·64	0·43	0·32	0·26	0·21
Annual dielectric losses, kWH. × 10^6	0·26	0·28	0·30	0·32	0·33
Total losses, kWH. × 10^6	0·90	0·71	0·62	0·58	0·54
	£	£	£	£	£
Cost of losses per annum	1 870	1 480	1 290	1 200	1 140
Interest and depreciation on cost of cable per annum	3 160	3 440	3 720	3 990	4 270
Total cost per annum	5 030	4 920	5 010	5 190	5 410

It will be seen from the table that at 22 000 and 33 000 volts the most economical size of conductor to employ is 0·30 sq. in., at 44 000 volts the best conductor is 0·25 sq. in., at 55 000 volts 0·20 sq. in., and at 66 000 volts 0·15 sq. in.

The influence of terminal-apparatus costs on the problem must now be considered. For this purpose the cost of the 33 000-volt apparatus required will be assumed as follows :—

Transformers	£11 000
Switchgear	£3 000

and the figures given in Table 21 again made use of in order to find the approximate costs at other voltages.

The final comparison may then be made as shown in Table 24, from which it will be seen that for the problem under consideration a 44 000-volt, 0·25 sq. in. cable is the best from an economic point of view. The current density in the conductor corresponding to the R.M.S. load is

$$\frac{I}{A} = \frac{118}{0{\cdot}25}$$
$$= 472 \quad \text{amperes per sq. in.}$$

TABLE 24.—*Comparison of Installation Costs at Various Voltages.* (See Problem—Art. 148.)

	Voltage.				
	22 000.	33 000.	44 000.	55 000.	66 000.
	£	£	£	£	£
Annual interest and depreciation on cable + cost of losses per annum	7 690	5 300	4 760	4 740	5 010
Annual interest and depreciation on cost of terminal apparatus	1 950	2 100	2 280	2 505	2 775
Total annual costs	9 640	7 400	7 040	7 245	7 785

It is very rare that excessive pressure drop gives any trouble in the case of underground lines thus necessitating a lower current density than that derived from economic considerations. This is on account of the shorter distances of transmission, and also on account of the smaller reactance of cables as compared with the reactance of an overhead line using the same size of conductors. It is necessary, however, to ascertain that the economic current density is not so high as to raise the temperature of the dielectric above the safe limit. This is particularly important in cases where the load factor of the system is small.

In the particular problem considered, the current flowing when maximum load of 15 000 kVA. is being transmitted is

$$\frac{15\,000 \times 1\,000}{44\,000 \times \sqrt{3}}$$
$$= 197 \text{ amperes,}$$

and comes well below the value which could be permitted if heating of the cable was the only factor to be taken into consideration. The voltage drop in the line at times of maximum load is about 2 %.

149. Choice of Frequency.—There are several different supply frequencies in existence at the present time but for new installations the standard frequency is 50 cycles with 25 cycles as a secondary standard. In America both 25- and 60-cycle supplies are used extensively, but the higher frequency is likely to predominate in the future. Many of the 25-cycle systems in operation were installed at a time when the design of converting apparatus for the higher frequencies had not reached a satisfactory stage. Now, however, by improved methods of design and construction—notably the use of higher peripheral speeds and commutating poles—rotary convertors for 50 or 60 cycles have been made practically equal to those for 25-cycle supply. As regards other apparatus, 50-cycle turbo-alternators are about 10 % cheaper than 25-cycle turbo-alternators, and 50-cycle transformers are about 25 to 30 % lighter and 25 % cheaper than those of the same output at the lower frequency.

One feature that should be considered is the influence of the frequency on the reactance and charging current of the circuit, both these quantities at 50 cycles being twice their value at 25 cycles. The high reactance of line and apparatus at 50 cycles tends to produce poor voltage regulation of the circuit, but has one advantage for the larger systems in that it limits short-circuit currents, and thus assists the circuit-breakers to function properly. The 50-cycle charging current also, in the case of long lines, and particularly where two or more sets of conductors in parallel are employed, may be so great as to limit the choice in transmission voltage. On the other hand, large charging currents can be allowed if synchronous phase modifiers are installed to maintain the line voltage constant.

In the absence, therefore, of special circumstances it appears

desirable to choose the higher frequency of 50 or 60 cycles as the case may be. The advantage of unification is to reduce the cost and simplify the manufacture of terminal apparatus due to the use of fewer designs. Furthermore, the adoption of one common supply frequency throughout the country facilitates the interchange of energy between different undertakings by removing the necessity of employing frequency changers as the link between interconnected systems.

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CHAPTER XI.

APPARATUS FOR THE PREVENTION OF DANGEROUS CURRENTS.

150. Oil Circuit-breakers.—The earliest methods of guarding against the effects of dangerous currents was by the employment of fuses in series with the circuit to be protected. This simple form of overload protection worked satisfactorily on the low-voltage small-power transmissions laid down in the early days of the electric supply industry, but it is manifestly unsuitable for present-day systems. High-tension fuses or fuse-switches, however, are used for small consumers supplied through a tapping from the main transmission line, owing to their low cost compared with the cost of oil circuit-breakers. They are also used on other circuits of small current-carrying capacity such as the pressure circuits of instruments.

Generally speaking, however, high-voltage alternating-current circuits are broken by means of oil circuit-breakers. These breakers possess the valuable characteristic of rupturing the circuit near to the zero point on the current wave. An arc is drawn out between the separating contacts of the breaker under the oil, and this arc is rapidly cooled and compressed by the surrounding oil. Naturally, the capability of resistance to rupture of the hot vapour column forming the arc is at a minimum when the current flowing is at zero, at which instant if the breaker be equal to its duty the circuit is finally opened. It will thus be seen that the circuit is broken at the most favourable moment, in strong contra-distinction to the case of the air-break circuit-breaker with which, on an alternating-current circuit, the tendency to rupture is strongest when the current is at its maximum.

Oil circuit-breakers are practically always of the automatic type, that is they are equipped with trip coils, which on the

occurrence of abnormal conditions in the main circuit are energised by means of relays. The mechanical design of the breaker is such that when it is closed considerable energy is stored in strong springs and the parts held together by a system of toggles, the latter being so constructed that only a slight pressure on a trigger is needed to open the breaker. When actuated by the protective relay, the trigger is tripped, thereby releasing the potential energy of the springs which causes the contacts to open in a small fraction of a second.

151. Protective Relays.—The protective relay may be defined as an electrical instrument interposed between the main circuit and the circuit-breaker in such a manner that any abnormality in the circuit acts on the relay, which in turn, if the abnormality is of a dangerous character, causes the breaker to function and relieve or protect the circuit and apparatus. In recent years great advances have been made in the design and construction of these instruments, and present-day types can be depended upon in practically all conceivable cases of electrical distress. The importance of automatic protective devices has grown with the size of transmission systems and their systematic employment is now almost a necessity. These devices protect the plant from damage in the event of heavy fault currents occurring, but their chief function is to preserve the continuity of the general supply by rapidly removing from the system faulty lines or apparatus. By the employment of selectively-acting protective relays, defective lines can be automatically located and cut out of circuit with the minimum amount of disturbance to the rest of the system, and in many cases while the fault is in its early stages.

A current flowing in an electric circuit becomes dangerous either when it increases in magnitude beyond the capacity of the conductor, or when the electric power corresponding to the current in question flows in the wrong direction, or when due to failure of the insulation the current follows a path other than the prescribed one. Hence automatic protective devices may be roughly classified as follows :—

1. Overload relays or devices which operate when the current exceeds a certain predetermined limit.

2. Reverse-power relays which operate when the flow of power at any instant is in the reverse direction to that which is normal at that instant.

3. Leakage relays which operate when current flows through other than the proper channels.

Generally speaking, protective devices used for the purpose of opening circuit-breakers have their connections divided into three circuits. The first is the primary winding of a current transformer which is connected in series with the main circuit to be protected. This primary winding very often consists of a single turn, or even of the main conductor merely linked through an iron core. The second circuit consists of the secondary winding of the current transformer and the relay-operating winding. The third, or tripping circuit, may be either D.C. or A.C. and consists of a source of supply, the trip coil on the circuit-breaker, and the relay stationary contacts. The operation of the relay is effected by another contact which is attached to the moving element of the relay. When the abnormality on the protected circuit reaches a certain predetermined magnitude, the moving element of the relay causes this latter contact to bridge the stationary contacts, thus completing the tripping circuit, energising the trip coil of the breaker, and causing the breaker to open.

One of the most important points in connection with the installation of protective devices is the source of supply for the tripping circuit. This supply should be absolutely reliable, and if possible independent of the system being protected. Undoubtedly the most dependable and suitable source is a good storage battery.

152. Constructional Details of Relay.—The majority of protective relays depend on the magnetic effect of the current for their operation and fall roughly into three classes; (1) the electromagnet type, utilising the attractive force of an electromagnet on an armature, (2) the solenoid-and-plunger type, making use of the sucking effect of an energised solenoid on an iron plunger, and (3) the induction type, employing the basic principles of the induction motor and watt-hour meter. Relays are made in a large variety of forms and a detailed description of the various makes in use is outside the scope of this work. However, for the sake of completeness, a short description of the design and construction of a typical modern induction-type relay will be given. It is generally acknowledged that relays based on the induction principle are the most accurate and reliable for

overload or reverse-power protection. The calibration has the valuable quality of permanency, and independent adjustment can be made within wide limits both as regards current and time.

The form of relay made by the Westinghouse Co. is illustrated in Fig. 82. The conductor to be protected forms the primary of a current transformer, and the secondary winding of this transformer is connected in series with the relay winding E. On the same core as E is another winding A, which supplies the pole-piece windings B and D by transformer action; the path of the magnetic flux being indicated by the dotted lines in the figure. The value of overload at which the relay will operate

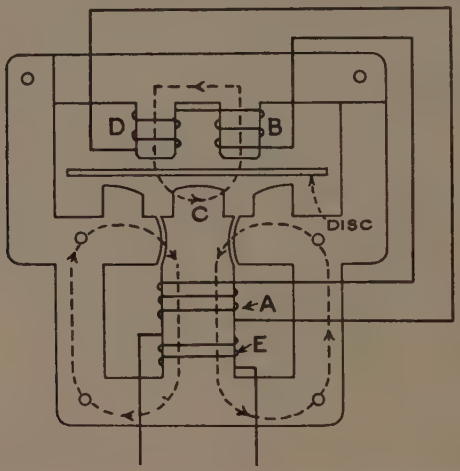

Fig. 82.—Magnetic and electric circuits of Westinghouse induction-type relay.

depends on the number of turns of E connected in series with the current transformer, and several tappings are taken from one end of E thus providing for an adjustable current setting. When current passes through E the disc K (shown in plan in Fig. 83) tends to rotate, but this movement is opposed by the spiral spring I fastened with its outer end to the permanent support M and its inner end to the shaft L which also carries the moving contact F. The rotation of the disc is damped by the permanent magnets O.

When sufficient current flows through the coil E to develop in the disc the necessary torque, it rotates until the contact F touches contact G, thereby completing the trip circuit. It is essential that the time of action of the relay for a given overload can be

adjusted, and to accomplish this the angle through which the contact *F* must travel is varied. Thus, if the contact must travel one-half revolution it may take two seconds, but the stop *H* can be arranged so that the contact need only make one-fourth of a revolution and then the time will be halved. Other positions of *H*, set by a small lever, enable any time from instantaneous to a maximum to be set.

Owing to the accuracy required in a relay the parts must be small, and delicately constructed. Consequently the contacts of the trip circuit cannot be of a heavy character and are not designed to open the trip circuit, either intentionally or unintentionally, once it has been established. This is an important

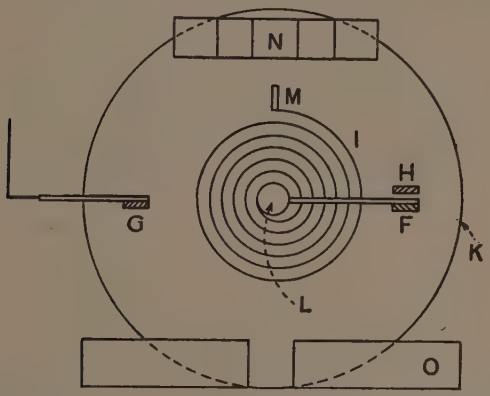

FIG. 83.—Top view showing position of contacts, etc., in Westinghouse relay.

point as the tripping circuits are as a rule highly inductive, and the D.C. arc which would naturally follow the opening of the circuit might persist for a considerable length of time, and result in serious damage to the contacts. For this reason it is necessary that the tripping circuit be opened by a small auxiliary switch fastened to the moving parts of the circuit-breaker, in such a manner that the opening of the breaker automatically opens the tripping circuit also.

A contactor switch connected in parallel with the relay contacts is usually provided for two reasons. In the first place, a circuit-breaker takes a definite period of time, say, 0·2 to 0·3 seconds to open after the relay contacts have closed and the trip coil of the breaker has been energised. Now if during this small

interval of time the overload disappears, the relay would instantly attempt to reset, and in so doing would open the tripping circuit before the auxiliary switch attached to the breaker could operate, thus resulting in severe burning of the relay contacts. In the second place, the overload might just be great enough to barely close the relay contacts. This weak closing might not allow sufficient current to pass through to operate the trip coil of the breaker, and the contacts would chatter and burn badly. By employing the contactor switch the weak fluttering contact is changed into a good positive contact, which ensures that the tripping circuit is kept closed until opened externally by the auxiliary switch on the breaker. A diagrammatic scheme of the contactor switch is shown in Fig. 84. The relay contacts *A*,

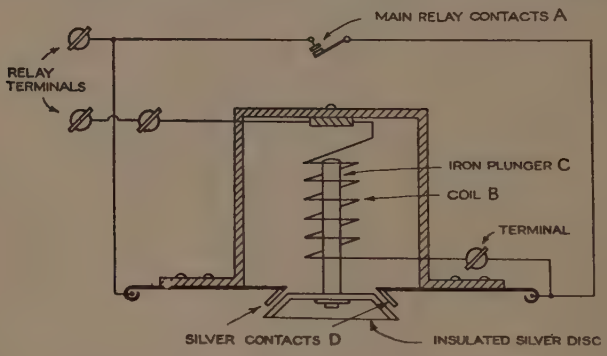

Fig. 84.—Contactor switch for use with relays.

when they close the tripping circuit of the relay, energise the small coil *B* which attracts an iron plunger *C* and closes the contacts *D* which are in parallel with *A*. Thus, it will be seen that even though the contacts *A* should open, the current will pass through *B*, and contacts *D* will stay closed until the auxiliary switch on the circuit-breaker opens the circuit; when this occurs coil *B* loses its pull and allows contacts *D* to open.

153. Overload Relays.—The application of plain overload relays for the protection of a three-phase line is illustrated in Fig. 85. The current transformers together with their respective relay-operating coils are connected in star, and on the occurrence of an excessive current in the line, either due to a short-circuit between conductors or between a conductor and earth, the circuit-breaker is immediately tripped out. On an unearthed system only

two current transformers and two relays are necessary. In this case it is obvious that an earth on one phase only cannot produce an overload, hence two or three phases must be affected, and at least one relay will function when the predetermined magnitude of current is exceeded.

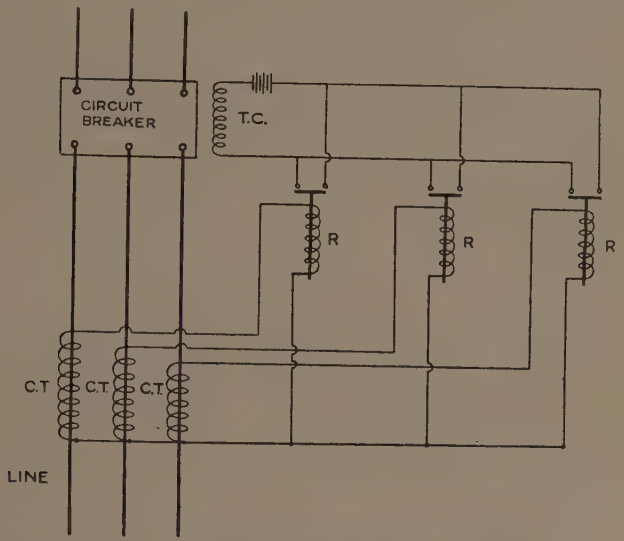

Fig. 85.—Plain overload protection for three-phase line.

154. Overload Relays with Series Tripping.—Occasionally it is desired to have some form of overload protection without incurring the expense of installing and maintaining a storage battery supply for tripping purposes. In this case the secondary current of the current transformer can be used as the emergency source, and a series tripping relay employed. The secondary of the transformer is connected in series with the trip coil on the circuit-breaker, but this coil is normally short-circuited by the relay. On the occurrence of an abnormal current in the line the contacts of the relay open, and the secondary current flows through the alternative path formed by the trip coil thereby energising it and opening the breaker. The diagram in Fig. 86 shows the connections for a single-phase line. With an unearthed three-phase system two current transformers and relays, and with an earthed three-phase system three transformers and relays, are required for complete overload protection. The drawback of this

method is that if the relay contacts become dirty, or make poor contact due to vibration, they may shunt current through the trip coil under normal load conditions and trip the circuit-breaker without cause. Hence it would be difficult to justify the omission of a storage battery or other separate source on installations of any importance.

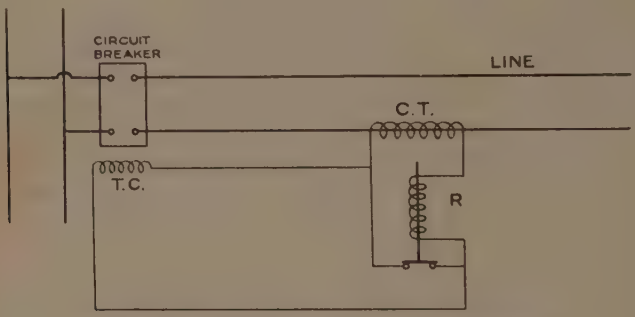

Fig. 86.—Overload protection with series tripping.

155. Time Delays.—For many purposes an instantaneously-acting, overload relay is undesirable, and a time lag is introduced therein. In the event of a momentary fault or temporary overload the circuit will then not be disturbed. The method of lagging or damping the moving element of a relay depends largely on the principle of operation. In the solenoid-and-plunger type some manufacturers employ a leather bellows with a small adjustable needle valve to allow the air to escape slowly. As the plunger attempts to rise the air is compressed in the bellows, thus retarding the movement. Other manufacturers use a dashpot with oil to retard the action. In the induction type of relay the time lag is obtained by setting back the fixed stop, so that the disc has to rotate through a greater angle before making contact. The permanent magnets also exert a damping effect.

In the event of series tripping being employed, a time lag can be imparted to the relay by connecting a fuse in parallel with the relay contacts. On the occurrence of an overload the contacts open, thus causing the excess current to pass through the fuse until the latter blows and the current is diverted through the trip coil. A cheaper arrangement still is to do away with the relay and simply employ a fuse connected in parallel with the

trip coil of the breaker, the trip coil being across the secondary of the series transformer.

The above methods of introducing a time lag into the action of the relay result in the production of relays having an inverse time limit. By this is meant that the time required for the relay to operate becomes shorter the greater the overload, the time lag varying approximately inversely as the current. An ordinary fuse gives protection of this character. Fig. 87 gives typical curves of an inverse-time-lag overload relay.

Another form of time lag used with relays is the definite time limit. As the name implies, in this type of protection there is a

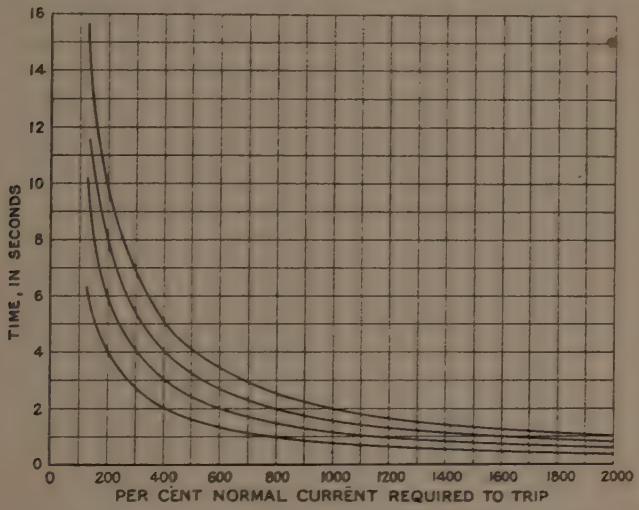

FIG. 87.—Typical time-load curves for inverse-time-lag relays.

definite time delay between the instant of disturbance and the closing of the relay contacts, and this delay in the action is in no way affected by the magnitude of the abnormality.

Both inverse-time-lag and definite-time-lag overload relays have their field of application, but on transmission systems they have been almost superseded by the compensated overload relay which will be described below.

156. Application of Overload Relays to a Radial Transmission System.—In any system of power transmission there will be found several circuit-breakers in series such as the generator breaker *A*, the main line breaker *B*, the receiving-station feeder breaker *C*, and the substation feeder breaker *D*.

This type of layout, shown diagrammatically in Fig. 88, is known as a radial system of transmission. Supposing now that each of these circuit-breakers are fitted with instantaneous overload relays it is exceedingly probable that a heavy short-circuit occurring on the consumer's premises would open the whole series right up to and including the sending-end station breakers, thus involving a general stoppage of supply. This would also be the case if the relays were of the inverse-time-lag type since at heavy loads, as shown by the curves, their action is almost instantaneous. Both these relays are unsuitable for a transmission system as they have no discriminative properties. The object to be aimed at is to open the circuit-breaker nearest to the fault, and all the others should remain unaffected so as to limit the area of the disturbance as much as possible.

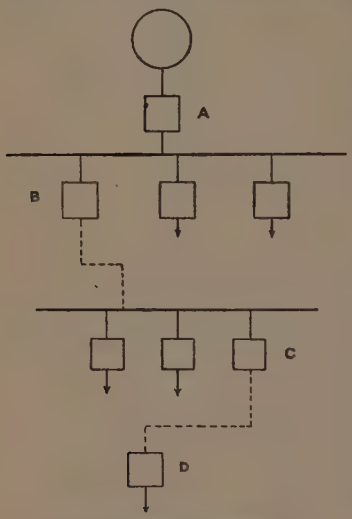

FIG. 88.—Radial system of transmission.

The first reliable method of ensuring the correct sequence of operation was to employ definite-time-lag relays. Thus *D* would be set to trip instantaneously, *C* would trip after 1 second, *B* after 2 seconds, and *A* after 3 seconds, no matter what the magnitude of the overload. This method has the undesirable feature that a 100 % overload is tripped as quickly as a short-circuit. Furthermore, on a large system having probably four or five subdivisions of the radial layout, the time delay close in to the generating station would become dangerously long as a short-circuit sustained for several seconds could do much damage.

To meet these difficulties the compensated overload relay has been developed and now enjoys a wide application. The relay is of the induction type but is supplied from a small transformer called a 'torque compensator' which is so designed that it becomes saturated with heavy currents and thus limits the disc speed to a predetermined maximum. By this means an inverse time lag can be obtained at moderate overloads, but with heavy overloads or

short-circuits there is a definite minimum time of operation, as shown by the curves of Fig. 89.

Modern circuit-breakers can be depended upon to open a circuit within about 0·25 seconds of the relay contacts closing, and allowing the same interval of time to compensate for any inaccuracies which may exist in individual breakers or relays, a reliable selective action can be obtained with differences of 0·5 seconds between consecutive relays. Generally speaking, it is not safe to sustain a short-circuit on the generators for longer than 2 seconds, and the last relay may be instantaneous. Hence, about five compensated overload relays may be used in series on a radial system,

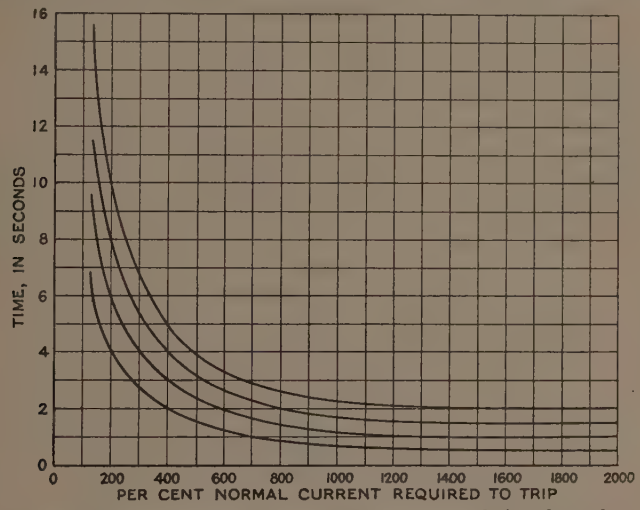

FIG. 89.—Typical time-load curves for compensated-time-lag relays.

and each division will not only be thoroughly protected, but will not cause interruption to any good feeder between the busbars supplying the defective feeder and the generators.

157. Protection of Parallel Lines by Overload and Reverse-power Relays.—In any important system of power transmission reliance is not placed on one single line or feeder. At least two lines are employed and are connected normally in parallel so as to share the load. These lines may or may not run on the same towers or the same right of way. Special methods of protecting such parallel lines are available which, in the event of a fault occurring, will select and isolate the defective line while the other instantly assumes the increased load.

The ordinary method of protection for parallel lines is to install time-lag overload relays at the sending end of the lines and instantaneous reverse-power relays at the receiving end, as shown in Fig. 90. If, now, a heavy fault or short-circuit occurs on one line at the point X, current will flow into it through B, and also through A, C, and D. The direction of power flow will be reversed through the relay on D, which will open. The excess current is then confined to B until its overload relay operates and trips the breaker, thus completely isolating the defective line.

This method of protection has the merit of simplicity, but until recently suffered from faults due to the employment of reverse-power relays. These relays have not always been of a

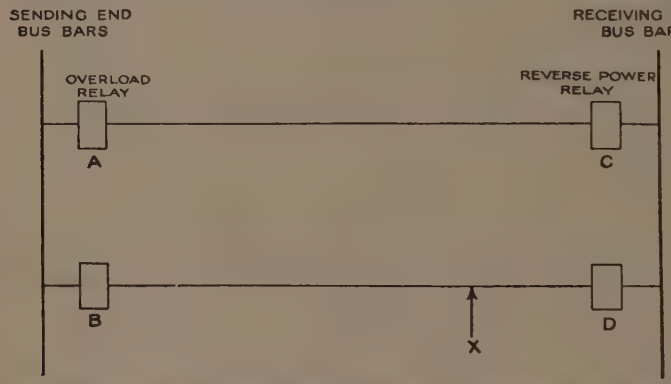

Fig. 90.—Protection of parallel lines by means of overload and reverse-power relays.

reliable character and the development of a satisfactory design has not been a simple proposition. It might appear at first sight an easy matter to place contacts on a wattmeter, the contacts being normally open, and closing on reversal of power. In the case of a short-circuit near the generating end of the line, however, the voltage will be very low even if the current is high. In fact, actual tests have shown that under such circumstances the voltage may drop down to 1 or 2 % of normal. Furthermore, in the case of three-phase systems, phase distortions are introduced on the occurrence of short-circuits on one phase only, or between one conductor and earth. These asymmetrical short-circuits may so distort the relation of current and voltage as to cause the angle between them to be almost 90 degrees. Conse-

quently the ordinary wattmeter movement has been found to be entirely inadequate, on account of there being insufficient torque to operate the mechanism when short-circuits have caused the voltage and power factor to drop to low values. Another disadvantage was that sudden momentary surges due to synchronising or other switching operations would trip out the breakers unnecessarily.

One of the latest and most reliable designs of reverse-power relay has been brought out by the Westinghouse Co., and is claimed to possess neither of the above disadvantages. Fig. 91 gives a diagram of the electrical connections. In the relay two

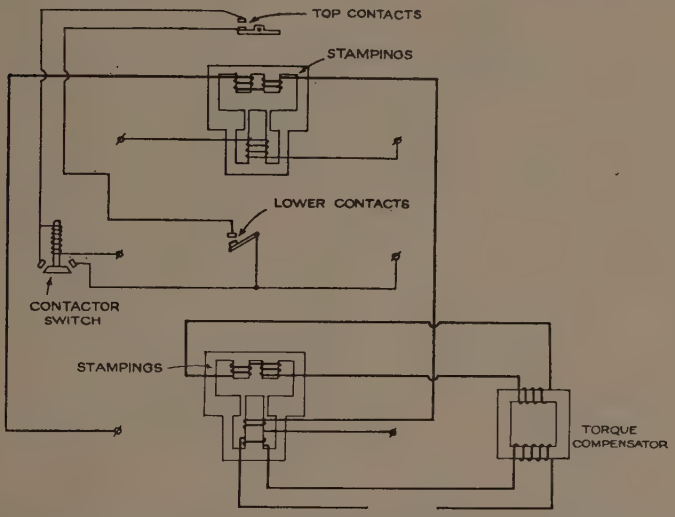

FIG. 91.—Magnetic and electric circuits of Westinghouse reverse-power relay.

entirely separate moving elements are employed, one being a simple overload element, and the other a wattmeter or directional element. The relay contacts of both these elements are in series. The directional element is made very quick-acting and sensitive, the least flow of power in the reverse direction being sufficient to close its contacts. The circuit-breaker, however, cannot be tripped unless the excess current is of such a magnitude as to close the overload-element contacts also. This prevents the breaker being tripped by switching surges or temporary reversals of energy due to hunting of substation machinery. On the other hand, with current in the normal direction, the directional-element

contacts are always open, so that if the overload-element contacts were closed by an excessive current the breaker would not be tripped. It is claimed that this type of relay selects between overload in the normal and reverse directions even though the pressure drops to 2 % of normal, and the power factor to 10 %. In connection with the diagram of Fig. 91 there should be noted the employment of the torque compensator previously mentioned in order to obtain a definite minimum time of operation for the overload element.[1]

158. Protection of Ring System.—When several substations are fed from the receiving-end station and their geographical

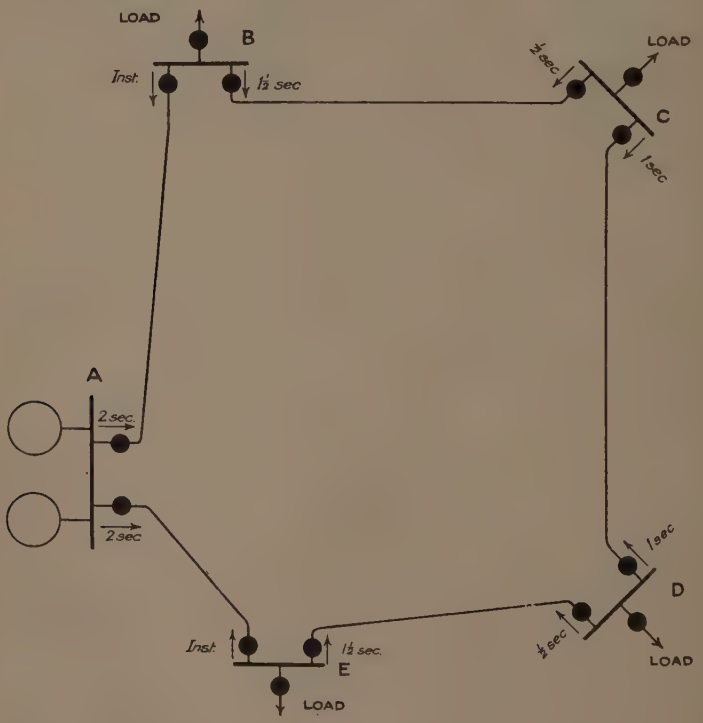

FIG. 92.—Protection of ring system.

situation is favourable, the ring system is one of the best means of securing uninterrupted supply with a minimum expenditure on transmission lines or feeders. An elementary diagram of such a system is illustrated in Fig. 92, where *A* is the receiving station, and *B*, *C*, *D*, and *E* are the substations. The lines running

between any two of the substations are known as interconnectors. In the lines AB and AE the flow of power must be always in the same direction, that is from the source outwards, but in the interconnectors BC, CD, and DE the flow may be in either direction varying with the load conditions. It will be readily seen that if a fault occurs on any of the lines, and the defective line can be promptly isolated, there will be no interruption of the supply to any of the substations.

The protective arrangements for such a system comprise time-lag overload relays at the main station, and time-lag reverse-power relays at both ends of the substations. The reverse-power relays are set so that they will only trip when an overload flows away from the substation which they protect. Going round the ring in the direction $ABCDE$, the relays on the further side of each station are set with decreasing time lags; for instance, $A = 2$ seconds, $B = 1{\cdot}5$ seconds, $C = 1$ second, $D = 0{\cdot}5$ second, and E is instantaneous. Going round the ring in the opposite direction, the relays on the outgoing sides would be set as follows: $A = 2$ seconds, $E = 1{\cdot}5$ seconds, $D = 1$ second, $C = 0{\cdot}5$ second, and B is instantaneous. Now remembering that the reverse-power relays are set so as to function only when power flows away from the substation at which they are installed, a study of the diagram will make it clear that a short-circuit occurring at any point on the system will cause to operate only the two circuit-breakers immediately adjacent.

When parallel lines are used between substations on a ring system the reverse-power relays will maintain the ring intact by isolating only the faulty line. In ordinary parallel line protection the reverse-power relays are set to act almost instantaneously in order to cut out the defective line as quickly as possible, but when applied to a ring system the relays must be set with a decreasing time lag as described.

159. Protection of Parallel Lines by Interlocked Relays.—The use of overload and reverse-power relays does not afford a complete solution of the problem of parallel-line protection owing to the fact that the reversal of current alone is not a criterion of failure of the line in which the reversal occurs. Such a reversal may be produced by surges of power fed back from rotary converting apparatus or other synchronous plant installed on the distribution system. In case of a fault occurring near the

generating station the operating voltage is suddenly reduced, and the synchronous machinery then feeds back energy into the fault, the current reversing in perfectly healthy lines. On the other hand, a high-resistance fault on a line would not cause any reversal of current, so that in this case the defective line would not be isolated.

An advance in the art of parallel-line protection takes advantage of the fact that under normal conditions lines of the same length and cross-section will take practically equal shares of the load current. A break-down on one line, however, will alter the relative current values, the faulty line carrying the heavier current. By interlocking the relays of the two lines either electrically, electromagnetically, or mechanically, the isolation of the faulty line can be secured.

Consider first of all the sending-end protection. To dis-

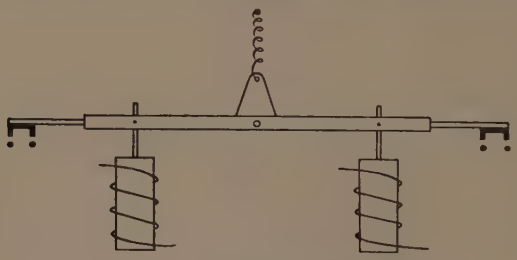

Fig. 93.—Balanced overload relay.

criminate between two parallel lines, interlocked overload relays of the type illustrated diagrammatically in Fig. 93 may be used. The two coils are operated respectively from series transformers on the two lines, and the balance arm of the relay is pivoted in its middle position. So long as the currents are equal the balance arm will be unaffected, but with an increase of current on one side the arm will be drawn down on that side against the action of the spring, and will close the corresponding pair of contacts to trip the circuit-breaker in the more heavily loaded line.

On the faulty line being disconnected the other takes the whole of the load, and some device is required to prevent its being tripped out. A satisfactory method of accomplishing this end is by means of small auxiliary switches on the circuit-breaker. This automatically converts the protection into time-lag overload protection on the line left in circuit.

In practice, a perfect balance of current between the lines is not obtainable, due not only to the line impedances varying, but also to dissimilarities in the characteristics of the current transformers. Hence, on a severe overload, there will be an appreciable excess current on one side or the other of the relay. For example, if the maximum short-circuit current or 'through' current was 50 times the normal-load current in each line, and the relay coils balance under normal load conditions within 2 %, then on dead short-circuit there will be a difference current equal to normal-load current, and the relays of course must not operate under these conditions. Hence, under these circumstances, a fault setting more than 100 % of the normal load of the line must be used.

This difficulty of balancing is completely eliminated by making use of the principle of biasing first suggested by Wedmore.[2] Each line is provided with a relay having two coils, one an

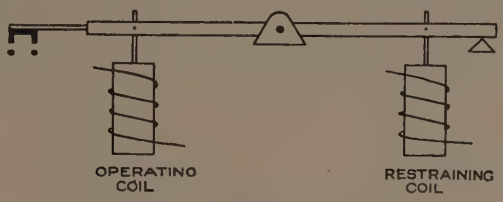

FIG. 94.—Biased overload relay.

operating coil, and the other a restraining coil, as shown in Fig. 94. The operating coil is excited from the line that is being protected, and the restraining coil from the line in parallel with it. The arrangement is biased in favour of the restraining coil to the extent of, say, 10 % by placing extra turns on the latter coil, or altering the position of the fulcrum of the balance arm, so as to make its effect more powerful than the operating coil by 10 %. The relay will then operate when the fault current amounts to 10 % of the current flowing in the main circuit whether this is normal load or overload. The devices may be set to operate instantaneously and a gradually developing fault removed while the leakage current is fairly small, but are uninfluenced by heavy 'through' currents due to a short-circuit on some more remote part of the system.

Fig. 95 shows the application of biased overload relays to the protection of two parallel lines. The current transformers are

connected for circulating current and so the points P and P' are normally at the same potential. Thus the transformer current normally circulates through the parallel path formed by the operating and restraining coils of the relays. If one line carries current in excess of that carried by the other, the major portion of this excess current flows through the local circuit formed by the operating coil of the relay protecting the faulty line and the balancing lead.

At the receiving end of parallel lines, protection can be secured by means of interlocked reverse-power relays. The application of the interlocking idea here renders the relays independent of

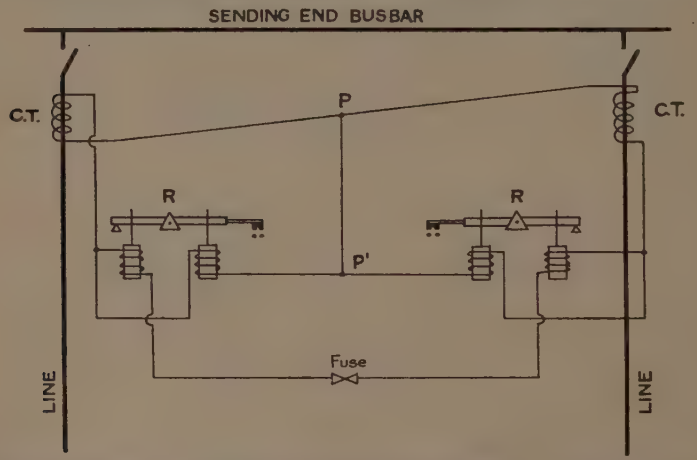

Fig. 95.—Protection of parallel lines at sending end by interlocked overload relays.

surges of power fed back from synchronous machinery on the system. Fig. 96 illustrates the employment of interlocked reverse-power relays arranged for series tripping. The current transformers in the two lines are so connected that current normally circulates between them and the relays, and trip coils are connected in shunt to them. No current, however, flows through the relays, which are connected in series between equipotential points. In case of unbalancing, the difference current flows through the relays in such a direction as to operate the relay on the faulty line and to restrain the other, so that the faulty line only is isolated.

The principle of interlocked overload and reverse-power relays

may be extended to several lines in parallel, and can also be applied direct to the various elements of a radial transmission system. Ring systems with parallel lines can be protected by installing interlocked overload relays at the main station and interlocked reverse-power relays at both sides of the substations.

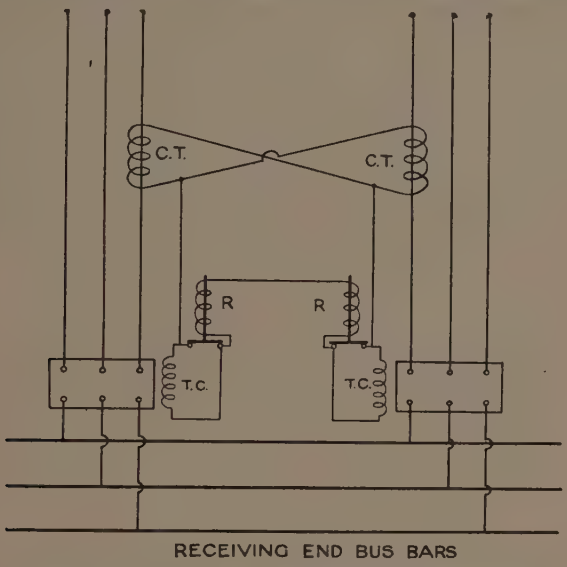

FIG. 96.—Protection of parallel lines at receiving end by interlocked reverse-power relays.

160. Core-balance Leakage Protection.—A number of methods are now in use whereby the leakage of current from an electric circuit is caused to automatically disconnect the faulty circuit from the system. Such leakage protective gear can only be used on earthed systems, as it is essential that any fault current from a conductor to earth should have an adequate return path to the system. On large systems, however, there may be no necessity to earth the neutral point direct, since the path provided by the capacitance of the conductors may have a sufficiently low impedance to ensure operation of the protective gear.

On a transmission line protected only by overload relays, a fault may not be of sufficient magnitude to operate the relays. Thus the leak may continue for a long time and do considerable damage before it ultimately develops into a short-circuit and is removed from the system. The use of leakage protective devices

under such circumstances ensures disconnection of the fault in its early stages.

Fig. 97 shows the use of a core-balance leakage relay to protect a line against faults to earth. The three conductors of the three-phase system are surrounded by the iron circuit of a current transformer. This has a single secondary winding which is connected to the relay. If there is no earth fault on the line the instantaneous sum of the three currents is always zero, and there is no resultant flux in the core of the current transformer no matter how much the load is out of balance. The sum of the three main currents, however, is no longer zero if an earth fault appears on the system, in which case a current is induced in the transformer secondary causing the relay to operate. By this

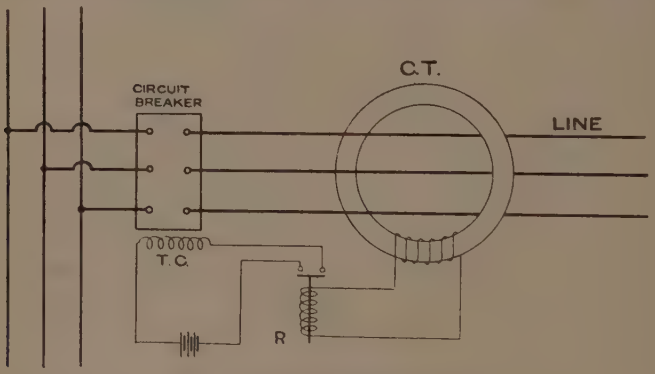

Fig. 97.—Core-balance leakage protection.

means it is possible to secure definite and instantaneous operation with a leakage current corresponding to 5 % of the full-load current in the circuit protected.

In its application to a multi-core cable a special form of core-balance transformer, the Ferranti-Field current transformer, can be used. Instead of having primary windings, the core of this transformer is in two halves and is clamped over the outside of the cable, first of all removing the armouring of the cable. The transformer is exactly the same in principle as illustrated in Fig. 97. It is remarkably easy to install and test, but cannot be made to respond to such small leakage currents as a transformer of the usual type having a wound primary.

Leakage protection is one of the simplest and probably the

cheapest form of protection against fault conditions, but cannot be regarded as adequate for a radial system of transmission requiring many relays in series owing to its lack of discrimination. It could, however, be made suitable for use on such a system if the neutral was earthed through a high resistance, so as to obtain smaller currents and powers on dead short-circuits to earth. Fault currents would then be definitely limited in value, and could be allowed to flow in the system for several seconds without much fear of damage. Under these circumstances definite time lags could be fitted to the leakage relays, and discrimination between relays in series with each other could be obtained.

161. Combined Core-balance and Overload Protection.—The core-balance protection outlined above suffers from the dis-

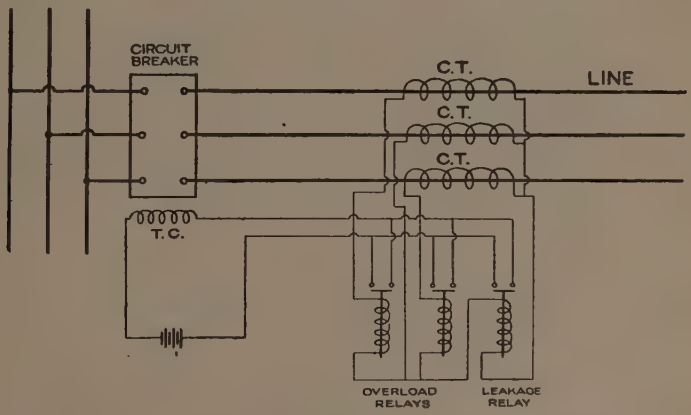

FIG. 98.—Combined core-balance and leakage protection.

advantage that if the fault or leakage occurs between phases the relay is inoperative. This disadvantage is overcome by the method of using three separate current transformers, due to Wedmore.[2] In this system of protection two overload relays and one leakage relay are connected as shown in Fig. 98. The two overload relays are sufficient to protect all three phases, while the leakage relay receives the resultant current from all three transformers, this being zero unless there is a leakage fault on the system. Hence protection is obtained against faults and short-circuits either to earth or between phases.

A simpler but less sensitive arrangement is to dispense with relays and employ three trip coils on the circuit-breaker, these

coils being connected in the same position as the three relay-operating coils in Fig. 98.

162. Core-balance Protection of Parallel Lines.—Simple leakage protective gear is unsuitable for use with parallel lines. This is seen from a consideration of Fig. 99. If an earth fault occurs on line *A* it is clear that the leakage current will flow not only along *A* but also in part along *B*. If both lines are equipped with leakage protective gear both lines will be disconnected—*i.e.* both the sound and the faulty one which is not desirable.

Possibly, as suggested by Garrard, this could be overcome by interlocking the leakage relays in a similar manner to that described in connection with overload relays, as the faulty line

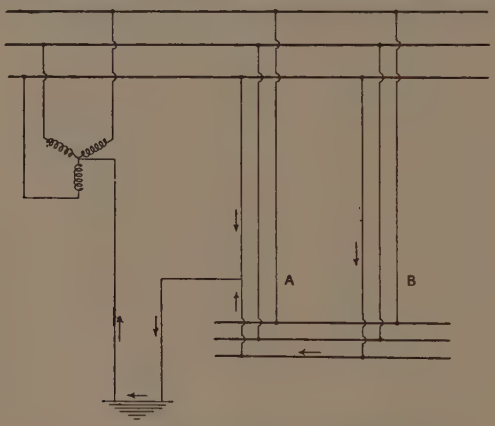

FIG. 99.—Effect of leakage on parallel lines.

would have the heavier leakage current. A biased relay could then be used if necessary, the leakage current through the protected line going through the operating coil, and that of the parallel line through the restraining coil.

163. Merz-Price Protection.—The fundamental principle on which this system of protection is based is that under healthy conditions the current entering one end of a line is equal to that leaving the other end. As soon as a fault occurs, however, this condition no longer holds, and the difference between the incoming and outgoing currents is arranged to operate a relay and isolate the defective line.

The application of this principle can be easily understood from the diagram given in Fig. 100, where the connections are shown for

the sake of simplicity as relating to a single conductor. Two similar current transformers are connected together by means of pilot wires, so that under healthy conditions the voltages of the two transformers are balanced against each other, with the result that no current flows through the relays connected in series with the secondaries of the transformers at each end. Should, however, the currents at either end become unequal due to a fault somewhere on the line, then the secondary voltages no longer balance and a current flows through the relays tripping the circuit-breakers at both ends of the line.

In actual practice, on a three-phase line each conductor has its own pair of current transformers and relays, these being connected by a three-wire pilot circuit. The other ends of the current transformers are connected in star, the neutral point being earthed at one end of the pilot circuit only, so as to prevent

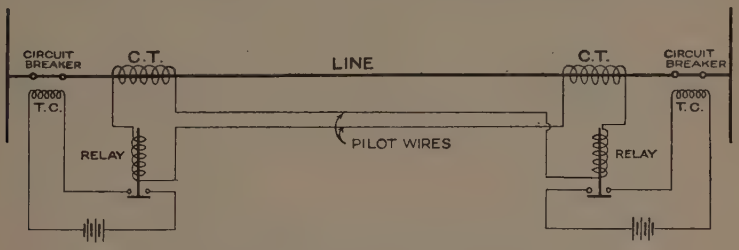

Fig. 100.—Principle of Merz-Price protection.

any current flowing between the two star points as this might operate the relays. This system has been widely employed for the protection of underground lines, and the pilot circuit then consists of a three-core, 7 / 0· 029, low-pressure cable, usually, but not necessarily, laid alongside the protected cable.

One of the disadvantages of the gear is that on large power systems a relatively high fault setting is necessary to ensure that it shall not operate under any circumstances unless a fault occurs on the line which it is protecting. One reason for this is the difficulty in obtaining current transformers to balance accurately when lines are carrying momentarily enormous currents due to a short-circuit on a more remote element of the transmission system. Furthermore, even if exact balance of the current transformers at each end is obtained, these heavy through currents may induce voltages of 1 000 volts or more in the current-transformer

secondaries, causing capacitance currents to flow in the pilot wires. The relays must therefore be set so that they will not operate with these capacitance currents, and the minimum fault setting that can be used in practice is of the order of 100 % normal load of the line.

This trouble caused by capacitance currents has been elimin-

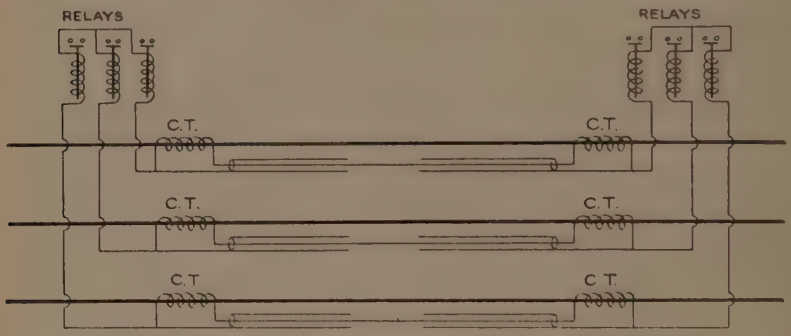

Fig. 101.—Merz-Price protection using compensated pilot cables.

ated by the introduction of the Beard-Hunter compensated pilot cable in which means are provided of shunting the capacitance currents from the relays. This action is obtained by surrounding each conductor of the pilot cable with a metallic screen or sheath which is divided at the centre of its length so as to form two conductors of equal length. When a heavy overload comes on, a high potential is induced in the transformer secondaries, but the resulting capacitance current, instead of flowing in the relays, flows in the local circuit formed by the sheath, transformer, and pilot wire. This circuit can easily be traced in Fig. 101 and a section of the shielded pilot cable is shown in Fig. 102. As a consequence, much more sensitive relay settings can be employed than with the ordinary pilot cable. Furthermore, owing to the screening effect of the metallic sheaths, it is possible to embody telephone wires in the pilot cable, thus enabling one large strong cable to be used instead of two relatively weak ones.

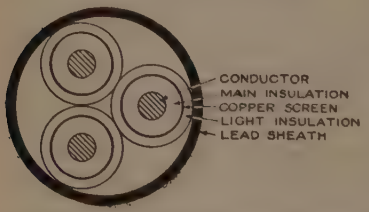

Fig. 102.—Section of Beard-Hunter compensated pilot cable.

164. Ferranti-Hawkins Protection.—This form of protection

is very similar to the Merz-Price system, the difference being that instead of three separate current transformers being employed at each end of the line one core-balance transformer is used to link with all three conductors, as illustrated in Fig. 97. The current transformers are of the Ferranti-Field type and are clamped over the three-core cable to be protected, one at each end. The secondary windings of the transformers are connected by means of a two-core pilot cable, and under normal conditions no potential difference exists between the various parts of the pilot-wire circuit. With a fault current to earth the vector sum of the currents in the main circuit at one end is no longer zero, and this causes a current to be induced in the secondary windings of the transformers thus operating the relays.

This gear is very simple and is also discriminative in action. It is, however, sensitive to earth faults only, and will not operate if faults occur between the cores of the cable. In practice these inter-core faults rapidly destroy the insulation to earth and as soon as this happens the gear will operate.

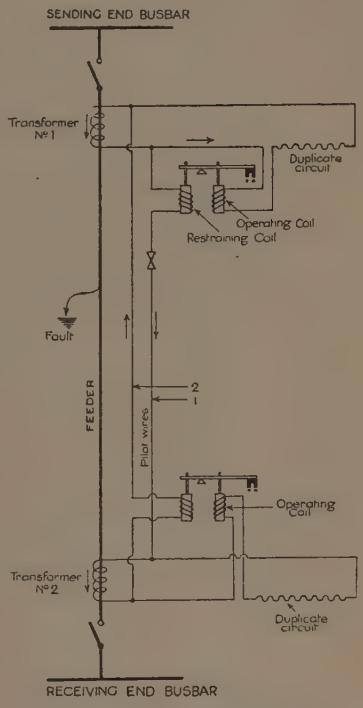

FIG. 103.—McColl biased line protection.

165. McColl Protection.—This system of protection in its application to one phase of a line is illustrated in Fig. 103. The protected conductor forms the primary of two current transformers, one at each end of the line, the secondaries being connected in series by means of two pilot wires. At each end of the line a beam relay is employed, the two restraining coils being connected in series with the transformer secondaries. Each secondary also feeds the operating coil of the beam relay through a duplicate circuit which is adjusted to have the same resistance as one of the pilot wires. The relay is

mechanically biased by having the fulcrum situated somewhat nearer the operating coil than the restraining coil, giving a greater leverage to the latter.

When the line is healthy the secondaries of the two transformers are circulating the same current, the two pilot wires and the two secondaries forming a closed circuit through which the current flows. Thus transformer No. 1 is responsible for the task of passing the current down one-half of the total system, that is, down one pilot wire. Now this transformer has also an alternative path for its current in the duplicate circuit connected to it which, as has been stated, is of the same resistance as the pilot wire. Hence equal currents flow round the two coils of each relay and the latter is quite stable.

If, however, a fault develops in the line, more current will flow through transformer No. 1 (this being the sending end of the line) than No. 2, and consequently its secondary will deliver a greater current than the other. There is thus an excess current that does not circulate through the distant transformer. There are two paths open to this additional current; one through its own duplicate circuit, and one through the duplicate circuit at the other end in series with the two lengths of pilot wire. It will be seen that the latter path has three times the resistance of the former, so that three-quarters of the extra current will pass through the operating coil of the nearer relay, and a quarter through the restraining coil of this relay and the two coils of the distant relay.

The result is that 75 % of the extra current affects the operating coil, and thus the relay tends to operate but is restrained by the bias of, say, 10 % which it has been given. If the total currents flowing through the two coils were calculated it would be seen that the effect of a 10 % leak is to increase by almost exactly 10 % the pull of the operating coil of the relay at the sending end of the line relatively to its restraining coil. The result of the 10 % leak is therefore that the relay just balances, and when this percentage increases, operates, completing the local circuit and tripping the circuit-breaker.

The above holds good whatever the load on the line, as the relay deals in percentages only. Hence a small fault setting down to 5 or 10 % of normal full load can be employed, while there is no risk of the relays tripping owing to heavy 'through' currents.

One advantage of this method of protection is that ordinary standard current transformers can be used. Also there is no trouble due to capacitance currents in the pilot wires as these currents flow through the restraining coils only, and actually produce a stabilising effect. Unlike the two pilot-wire systems described above, a current is constantly circulating through the pilot circuit under normal conditions, so that if by any mischance this should be open-circuited, both relays will at once operate. Frequent inspection and testing of the pilot circuit is therefore unnecessary.

Also, due to the fact that the secondary or pilot circuit carries a current which is a function of the load current in the line, the system can be made to give additional overload protection. This feature may be obtained by the simple expedient of inserting fuses in circuit with the pilot wires. In the event of a relay or circuit-breaker failing to operate after a short-circuit on some remote part of the system, the large sustained current flowing in the pilot wires will blow the fuses and divert all the current of each transformer through the operating coil of its relay, thus putting an end to the destructive effects.

166. Merz-Hunter Split-conductor Protection.—This system is a successful attempt to secure the benefits of a balanced method of protection without the necessity of employing pilot wires. The principle of operation depends on the fact that two conductors of equal length and impedance when connected in parallel will share the load equally, provided that the insulation of the system is sound. When a fault develops on one conductor it will carry more current than the other, and this inequality of currents is arranged to operate a relay and thus isolate the defective line.

In applying this principle each conductor of the line is divided into two sections or 'splits' which need be only lightly insulated from each other. For an overhead line each split may be carried by separate insulators, in which case the system really becomes a form of parallel-line protection. When applied to underground lines specially constructed cables are used, generally of one of the three types illustrated in Fig. 104. Great care must be taken to ensure equality of impedance between the splits, and if necessary this is obtained by arranging for transpositions at certain of the joints.

The application of the protective gear to a three-phase line is

illustrated in Fig. 105. At both ends of the line the two splits forming one phase are connected in series with a differentially-

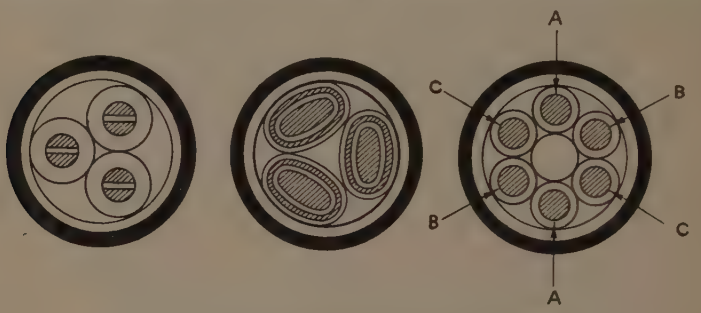

Fig. 104.—Types of three-core split-conductor cables.

wound current transformer. The single secondary winding of the transformer is connected to the relay-operating coil. It is readily seen that while the currents in each of the splits are equal there

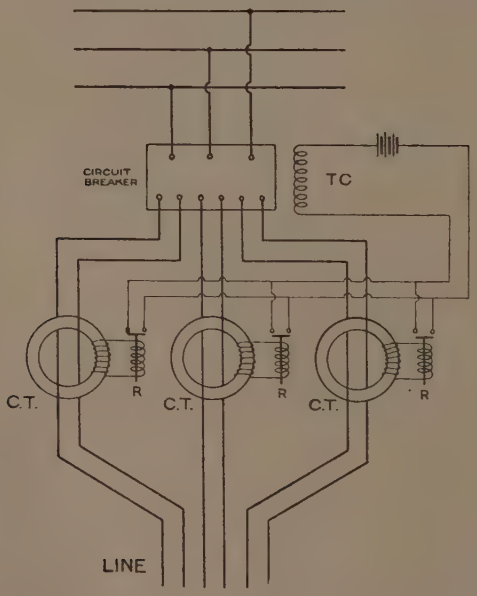

Fig. 105.—Merz-Hunter split-conductor protection.

is no resultant flux in the core of the transformer, but as soon as dissimilar currents flow (due to a fault developing), a secondary current flows through the relay, and if sufficiently great will trip

the circuit-breaker. In the best arrangement of the system special circuit-breakers are installed at the line termini so that the splits are not connected until they reach the busbar side of the breaker. The reason for thus carrying the splits right through the circuit-breaker will be clear from the diagram of Fig. 106 where one phase of the line is shown as having a fault very near the receiving end. It will be seen that the current in the one half winding of the receiving-end current transformer has reversed, so that its core will become magnetised and the receiving-end circuit-breaker will trip. The currents in neither of the windings of the sending-end transformer will have reversed, and supposing that the splits have not been taken right through the receiving-end circuit-breaker, will continue to flow through both splits in parallel into the fault. Now on a long line the impedances of the two splits up to the fault would be practically equal, and the inequality of current at the sending end would not be

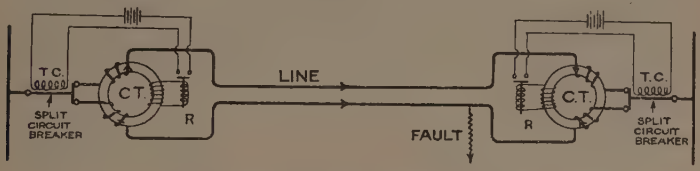

Fig. 106.—Use of split-contact circuit-breaker with Merz-Hunter system of protection.

sufficient to trip the circuit-breaker at this end. If, however, the splits are carried right through the circuit-breaker this condition cannot arise, for as soon as the receiving-end circuit-breaker is tripped the two splits are isolated from one another at this end, and then the current flowing in the one split to feed the fault will operate the relay and trip the sending-end breaker, thus completely isolating the defective line.

An alternative arrangement of the system, in cases where it is desired to use the ordinary type of circuit-breaker, is to install special high-reactance current transformers which accentuate the inequality of currents in the sound and faulty splits. It is found, however, that better results are obtained by using the split-contact circuit-breaker, and this is now to be considered the standard practice.

167. Callender-Hunter Four-conductor Protection.—This system is a combination of the Merz-Price and Split-conductor

systems. One phase (B in Fig. 107) is protected on the split-conductor system, whilst the other phases (A and C) are protected on the Merz-Price system, using the splits of phase B instead of pilot wires. In the diagram, D and E are the split-conductor transformers for phase B, and F and G are balancing transformers with which all three phases are interlinked. The relays are actuated by secondary windings on D and E.

The action of the arrangement for various kinds of faults may be explained as follows :—

Faults from Unsplit Phases to Earth or to Each Other.—Take the case of phase A—while the same current leaves A at one end as enters it at the other equal fluxes are produced by this current in the cores of B and B'. These fluxes produce e.m.fs. at each end of B and B' but the connections are such that these

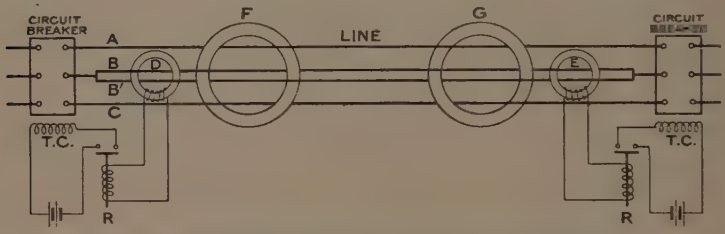

Fig. 107.—Callender-Hunter four-conductor protection.

e.m.fs. are in opposition. As a consequence the normal load currents in B and B' are undisturbed.

In the event of a leakage from A, either F or G will produce the greater e.m.f. and tend to circulate a current in the closed circuit formed by B and B', with the result that the equality of currents in the primaries of transformers D and E will be upset and the relays R operated. A similar sequence of operations would occur in the event of a leakage from C, and also in the event of a leakage from A to C.

Faults from Unsplit Phases to Split Phase.—In this case the fault current produces a direct unbalancing of current in B and B', and although the transformers F and G may accentuate the inequality of currents in B and B' this action is not necessary to ensure operation of the relays on the occurrence of such a fault.

Faults from Splits to Earth.—In the event of leakage from

either B or B' to earth a direct unbalancing of the currents is produced and the relays are operated.

Faults between Splits.—The balancing transformers F and G have the effect of tripping the relays in the event of a fault between B and B'. When A or C or both of them are carrying load a flux exists in the cores of F and G. These fluxes produce e.m.fs. in B and B' which are normally opposed and equal, and therefore do not disturb the currents carried by B and B'. In other words, the effect of balancing transformer F is to induce an e.m.f. in B and B' at one end, and the effect of balancing transformer G is to induce a precisely similar but opposite e.m.f. at the other end.

If now a fault occurs between B and B' at any point intermediate of the two ends the two e.m.fs. are no longer opposed but can circulate current each in its own local circuit, with the result that the relays are caused to operate.

It is clear from the above that besides being discriminative the protective arrangement operates for any form of leakage which may occur within the line, whether it be from phase to earth, phase to phase, or between splits.

168. Ideal Characteristics of Protective Gear.—The requirements of an ideal system of line protection can be summarised as follows :—

1. The protection should discriminate with absolute certainty, *i.e.* it should be certain of action on faulty cables and overhead lines, and inoperative with equal certainty on healthy sections.

2. The protection should be universally applicable, *i.e.* it should be suitable for use on all possible arrangements of lines whether these comprise separate lines, or form interconnectors on complicated networks, or consist of branched lines without switchgear at the points of junction.

3. The protection should be as instantaneous in isolating a faulty line as it is physically possible to be.

4. The protection should not interfere with complete freedom in laying-out, extending and operating the transmission and distribution system.

The only forms of protective gear which have shown in practice that they can make any claim to fulfil these requirements are those based on the balanced principle of protection, and exemplified in the systems described in Arts. 163 to 167.

169. Beaver Test-sheath Protection.—If a cable is furnished with an auxiliary sheath, lightly insulated from the outer lead sheath, it is clear that any fault penetrating the cable from without will reach the auxiliary sheath before it attacks the main insulation. This feature is utilised in the protective system described below, due to Beaver.

The system makes use of a special three-core cable wherein a test sheath is provided, surrounding the cores but insulated therefrom, and also lightly insulated by a hygroscopic material from the lead sheath which in turn surrounds it (*see* Fig. 108). A battery is connected between the two sheaths, and an incipient

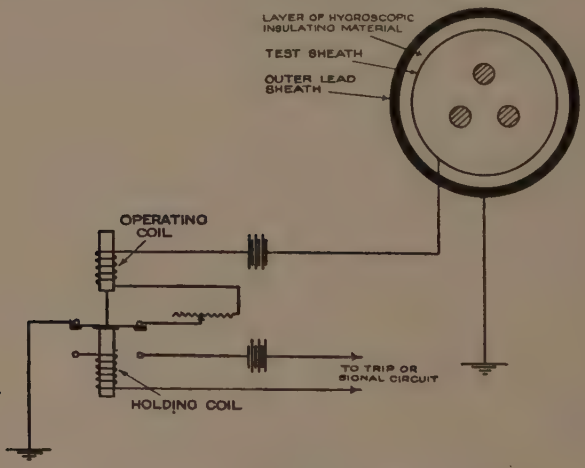

FIG. 108.—Beaver's cable protective system.

break-down of the insulation can be detected by an alarm device, or may cause a tripping relay to operate and disconnect the cable. There is thus a possibility of locating a fault in its early stages, the cable being then removed from service and repaired before complete break-down occurs.

Many other systems have also been developed in which auxiliary conductors in the form of sheaths are inserted in the cables.[3]

170. Short-circuit of Alternators.—In applying any protective scheme the calculation of short-circuit current values is necessary, as a basis of relay settings. In this connection it is unfortunate that the term 'overload' has come into use in refer-

ence to sectionalising transmission systems, since it implies that the relays should be set to operate at a value determined by the normal load on the line. A short-circuit on a system, however, produces an abnormal condition which has no relation to normal loads and overloads, and in order to obtain selective action from overload and reverse-power relays it is necessary to set the form the currents flowing under this condition.

Before describing methods of calculating these abnormal currents it appears desirable to discuss briefly the phenomena occurring when an alternator is short-circuited.

If the impedance of the load circuit of an alternator, with normal excitation, is gradually reduced to zero, the short-circuit current is limited only by the synchronous impedance of the armature. Neglecting the armature resistance, which is very small in comparison with the synchronous reactance, the short-circuit current is

$$I_{sc} = \frac{E}{x_s}. \quad . \quad . \quad . \quad . \quad (186)$$

x_s is made up of two components: x_l which represents the effect of the armature leakage flux, and x_r which represents the effect of armature reaction.

The value of I_{sc} is generally about two or three times full-load current, although turbo-alternators are now being built which have a permanent short-circuit current of only one or one and one-half times full-load current.

In the case of a sudden short-circuit the initial current will be much larger than the permanent value I_{sc}, because the component x_r of the synchronous reactance, which represents the effect of armature reaction, does not act instantaneously to limit the current. It represents a change in the flux which interlinks with the field circuit of the machine, and on account of the inductance of the field winding with its large number of turns, this change of flux cannot take place instantaneously but may take several seconds to complete. Again neglecting the armature resistance, the initial short-circuit current is limited only by the inherent or leakage reactance x_l and is

$$I_{sc} = \frac{E}{x_l}. \quad . \quad . \quad . \quad . \quad (187)$$

The amplitude of the current gradually decreases as the armature reaction reduces the flux in the field and the generated

e.m.f. E, until the permanent value I_{sc} is reached. Fig. 109 is a typical curve for an alternator having a leakage inductance of 10 %.

It will be seen that the initial short-circuit current bears to the permanent short-circuit current the ratio

$$\frac{I'_{sc}}{I_{sc}} = \frac{x_s}{x_l} \quad . \quad . \quad . \quad . \quad . \quad (188)$$

In machines of high self-inductance and low armature reaction the ratio will have a moderate value, but in large low-frequency turbo-alternators with low self-inductance and high armature

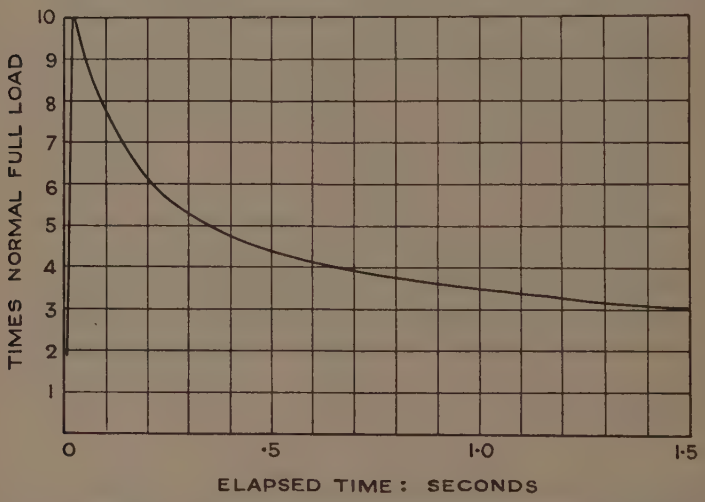

Fig. 109.—Typical curve showing R.M.S. short-circuit current of alternator in terms of full-load current.

reaction the ratio may be very large. In extreme cases the initial short-circuit current may reach twenty or thirty times full-load current.

It should be noted that I'_{sc} will represent the initial short-circuit current only when the short-circuit occurs at the maximum point of the generated e.m.f. wave as illustrated in Fig. 110 (a). Neglecting the armature resistance, the generated e.m.f. is consumed by the back e.m.f. of armature inductance $= -L\frac{di}{dt}$. The cross hatched area under the e.m.f. wave is

$$\int_0^{\sqrt{2}E} e\,dt = \int_{\sqrt{2}I'_{sc}}^{0} -L\frac{di}{dt}\,dt$$
$$= \sqrt{2}\,I_{sc}L,$$

and it causes the current to increase from zero to $\sqrt{2}I_{sc}$. The succeeding quarter-wave of e.m.f. is negative and reduces the current to zero again ; it then reverses and grows in the opposite direction to a value slightly less than before, since the armature reaction will have caused a slight reduction in the flux and the generated e.m.f. The current is symmetrical about the zero line, and its amplitude decreases until the reduction of flux by armature reaction is complete, after which it remains constant. The

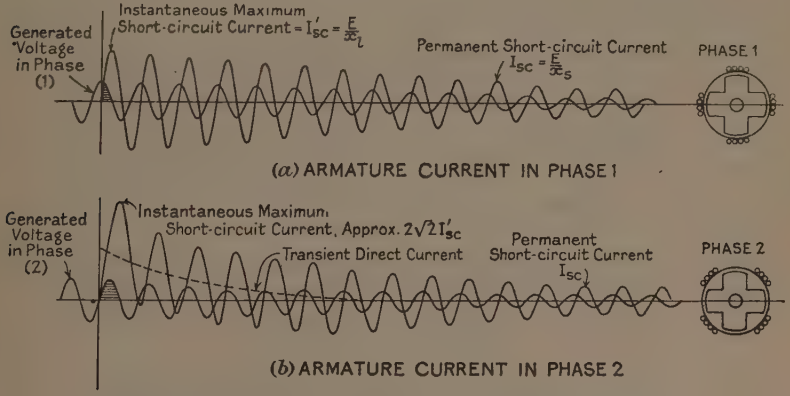
Generated Voltage in Phase (1)
Instantaneous Maximum Short-circuit Current = $I'_{sc} = \frac{E}{x'_l}$
Permanent Short-circuit Current $I_{sc} = \frac{E}{x_s}$
PHASE 1
(a) ARMATURE CURRENT IN PHASE 1
Generated Voltage in Phase (2)
Instantaneous Maximum Short-circuit Current, Approx. $2\sqrt{2}I'_{sc}$
Transient Direct Current
Permanent Short-circuit Current I_{sc}
PHASE 2
(b) ARMATURE CURRENT IN PHASE 2

FIG. 110.—Short-circuit currents in a two-phase alternator.

transition from the initial to the permanent value will take from a few hundred to thousands of cycles, depending on the inductance of the field winding. In Fig. 110 (*a*) it is represented as covering only a few cycles.

Fig. 110 (*b*) shows the short-circuit current in the second phase of the machine. The short-circuit occurs at the zero point of the generated e.m.f. wave, and the positive area under the wave is twice as great as before and builds up the current to double the value in (*a*). The current then decreases, and it alternates, not about the line of zero current as before, but about a line starting above the zero line by an amount $\sqrt{2}I'_{sc}$ on the current scale and sloping down to coincide with the zero line after a few cycles.

The armature current may be considered as being composed

of a transient direct current, shown as a broken line, and superimposed on it the alternating current I'_{sc} with its decreasing amplitude. The R.M.S. value of the current wave at any instant is the square root of the sum of the squares of the value of the direct component and the effective value of the alternating component at that instant.

171. Calculation of Short-circuit Currents.—In determining the current produced by a short-circuit on a transmission system it is customary to neglect the resistances of lines and apparatus and make all calculations with reactances alone. This approximation is justifiable as the reactances involved are usually large compared to the respective resistances, so that the impedance is only slightly greater than the reactance numerically. Furthermore, this method gives values of short-circuit current rather larger than actually obtain, and consequently the results are on the safe side. In the case of underground lines, however, the resistance of the cable is often larger than the reactance and should be taken into account.

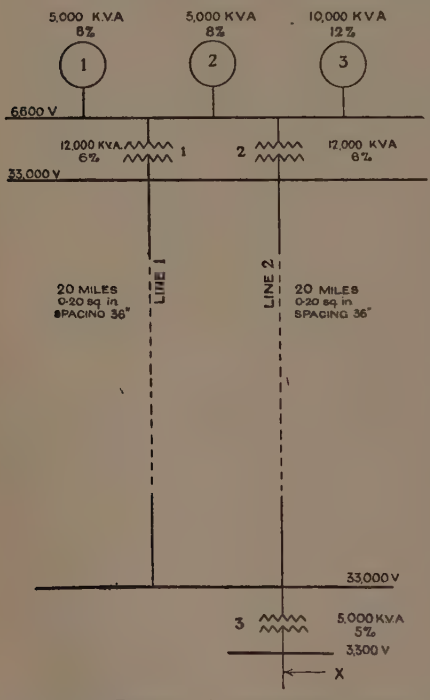

FIG. 111.—Layout of transmission system to illustrate example on calculation of short-circuit currents.

The ordinary method of calculation would be to express the reactance of all generators, transformers, and lines in ohms to neutral, and by adding up the reactance from the point of short-circuit to and including the generators, and dividing the sum into the voltage to neutral, the initial value of the short-circuit current would be found. In practice, however, the reactance of most apparatus is expressed on a percentage basis, and furthermore, several different voltages are usually found on the same

power system, so that it is found more convenient to use the percentage method of calculation described by Wilson.[4]

In this method all reactance of generators, transformers, lines, or other apparatus is expressed as per cent. reactance at a common kVA. base arbitrarily selected. The various reactances are converted up or down as the case may be to this base. The combined reactance from the generator neutrals to the point of short-circuit is determined, and it is assumed that the entire voltage of the generators is used between these two points. The initial short-circuit kVA. is then

$$\frac{\text{kVA. base}}{\text{percentage reactance to point of fault}}. \qquad (189)$$

To illustrate the application of this method the layout given in Fig. 111 will be assumed and the short-circuit current at the point X calculated.

Generator No. 1 rated at 5 000 kVA. inherent reactance 8 %.
Generator No. 2 rated at 5 000 kVA. inherent reactance 8 %.
Generator No. 3 rated at 10 000 kVA. inherent reactance 12 %.

Taking as base 10 000 kVA.

$$\text{Reactance of No. 1} = \frac{10\,000}{5\,000} \times 8\,\% = 16\,\%,$$

$$\text{Reactance of No. 2} = \frac{10\,000}{5\,000} \times 8\,\% = 16\,\%,$$

$$\text{Reactance of No. 3} = \frac{10\,000}{10\,000} \times 12\,\% = 12\,\%,$$

and the combined reactance of the three generators

$$= \frac{1}{\frac{1}{16} + \frac{1}{16} + \frac{1}{12}}\,\%$$
$$= 4{\cdot}8\,\%.$$

Transformer No. 1 rated at 12 000 kVA. reactance 6 %.
Transformer No. 2 rated at 12 000 kVA. reactance 6 %.

At a base of 10 000 kVA.

$$\text{Reactance of No. 1 or No. 2} = \frac{10\,000}{12\,000} \times 6\,\% = 5\,\%,$$

and the combined reactance of the two transformers

$$= \frac{1}{\frac{1}{5} + \frac{1}{5}} \%$$
$$= 2{\cdot}5\ \%.$$

Line No. 1, 20 miles long, 0·20 sq. in. stranded copper, diameter 0·579 inches, spacing 36-inches delta.

Reactance of conductor

$$= 1{\cdot}654 \times 2\pi \times 50 \times 20.$$
$$= 10{\cdot}39 \quad \text{ohms.}$$

At 10 000 kVA. base (3 333 kVA. per phase) the current is

$$\frac{3\,333 \times 1\,000}{\frac{33\,000}{\sqrt{3}}} \text{ amperes,}$$

the reactance pressure drop is

$$\frac{3\,333 \times 1\,000 \times 10{\cdot}39}{\frac{33\,000}{\sqrt{3}}} \text{ volts,}$$

and the percentage reactance is

$$\frac{3\,333 \times 1\,000 \times 10{\cdot}39 \times 100}{\frac{33\,000}{\sqrt{3}} \times \frac{33\,000}{\sqrt{3}}}$$
$$= 9{\cdot}54\ \%.$$

Line No. 2 has a similar percentage reactance and the combined reactance of the two lines is

$$\frac{1}{\frac{1}{9{\cdot}54} + \frac{1}{9{\cdot}54}} \%$$
$$= 4{\cdot}8\ \%.$$

Transformer No. 3 rated at 5 000 kVA. reactance 5 %. On a base of 10 000 kVA.,

$$\text{Reactance} = \frac{10\,000}{5\,000} \times 5\ \% = 10\ \%.$$

The total reactance up to point of short-circuit

$$= 4{\cdot}8 + 2{\cdot}5 + 4{\cdot}8 + 10{\cdot}0$$
$$= 22{\cdot}1\ \%.$$

The kVA. at the point of short-circuit

$$= \frac{10\,000 \times 100}{22{\cdot}1}$$

$$= 45\,200 \quad \text{kVA.}$$

This is the initial value of the short-circuit and the current corresponding thereto is 2 380 amperes. The current to be ruptured by the circuit-breaker will be lower than this, being intermediate in value between the initial and permanent short-circuit currents. Its actual value will depend on the time-lag setting of the protective relay, and the time lag introduced by the operation of the circuit-breaker, *i.e.* between the energising of the trip coil and the separation of the breaker contacts.

In order to estimate under these conditions the actual short-circuit current at rupture, the figures given in Table 25 may be employed. These figures represent average values taken from a large number of cases and are very useful for quickly ascertaining the short-circuit current when machines of varying reactance are employed, or when there is considerable impedance external to the generators. The table was prepared after a conference between the leading switchgear manufacturers in America, and was given in a paper by Hewlett, Mahoney, and Burnham.[5]

The numbers given are factors by which the normal full-load current must be multiplied to get the short-circuit current at any given instant. The percentage reactance figures represent the total reactance in the circuit, including that of the generator, up to the point at which the short-circuit takes place. At the shorter time intervals allowance has been made for possible doubling-effect.

In calculating the possible short-circuit current all synchronous machinery connected to the system must be taken into account, and not merely the total capacity of the generating plant. A rotary convertor or synchronous phase modifier possesses sufficient flywheel-effect to enable it to supply current as a generator during such times as an oil circuit-breaker requires to operate.

Table 25 is based on the assumption that at the time of short-circuit the generators are carrying full-load current at 80 % power factor, and that no automatic voltage regulators are installed. Furthermore, all values of reactance up to 20 % are considered to be inherent to the generators. No serious error

TABLE 25.—*Short-circuit Current Factors.*

Reactance.	8 %.	10 %.	12 %.	15 %.	20 %.	30 %.	40 %.	50 %.	60 %.	75 %.	100 %.	125 %.	150 %.
Elapsed Time in Seconds from Start of Short-circuit.	Current Factors expressed as number of times Full-load Current.												
0·05	13·91	11·16	9·59	7·68	6·04	4·03	3·01	2·40	2·00	1·58	1·17	0·92	0·77
0·08	11·78	9·54	8·25	6·66	5·27	3·59	2·74	2·21	1·86	1·50	1·13	0·90	0·76
0·10	10·94	8·89	7·68	6·23	4·97	3·41	2·63	2·13	1·81	1·46	1·11	0·89	0·76
0·15	9·16	7·54	6·57	5·40	4·38	3·08	2·42	2·00	1·71	1·41	1·09	0·89	0·76
0·20	8·24	6·80	5·97	4·95	4·06	2·92	2·30	1·92	1·66	1·38	1·08	0·88	0·76
0·25	7·55	6·28	5·54	4·63	3·82	2·79	2·23	1·87	1·63	1·36	1·07	0·88	0·76
0·30	7·03	5·88	5·19	4·39	3·67	2·70	2·18	1·84	1·60	1·34	1·06	0·88	0·76
0·40	6·27	5·30	4·74	4·03	3·40	2·57	2·10	1·79	1·57	1·32	1·06	0·87	0·76
0·50	5·74	4·91	4·40	3·80	3·23	2·48	2·04	1·75	1·54	1·31	1·05	0·87	0·76
0·70	4·99	4·34	3·93	3·45	2·98	2·34	1·96	1·70	1·51	1·29	1·04	0·87	0·76
1·00	4·25	3·77	3·47	3·11	2·73	2·21	1·88	1·65	1·48	1·27	1·04	0·87	0·76
1·50	3·63	3·31	3·08	2·82	2·53	2·10	1·81	1·61	1·45	1·25	1·03	0·87	0·76
2·00	3·20	2·98	2·82	2·63	2·39	2·03	1·77	1·58	1·43	1·24	1·02	0·87	0·76

will be introduced, however, if the figures are applied to cases where only part of this reactance is inherent, and part external.

When automatic voltage regulators are used they will increase the excitation on the occurrence of a short-circuit in the endeavour to keep up the voltage. The effect of this increased excitation is not noticeable during the first 0·5 second but thereafter the short-circuit current will be higher, and the permanent short-circuit current after, say, 2 seconds will be about 50 % higher than the values given by Table 25. This condition only holds, however, when the fault is close to the generator terminals. If the short-circuit occurs at a remote point in the system, so that the normal voltage can be maintained at the busbars, the value of short-circuit current is not increased since the limiting factor is the reactance of the line and the connected apparatus.

As an illustration of the use of Table 25 for the determination of relay settings, suppose that the low-tension circuit-breaker of Transformer No. 3 (Fig. 111) is furnished with a relay intended to operate under short-circuit conditions after an interval of 1 second. The relay must then be set for the current value at the end of this interval. First of all it is necessary to find the reactance at the point of short-circuit, based on the total generator capacity. Converting the initial short-circuit kVA. to a percentage reactance based on the total generator capacity of the system (20 000 kVA.) gives a percentage reactance of

$$\frac{20\,000}{45\,200} \times 100 = 44{\cdot}2\ \%.$$

Interpolating the values given in Table 25 for 44·2 % reactance and 1 second gives the number of times full-load current as 1·78. Therefore the short-circuit current after a time lag of 1 second is

$$\frac{1{\cdot}78 \times \text{gen. kVA. per phase} \times 1\,000}{\text{voltage to neutral}},$$

$$= \frac{1{\cdot}78 \times 6\,667 \times 1\,000}{\frac{11\,000}{\sqrt{3}}}$$

$$= 1\,870 \quad \text{amperes.}$$

172. Effects of Short-circuit Currents.—With the great increase in size of generating stations, the power liberated on the

occurrence of a short-circuit is enormous, and it is increasingly difficult and expensive to provide circuit-breakers capable of rupturing the current. Thus, besides the installation of protective relays for tripping the breakers on the occurrence of a fault, it has become necessary to provide means for positively limiting the magnitude of the current flowing.

For this purpose current-limiting reactors are often employed. These reactors consist of large reactance or choking coils connected permanently in circuit, and under normal conditions only entail a small loss of power. When a short-circuit occurs, however, they limit the current to a safe value, and tend to localise the effect of the fault by restricting the amount of power that can flow into the defective section from other parts of the system. They also protect the generating, transforming, and transmitting apparatus from damage due to the thermal and electro-mechanical effects of very heavy currents.

When a short-circuit takes place on a transmission line heat is generated in the conductors very rapidly, and there is little time for radiation, convection, or conduction.

Hence if I = R.M.S. current flowing in amperes,
r = Resistance per centimetre of conductor in ohms,
G = Weight per centimetre of conductor in grammes,
S = Specific heat of conductor material = 0·095 in the case of Copper,
and g = Specific gravity of conductor material = 8·9 in the case of Copper,
the temperature rise per second in degrees C. is

$$\frac{0{\cdot}24I^2r}{GS},$$

or as

$$G = gA \times (2{\cdot}54)^2,$$

and

$$r = \frac{\rho}{A \times (2{\cdot}54)^2},$$

where ρ = resistivity of the conductor material,
and A = cross-sectional area in square inches,
this may be written as

$$\frac{0{\cdot}24I^2\rho}{(2{\cdot}54)^2 \times (2{\cdot}54)^2 \times A^2Sg}. \quad . \quad . \quad (190)$$

In the case of Copper the temperature rise is

$$\frac{0{\cdot}012I^2}{A^2 \times 10^6} \text{ deg. C. per second.} \quad . \quad . \quad (191)$$

Besides this heating effect there are mechanical forces set up between the conductors which have been short-circuited. These forces assume special prominence in the case of underground lines owing to the close interaxial spacing of the conductors, and cables have been known to burst their paper and lead sheathing on the occurrence of a short-circuit.

Consider first the case of two parallel conductors carrying a current I amperes in opposite directions, the interaxial spacing being x centimetres. The repulsive force between the conductors is

$$F = \frac{0{\cdot}2I^2}{x} \text{ dynes per centimetre length,}$$

or changing into more practical units

$$F = \frac{5{\cdot}4I^2}{d \times 10^7} \text{ lbs. per foot,}$$

where d is the distance between centres of conductors in inches.

If the current instead of being continuous is alternating, the R.M.S. value being I, the mechanical stress exerted between the two conductors pulsates between zero and the maximum value—

$$F = \frac{1{\cdot}08I^2}{d \times 10^6} \text{ lbs. per foot.} \quad . \quad . \quad (192)$$

In a three-phase circuit the actual currents that will develop at a short-circuit, and the consequent mechanical stresses between conductors, depend upon the nature of the fault. Thus the short-circuit might occur between one conductor and earth, between two adjacent conductors, or across all three phases simultaneously. The forces developed also depend upon the method of arrangement of the conductors. Exact formulæ for various cases will be found in a paper by Robinson.[6]

173. Current-limiting Reactors.—The reactance of these pieces of apparatus, like that of generators and transformers, is usually expressed as a percentage. Thus a 5 % reactor is one in which the voltage drop when carrying full rated current is 5 % of the voltage to neutral of the circuit in which it is placed. In case of short-circuit the current flowing through a reactor of this value

will be high, but (neglecting any initial doubling effect) will always be limited to a magnitude not exceeding 20 times full-load current.

Supposing that an iron core was used in the construction of a reactor the normal magnetic flux density would have to be kept very low so as to avoid saturation with a m.m.f. of many times normal. Otherwise the protective effect of the reactor would compare badly with the unavoidable pressure drop at normal load. This low flux density makes the use of iron uneconomical in reactors, and even if its use were electrically permissible an iron magnetic circuit would be much larger than an air circuit.

There is considerable variation in details of construction of these reactors. One of the most successful types utilises bare conductors embedded in concrete supports which are cast round the coil, and ensure a very rigid construction. This is necessary on account of the large forces which are suddenly called into play when the first rush of current occurs.

Another novel design of reactor consists of a coil of paper-insulated, lead-sheathed, three-core cable cast in concrete. The conductors are connected in series forming one complete coil, and the lead is broken at several places to prevent eddy-current losses in the sheath. There are no exposed conductors and therefore there is no danger from shocks under working conditions; also no special chambers have to be provided to house the reactor. Where space is a consideration the reactor may be buried in the ground.

The losses in reactors are due to the I^2R and eddy-current losses in the conductors and to reduce the latter losses the conductors are stranded. The total loss usually varies from 2 to 5 % of the kVA. rating of the reactor.

As previously pointed out, an ordinary iron-core reactor cannot be economically employed owing to the iron becoming saturated at increasing values of load and thus having a much lower reactance at short-circuit than at normal load. A special type of reactor recently introduced known as the saturated-core reactor has precisely opposite characteristics.[7] In this apparatus the iron core is excited by an auxiliary D.C. winding, and by suitable design the ratio of the short-circuit reactance to the normal-load reactance may be made as high as three to one. In other words, if the reactance under normal conditions was

10 %, the reactance under short-circuit would be about 30 %. This highly desirable rising-reactance characteristic of the saturated-core reactor is obtained at the expense of higher losses and costs, and so this type of reactor will only be used in special cases.

174. Location of Reactors in System.—Reactors may be used in series with generators or transmission lines, or inserted between various busbar sections.

If reactors are put in series with each generator, as in Fig. 112 (*a*), the current is limited on short-circuit no matter whether

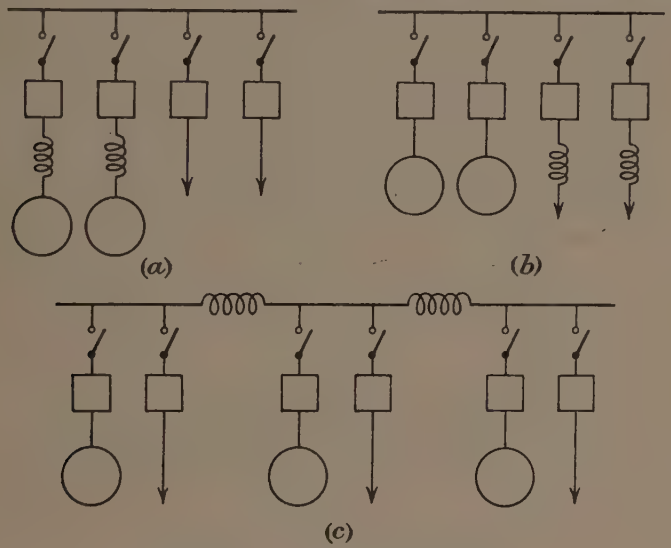

FIG. 112.—Location of Reactors.

the fault occurs on the busbars or on the line. The protection afforded, however, in case of a line short-circuit is less than with a separate line reactor, as the whole system is affected by any drop in voltage. When new machines are being built the usual practice is to incorporate the necessary reactance in the design of the machine itself, so that it can stand repeated short-circuiting without damage. As a rule, generator reactance is only used with high-voltage machines, or in series with old machines with the object of protecting the windings from excessive mechanical shock. In such cases the reactor may have a value of 5 to 10 %.

Since the vast majority of faults occur on the transmission

system, reactors in series with the line, as shown in Fig. 112 (*b*), are most effective, and localise the trouble to the single line on which the fault occurs. To obtain a given degree of protection, *i.e.* to limit the current on short-circuit to a definite figure, less reactance is required in the line than elsewhere. When used, line reactors are usually of the order of from 3 to 5 % based on the normal load. As in the case of generators, transformers for high-voltage transmission systems are built whenever possible with a comparatively high internal reactance, so that external line reactors are not required.

In both these cases the current normally passes through the reactor, involving not only additional losses but also a varying voltage drop according to the load. With reactors installed in the run of busbars between groups of generators, as shown in Fig. 112 (*c*), only as much current passes from one section to another as is necessary to equalise the loading of the generators. Thus losses are reduced and voltage regulation improved. The effects of a short-circuit are also largely confined to those circuits supplied from the bus section to which is connected the line in trouble. As busbar reactors do not normally carry much current they are frequently as great as 25 % based on the capacity of one group of generators. The busbar reactor arrangement is probably more extensively used than any other, often by itself but sometimes in conjunction with individual or group line reactors.

175. Experimental Methods of Determining Short-circuit Currents.—A knowledge of the magnitude of the short-circuit current at various points on a power system is essential for many purposes. For example, this information is required in dealing with the following problems :—

1. Selection of oil circuit-breakers of the required interrupting capacity.

2. Determination of the size of current-limiting reactors.

3. Determination of relay settings in relay systems depending on selective action from overload and directional relays.

4. Calculation of mechanical stresses in the structural elements of apparatus subject to heavy electromagnetic forces.

The size and complexity of present-day systems, however, have reached such a point that the determination of short-circuit values by mathematical methods is extremely lengthy and laborious. Hence attention is now being given to experimental methods of

solution which give results quicker and of sufficient degree of accuracy.

The most extensively used of these experimental methods is the short-circuit calculating table, which is an apparatus in which the actual A.C. system is represented by an approximately equivalent D.C. network, reactances being represented by resistances, etc. In the later forms of tables any particular problem can be set up rapidly by an arrangement of telephone jacks and cord circuits. The magnitude of the currents in the various circuit branches of this equivalent D.C. network can then be read off, and converted, by the use of the proper factor of proportionality, into the corresponding A.C. currents of the original system. According to Schurig [8] the results obtained are safely under 20 %, and usually under 10 % in error.

A later and more elaborate method of attack is to build up a miniature three-phase power system corresponding to the original as closely as possible.[9] Actual generators and transformers are used and loads are made up of conveniently-arranged lumped units of inductance, capacitance, and resistance. A miniature system of this character has been in use at the Schenectady works of the General Electric Co. since 1919, and found of great service. It should be noted that only symmetrical short-circuits can be determined by the short-circuit calculating table. With the miniature system, however, other types of short-circuit, *i.e.* those with unequal current in the various phases, can be readily dealt with. Results obtained on the miniature system are usually within 5 % of the true value. The error is introduced by switching in measuring instruments, and by the difficulty of making the large number of simultaneous readings required.

In the miniature A.C. system the scale of representation for the sake of convenience and economy should be as small as possible. In the Schenectady artificial system power stations are represented by generators of 3·75 kW. and it has been found inadvisable to use less powers than this as the metering errors then assume serious proportions. More recently, however, Spencer and Hazen [10] have designed and constructed a miniature A.C. network on a smaller scale more suited for the laboratory. In this model all rotating apparatus has been eliminated, generators being represented by phase-shifting transformers. Generating-station powers of only 100 watts are used, and by

taking suitable precautions when metering, notably by the use of vacuum-tube voltmeters, very accurate results can be obtained.

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[4] H. R. Wilson, 'Approximate Method of Calculating Short-Circuit Currents in an Alternating-Current System,' *Gen. El. Rev.*, Vol. 19, p. 475 (1916).

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[8] O. R. Schurig, 'Experimental Determination of Short-circuit Currents in Electric Power Networks,' *Trans. A.I.E.E.*, Vol. 42, p. 10 (1923).

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CHAPTER XII.

PRESSURE RISES.

176. Natural Impedance of a Circuit.—Before discussing the origin of pressure rises in transmission systems, it is necessary to examine briefly the fundamental properties of an oscillating circuit. Consider the case of an electric circuit in which an oscillation of energy is occurring, the power losses in the circuit being negligible. It is obvious that at the instant when the current wave passes through zero value the whole of the energy must be stored in the electrostatic field, and similarly, at the instant when the pressure wave passes through zero value the whole of the oscillating energy must be stored in the electromagnetic field. As it is the same stored energy appearing alternately as dielectric and magnetic energy, it follows that

$$\frac{Ce^2}{2} = \frac{Li^2}{2},$$

or

$$\frac{e}{i} = \sqrt{\frac{L}{C}} = Z_0. \quad . \quad . \quad . \quad (193)$$

The quantity Z_0 is thus seen to be of the nature of an impedance, and is expressed in ohms. It is generally known as the natural impedance, or surge impedance, of the circuit and its reciprocal Y_0 is the natural or surge admittance.

In the case of an overhead transmission line the conductor inductance is

$$L_0 = \left(0{\cdot}80 + 0{\cdot}741 \log_{10} \frac{d}{r}\right)10^{-3} \text{ henries per mile, } . \quad (9)$$

and the corresponding value of capacitance is

$$C_0 = \frac{0{\cdot}0388}{\log_{10} \frac{d}{r}} \times 10^{-6} \text{ farads per mile. } . \quad . \quad (14)$$

Neglecting the first term in the inductance formula, which is usually quite small in comparison with the second term, the natural impedance of an overhead line is closely given by

$$Z_0 = \sqrt{\frac{L_0}{C_0}} = 138 \log_{10} \frac{d}{r} \text{ ohms.} \quad . \quad . \quad . \quad (194)$$

For commercial lines Z_0 usually lies between 400 and 600 ohms.

In underground cables the capacitance is much larger relatively to the inductance, and the natural impedance has a smaller value which may be about one-tenth of the value for overhead lines.

A knowledge of Z_0 is exceedingly useful as it enables a calculation of the transient voltages and currents which may occur in a circuit. For instance, if a line carrying a current i has this current suddenly interrupted, the maximum value of the oscillating voltage produced thereby is iZ_0.

If one conductor of an unearthed system is earthed the maximum transient current flowing to earth is eY_0, where e is the voltage to earth.

If lightning strikes a line and the high-frequency break-down voltage of the insulators is e, the maximum discharge current in the conductors is limited to eY_0.

Power-transmission systems are always complex in character, that is, they consist of sections or elements—such as generator, transformer, transmission line, and load—having different electrical constants. In such a compound circuit, oscillations harmless in one part of the circuit may reach a dangerous condition in other parts, due to the variation in Z_0. For example, consider the case of a long transmission line one section of which is laid underground. An oscillating current in the cable, of little importance in itself, may give rise to dangerously high oscillating voltages when it enters into the overhead part of the line, owing to the far higher natural impedance of the latter.

Similarly, if a transformer is connected across the end of an overhead line. The natural impedance of such a transformer may be between 2 000 and 4 000 ohms, which is very much higher than that of the line itself. Consequently an oscillating current in the line, which only gives rise to moderate oscillating voltages therein, may produce destructive voltages in the windings of the transformer.

A consideration of the relative values of Z_0 in the different parts of a transmission system thus gives considerable information on the relative danger and protective action of the parts on each other. It also explains why reactive apparatus, such as transformers, are so much more liable to suffer from the disruptive effects of transient voltages than the lines to which they are connected.

177. Oscillation Frequency of a Line.—The rate at which the oscillating energy will pass back and forth between the magnetic and the dielectric fields is entirely independent of the frequency of the power current in a transmission line. From formula (193) we have

$$e = i\sqrt{\frac{L}{C}} \quad . \quad . \quad . \quad . \quad (195)$$

but since i may be considered as the charging current of a condenser of capacitance C with a voltage e across its terminals, we can also write

$$e = \frac{i}{2\pi f C} \quad . \quad . \quad . \quad . \quad (196)$$

Equating (195) and (196) gives

$$f = \frac{1}{2\pi\sqrt{LC}} \quad . \quad . \quad . \quad . \quad (197)$$

the usual formula for the period of oscillation of a circuit having massed capacitance and inductance. This formula requires modification when applied to a transmission line where the inductance and capacitance are distributed, as may be seen from the following considerations.

The velocity of propagation of an electric wave along an overhead conductor is approximately the same as the velocity of light or, say, 186 000 miles per second. Hence, if an oscillating current of frequency 1 000 occurs in the conductor, the wave length of the oscillation will be

$$\frac{186\,000}{1\,000} = 186 \quad \text{miles.}$$

In Fig. 113 the curve shows how the instantaneous value of the current varies along a line at one particular instant, *viz.* when the current at P has its maximum value. Q is the nearest point at which the current has exactly the same value as at P,

and PQ is the wave length of the oscillation. In exactly the same way the instantaneous value of the voltage varies along the line, repeating itself every 186 miles. Hence the inductance voltage and capacitance current in each little element of the conductor due to the oscillation also varies progressively along the line, and the total inductance voltage and capacitance current are obtained by adding vectorially the values in each little element. The net result is to reduce the values of conductor inductance voltage and capacitance current in the ratio of the diameter of a semi-circle to the length of the arc. This is exactly analogous to the calculation of the magneto-motive force of a distributed armature winding (n turns, i amperes) which has a value of $\frac{2}{\pi}ni$, not ni, ampere turns.

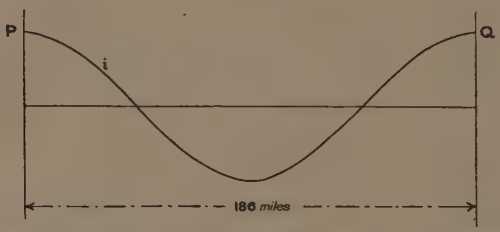

FIG. 113.—Instantaneous current at different points on a long transmission line.

In the case, therefore, of a transmission line, formula (196) must be modified and written

$$e = \frac{i}{4fC}.$$

The surge impedance is unaffected by the distributed character of the inductance and capacitance, so as before

$$e = i\sqrt{\frac{L}{C}}. \quad . \quad . \quad . \quad . \quad (195)$$

Combining the two equations gives for the frequency of oscillation of the line

$$f = \frac{1}{4\sqrt{LC}}. \quad . \quad . \quad . \quad . \quad (198)$$

This is the lowest or fundamental frequency of oscillation, and the corresponding wave length of the oscillation is four times the length of the line.

Besides this fundamental frequency higher harmonics may exist in a line oscillation. This can be seen as follows:—

At any point in an oscillating line the effective power is always zero, since voltage e and current i are in quadrature. The instantaneous power ei, on the other hand, is not zero, as equal amounts of power flow first one way and then the other way. Across the ends of the line, however, no energy can flow, otherwise the oscillation would not be limited to the line. This means that at both ends of the line the instantaneous power must be zero.

Thus two cases are possible:—

1. The current is zero at one end, and the voltage zero at the other end.

2. Either the current is zero at both ends, or the voltage is zero at both ends.

In the first case $i = 0$ at one end, $e = 0$ at the other end, so that the line must be open-circuited at one end and short-circuited at the other. The potential and current distribution in the line must then be as shown in Fig. 114. That is, the length of the line represents a quarter wave or an odd multiple thereof.

The frequency of oscillation under these conditions is

$$f = \frac{(2n - 1)}{4\sqrt{LC}}, \quad . \quad . \quad . \quad . \quad (199)$$

where n is any integer.

In the second case $i = 0$, or $e = 0$, at both ends, the line must be open-circuited or short-circuited at both ends, and the current and voltage distribution are as sketched in Fig. 115. The line now represents a half-wave length or a multiple thereof, and the frequency of oscillation is

$$f = \frac{n}{2\sqrt{LC}}. \quad . \quad . \quad . \quad . \quad (200)$$

The velocity of propagation of the oscillation is readily found by multiplying the frequency of oscillation by the wave length corresponding thereto.

Thus the fundamental frequency of oscillation is

$$f = \frac{1}{4\sqrt{LC}}$$

$$= \frac{1}{4l\sqrt{L_0C_0}}, \quad . \quad . \quad . \quad . \quad (201)$$

where l = length of line in miles,
L_0 = inductance of conductor per mile,
and C_0 = capacitance of conductor per mile,
and the corresponding wave length is $4l$, so that the velocity of propagation is

$$v = \frac{1}{4l\sqrt{L_0C_0}} \times 4l$$
$$= \frac{1}{\sqrt{L_0C_0}} \quad . \quad . \quad . \quad . \quad . \quad (202)$$

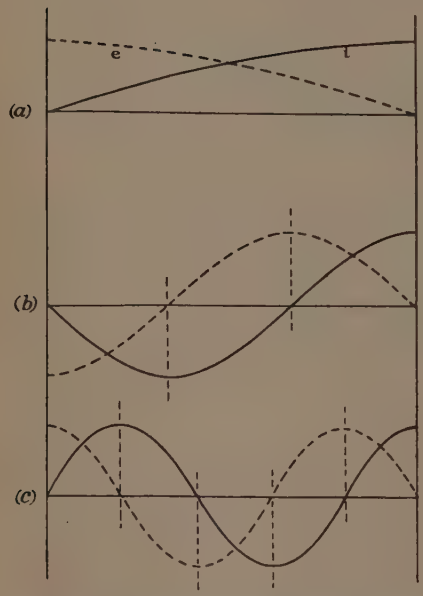

FIG. 114. — Instantaneous voltage and current in oscillating line. Line open-circuited at one end and short-circuited at the other.

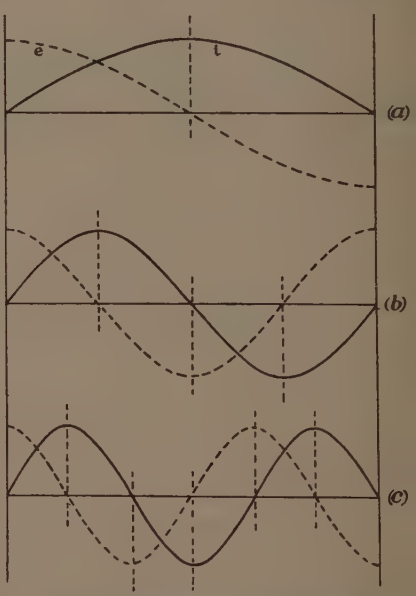

FIG. 115. — Instantaneous voltage and current in oscillating line. Line open-circuited or short-circuited at both ends.

The velocity of propagation v is reduced slightly below the velocity of light owing chiefly to the loss of power in the conductors, and the increase of inductance by the magnetic field inside of the conductor (which would not exist in a conductor of perfect conductivity). If, however, the value of the external inductance per mile of conductor is employed in formula (202), the quantity $\frac{1}{\sqrt{L_0C_0}} = 186\,000$, and the relationship can be used

for the calculation of the capacitance from the inductance and *vice versa*.

178. Travelling Waves.—The effect of a lightning discharge or similar phenomenon is to impress suddenly on a part of a transmission line a quantity of electric energy, which moves along the line at approximately the speed of light as a travelling wave or impulse, until it has been diverted from the line or its energy dissipated. The amplitude of the wave may not be high, nor yet of an oscillating character, yet by reason of the steepness of the wave front it may have a very destructive effect by piling up the voltage locally in the windings of reactive apparatus. It is essential to obtain a clear idea of the nature of such steep-fronted travelling waves as they are very common in transmission systems, being produced by other causes besides atmospheric disturbances.

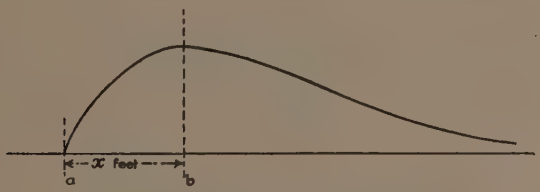

FIG. 116.—Travelling wave or impulse.

In travelling waves the source of danger is the high rate of change of voltage with the time, and the correspondingly high voltage gradient along the conductor, and it is convenient to compare oscillatory and non-oscillatory effects in this respect by speaking of the 'equivalent frequency' of non-oscillatory transients. Fig. 116 represents a non-oscillatory voltage wave or impulse travelling along a conductor from right to left. At the point a the voltage of the wave is zero at the instant figured, while the voltage at the point b is the crest value of the wave. As the wave moves along, the voltage at a will rise from zero to b, the time taken being equal to the time taken by the wave to travel a distance equal to ab. If the distance ab is x feet, the time taken for the voltage at any point of the line to rise from zero to b is

$$\frac{x}{186\,000 \times 5\,280} \text{ seconds.}$$

A representative value for x for waves due to lightning disturbances is 1 000 feet, for which the time is

$$\frac{1\ 000}{186\ 000 \times 5\ 280} = 1{\cdot}02 \times 10^{-6} \text{ seconds.}$$

Now, in an oscillating wave, the time taken for the voltage to rise from zero to a maximum corresponds to one-quarter of the periodic time. Hence the effect of the impulse for which the distance ab is equal to 1 000 feet will be somewhat similar to that of an oscillating wave of periodic time $= 4{\cdot}08 \times 10^{-6}$ seconds, which corresponds to a frequency of 245 kilocycles per second.

179. Reflection of Travelling Waves.—Electric travelling waves, like waves in water or air, are reflected back whenever they come up against an obstacle. More generally, they are partially reflected and partially transmitted. Such a phenomenon occurs if the nature of the circuit suddenly changes, as for

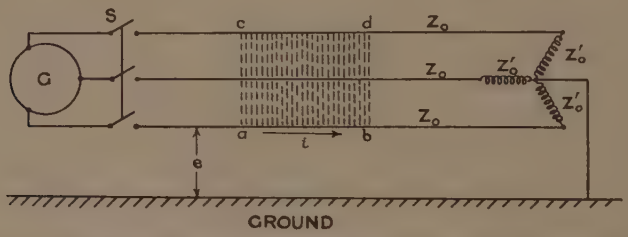

FIG. 117.—Voltage and current waves travelling along a transmission line.

example at the junction of an overhead and underground line, or at terminal stations where apparatus is connected across the line.

In order to fix ideas, consider the arrangement depicted in Fig. 117. Here G is a three-phase generator from which a voltage to neutral e is suddenly impressed on a transmission line by closing, and then immediately opening, a three-pole switch S. Assuming that the resistance of the conductors is negligible, and that the operation of closing and opening the switch is performed very quickly, a rectangular impulse will be produced in the line, the length ab of the impulse depending upon the time during which the switch was closed. The current i flows in the conductors between the points a and b, but neither forward of b nor backward of a. It carries with it the system of magnetic and dielectric flux lines in the space comprised between the planes ac

and bd; the complete energy impulse of voltage e and current i travelling away from G with the velocity of light. The remote end of the line is closed by a circuit having a different natural impedance Z'_0 to that of the line Z_0. On arrival at the transition point, partial reflection and partial transmission of the wave will occur.

It is evident that at the transition point the potential will be the sum of the incoming and reflected waves, while the total current will be given by the difference of the incoming and reflected current waves. Calling e_0 and i_0 the amplitudes of the transmitted voltage and current waves, and e_1 and i_1 those of the reflected waves, the following relations hold at the transition point:—

$$e_0 = e + e_1, \quad . \quad . \quad . \quad . \quad (203)$$

and

$$i_0 = i - i_1. \quad . \quad . \quad . \quad . \quad (204)$$

Since

$$i = \frac{e}{Z_0},$$

$$i_0 = \frac{e_0}{Z'_0}$$

and

$$i_1 = \frac{e_1}{Z_0},$$

the amplitude of the transmitted voltage wave is

$$e_0 = \frac{2Z'_0}{Z_0 + Z'_0}e, \quad . \quad . \quad . \quad . \quad (205)$$

and of the reflected voltage wave

$$e_1 = \frac{Z'_0 - Z_0}{Z_0 + Z'_0}e. \quad . \quad . \quad . \quad . \quad (206)$$

Similarly the amplitude of the transmitted current wave is

$$i_0 = \frac{2Z'_0}{Z_0 + Z'_0}i, \quad . \quad . \quad . \quad . \quad (207)$$

and the reflected current wave

$$i_1 = \frac{Z'_0 - Z_0}{Z_0 + Z'_0}i. \quad . \quad . \quad . \quad . \quad (208)$$

The following three cases are of special interest:—

1. Line open-circuited ($Z'_0 = \infty$).
2. Line short-circuited ($Z'_0 = 0$).
3. Line closed through a non-inductive resistance of value $R_1 = Z_0$.

Case 1.—If the line is open-circuited at the remote end, it is obvious that when the wave reaches this point it cannot flow any further but is reflected, the voltage and current of the reflected wave being of the same values as in the original waves because the energy remains constant. The total current of the incoming and reflected wave must, however, be zero, on account of the open-circuited line, and the whole energy is therefore stored at this point in the electrostatic field. The reflected current wave must therefore be reversed and its value is i, while the value of the voltage at the end of the line, where the incoming and reflected waves overlap, is equal to $2e$ as shown in Fig. 118. On arrival

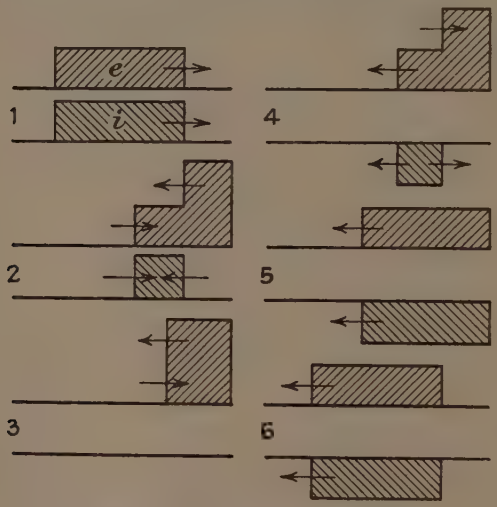

FIG. 118.—Reflection of waves at end of open-circuited line.

at the point where the reversal is complete the voltage will again have its original value of e volts.

Case 2.—With a short-circuited line, complete reflection of the wave will also occur, but it will now be the voltage wave instead of the current wave that is reversed. Due to the short-circuit the total voltage of the incoming and reflected voltage wave must be zero, and the whole energy is therefore stored at this point in the electro-magnetic field. The total current at the end of the line, where overlapping occurs, must be $2i$, and on arrival at the point where reversal is complete is i as before. This state of affairs is represented by Fig. 119.

Case 3.—Comparing the two previous extreme cases, it is evident that there must be a particular intermediate value of Z'_0 which will absorb the wave pulse and prevent reflection. Examination of formulæ (206) and (208) will show that when the end of the line is closed by a non-inductive resistance of magnitude equal to Z_0, reflection cannot occur, because both voltage and current waves will enter the resistance unchanged. If $R_1 > Z_0$ there is a partial reflection with transient rise of pressure and if $R_1 < Z_0$ there is a partial reflection with transient rise of current.

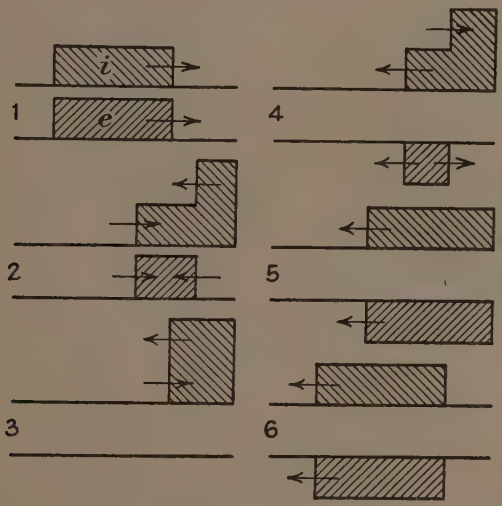

FIG. 119.—Reflection of waves at end of short-circuited line.

180. Origin of Pressure Rises.—There will now be considered in more detail a few of the ways in which dangerous pressure rises may be set up in transmission systems. These abnormal transient voltages are of comparatively frequent occurrence, particularly in overhead systems, and the problem of preventing damage to transmission lines and connected apparatus demands serious consideration.

In general, pressure rises may be caused by conditions external, or internal, to the system. A convenient classification is as follows:—

External Causes.—These are mainly due to lightning and only give trouble on overhead or partially-overhead systems.

1. Direct lightning stroke.

2. Electromagnetically-induced currents due to a lightning discharge in the immediate vicinity of the line. Such a discharge is called a 'side stroke.'

3. Electrostatically-induced charges on the conductors due to the presence of thunderclouds.

4. Electrostatic charges imparted to the line by the friction of dust or dry snow blowing past the conductors, or by changes of line altitude.

Internal Causes.—These are mainly due to oscillations set up by sudden changes of circuit conditions or by resonance. Trouble of this character may be experienced both on overhead and underground systems.

1. Switching operations on unloaded line.

2. Sudden opening of loaded line, particularly under short-circuit conditions.

3. Resonance.

4. Arcing grounds.

It should be noticed that the external causes produce over-voltages between line and earth, whereas the internal causes produce over-voltages between conductors. Also any oscillating potentials set up in a line by the external causes have an enormously high frequency of the order of a million cycles per second, whereas those produced by the internal causes have usually a comparatively moderate frequency ranging between several hundred and several thousand cycles per second. Arcing grounds form an exception to this rule, inasmuch as they set up oscillations between line and earth, the oscillations having either moderate or high frequency depending on the distance of the fault from the line termini.

Experimental investigations into these pressure rises have been very difficult on account of the high voltages and frequencies involved. Recently, however, there have been developed two instruments, *viz.* the Dufour cathode-ray oscillograph and the klydonograph, which are proving of great value in the study of these transient phenomena. The Dufour oscillograph renders it possible to secure oscillograms showing the voltage, current, and time relations of an electrical transient which may occupy a time of 1 / 100 000 000 of a second or less. The klydonograph is a surge recorder which gives a direct indication of

the polarity, and the approximate magnitude of an over-voltage, and an approximate indication of its wave front. Both these instruments are being pressed into service to obtain more definite information as to the nature of pressure rises, and the protective values of lightning arrestors and other similar apparatus.[1]

181. Direct Lightning Stroke.—Lightning discharges have been divided by Lodge[2] into two kinds, namely, *A* and *B* strokes. The *A* stroke is illustrated in Fig. 120 (*a*). A charged cloud near the surface of the earth induces a charge of opposite sign on all upstanding objects, such as church spires, chimneys, etc. The

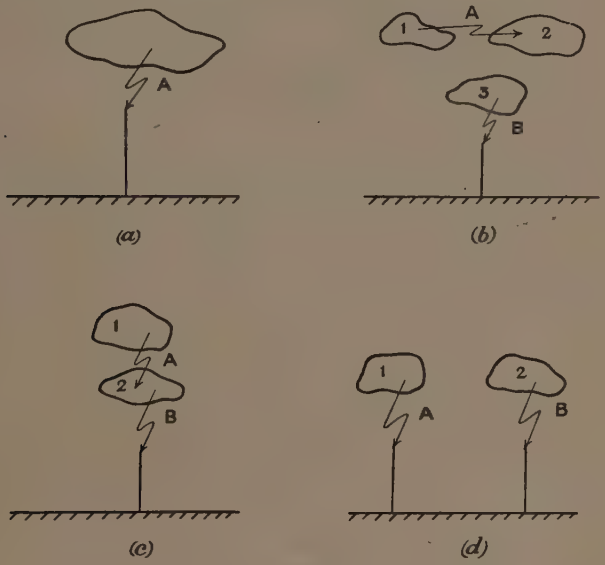

FIG. 120.—Types of lightning discharges.

electrostatic stress at the upper end of these objects is very great, and the air in the neighbourhood is rapidly ionised, streams of charged particles being repelled from all sharp corners and edges. This produces a gradual lowering of the resistance of the discharge path between the cloud and the conductor, until eventually a disruptive discharge occurs which tends to equalise the potentials of the two bodies. The distinguishing characteristic of this type of stroke is the length of time taken to produce, and the fact that when it occurs it is usually directed towards the highest and most sharply pointed object in the neighbourhood.

The *B* stroke is an impulsive rush of electricity occurring when the potential difference between the cloud and the earth is established almost simultaneously. It is generally induced by a previous *A* stroke. Thus in Fig. 120 (*b*), if an *A* stroke takes place between the two charged clouds 1 and 2, the cloud 3 may be left with a greater potential than the air can withstand, and a *B* stroke thus occurs to earth. This type of stroke takes place with absolute suddenness and does not seem to obey any definite laws like the *A* stroke does. It may shatter a building, ignoring a neighbouring well-designed lightning conductor, or strike the ground even if there are tall objects such as trees or chimneys near by. The *B* stroke is in consequence far more dangerous than the *A* stroke. Figs. 120 (*c*) and 120 (*d*) show other ways in which a *B* stroke may be produced. In Fig. 120 (*c*) an *A* stroke occurs between clouds 1 and 2, the latter being near the earth's surface, and a *B* stroke is thereby induced between the lower cloud and earth. In Fig. 120 (*d*) the *A* stroke from 1 to earth causes the *B* stroke from 2 to earth.

No device has yet been developed which is quick enough in action, or which has sufficient discharge capacity, to protect a transmission line from the effects of a direct stroke. Fortunately, such an event is of comparatively rare occurrence, and the damage caused thereby is more or less local. If a line is struck in the open country the discharge flows along in both directions, shattering insulators and even wrecking poles until its energy is dissipated in the paths to earth thus formed. If, however, the stroke occurred close to a terminal station, directly-connected apparatus would also be damaged.

182. Side Stroke of Lightning.—Very little is known regarding the quantitative aspects of lightning discharges, the difficulty of research work along these lines being readily appreciated. Peek,[3] however, has measured on actual transmission lines pressure rises due to lightning as high as 500 000 volts, and insulator flash-overs by lightning have occasionally indicated transient voltages as high as 1 500 000. Since the actual voltage induced in the line is only a small percentage of the pressure of the discharge itself, the latter is probably of the order of one hundred million volts. Enormous power is developed as the current in the discharge may amount to from 10 000 to 100 000 amperes. On the other hand, as the duration of the phenomenon

is apparently only a few micro-seconds, the actual quantity of electricity involved may be quite small.

In the past there has been considerable doubt as to whether lightning was oscillatory or unidirectional, but the high resistance of the discharge path due to radiation was considered to be a factor which would prohibit phenomena of the former character. Recent experimental evidence obtained by means of the klydonograph,[4] however, seems to prove definitely that lightning discharges are unidirectional in character, consisting of impulses having a steep wave front. The equivalent frequency of these impulses is of the order of 100 kilocycles per second. Occasionally, highly damped oscillations of one or two cycles are found on earthed-neutral systems, but these are apparently due to line flash-over following the lightning discharge, and these disturbances are really similar to unidirectional impulses in their effects.

When a side stroke occurs near a transmission line, corresponding high-frequency currents are induced in the line conductors. If these currents are not diverted they travel along the line, and will set up destructive pressure rises on entering the windings of electrical machinery connected at the line termini. Convincing evidence of the high-frequency nature of these currents is found in the way the inductance of a very small bend or loop in the run of an ordinary lightning conductor will cause a discharge through an alternative path of much higher resistance. For instance, where the conductor is taken over the coping of a building a discharge will invariably pass through the masonry, and not along the copper conductor.

183. Electrostatically-induced Charges.—A cloud electrostatically charged, and lying above a transmission line, will induce in the adjacent section of the line a corresponding charge of opposite sign known as a bound charge. This charge has its maximum value immediately below the cloud and then gradually tails away as shown in Fig. 121. At the far end of the line, charges of like sign to that in the cloud are located. So long as the cloud remains stationary this static distribution on the line persists. It should be noted that while this state of affairs holds, no oscillations will be set up, but the potential of the line immediately below the cloud may rise to such a value as to cause a 'spill-over' at the insulators. A static over-potential is therefore a rather dangerous phenomenon. If now the cloud is suddenly

discharged, the charges of opposite sign induced on the conductors will rush toward one another as a travelling wave or impulse. The reactance of the line prevents the freed charges spreading instantly, with the result that they often find a path to earth over the adjacent insulators.

FIG. 121.—Transmission line with electrostatically-induced charge due to presence of thundercloud.

184. Electrostatic Charging of Line Due to Changes in Altitude, or Friction Effects.—The atmospheric equipotential surfaces have a tendency to crowd together in mountainous districts, the surfaces of high potential coming very much nearer the ground in these regions than over the plains. The result is that a gradually-ascending transmission line passes progressively from regions of low, to regions of high, potential. This leads to the conductors being gradually charged up and the potential to earth being increased. A pressure rise of the order of 40 000 volts due to this cause has been observed on a low-voltage line.[5]

The wind blowing past the conductors, particularly if carrying in suspension sand or dry snow, is also often responsible for charging the line electrostatically. A rise of 13 000 volts produced in this way has been measured on a telegraph line.

In both these cases the phenomenon is entirely static, and it is unlikely that travelling waves of any importance would be set up

185. Switching Operations on Unloaded Line.—On connecting an open-circuited line to the terminals of a generator, travelling waves are set up which rapidly charge the line. On reaching the open end of the line, as previously shown, these waves are totally reflected, and overlapping with the incoming waves momentarily raise the voltage of the line to twice that of the impressed voltage at the instant of switching-in. Hence the maximum potential attained by the line at starting cannot be greater than twice the maximum generator voltage E_m. In prac-

tice, owing to the dissipation of energy, the potential attained by the line would be somewhat less than this theoretical maximum.

Analogous phenomena are produced on switching-out an open-circuited line. Travelling waves are set up which raise the potential of the line momentarily to a value not exceeding twice the generator voltage at the instant of disconnection.

The considerations thus far discussed do not take into account any characteristics of the arcing which always takes place at the contacts of a circuit-breaker, due to the finite speed of closing or opening. This phenomenon affects results considerably. For instance, suppose that the line is being opened. The arc between the circuit-breaker contacts first extinguishes at the zero point of the charging-current wave, which will, of course, be the crest of the voltage wave E_m. This leaves the line charged to a maximum of $+ E_m$. The circuit-breaker contacts are assumed to be separating continuously until at the middle of the next cycle, when the impressed voltage is $- E_m$, the voltage across the contacts is $2E_m$. There will be no restriking of the arc at this instant if the contacts are too far apart to break down at $2E_m$. If not, arc-over will occur, and there will be a resulting high-frequency oscillation about the point of the applied voltage $- E_m$ with an amplitude of $2E_m$, and the momentary rise of potential measured from zero will amount to $- 3E_m$. The oscillation will rapidly damp down, leaving the line charged to a potential of $- E_m$. Arc-over may again take place during the next cycle, but the potential can never be higher than $3E_m$. This case is the commonest met with in practice, but it should be noted that when very small charging currents are involved it is possible for the arcing to be extinguished at the zero point of the *high-frequency* current wave, and to be re-established at the crest of the 50-cycle voltage wave. This would lead to higher potential rises than $3E_m$.

Although circuit-breaker opening conditions have been discussed, it is obvious that closing operations accompanied by arcing lead to rises of the same magnitude.

A thorough experimental investigation of the subject has been made by Cox, McAuley, and Huggins,[6] and their chief conclusions are :—

1. Switching surges are usually unidirectional. When oscillatory they are highly damped and therefore of short duration.

2. Less than 50 % of all switching operations create a disturbance.

3. Pressure rises due to switching may reach 6 times normal operating crest voltage of the line, but 93 % of all produced are less than 3 times normal. On cable systems, 99 % are less than 3 times normal voltage.

4. No switching operations cause pressure rises high enough, or with a duration long enough, to affect the safety of properly insulated lines.

186. Sudden Opening of Loaded Line.—If a transmission line is suddenly opened while carrying load, a transient pressure rise is produced of value

$$e = iZ_0, \qquad (193)$$

where i is the instantaneous value of the current at rupture, and Z_0 the natural impedance of the line. Thus, if the natural impedance of the line is 400 ohms, and the instantaneous current at rupture 200 amperes, the transient rise of pressure however rapidly the current is interrupted cannot possibly exceed 80 000 volts, because this is the maximum value of the pressure wave necessary to store in the electrostatic field the whole of the energy stored in the magnetic field at the instant of rupture. If E_m is the crest value of the operating voltage, the maximum potential to which the line may be subjected is E_m + 80 000 volts.

Since the transient pressure rise produced by the sudden interruption of load is not a function of the line voltage, it follows that low-pressure transmission systems are liable to overvoltages of the same magnitude as high-pressure systems. The higher safety factor of the insulation on low-pressure systems is thus justified.

The worst case that could happen would be the interruption of the short-circuit current of the system at its crest value, when pressure rises of several hundred thousand volts might be produced. Fortunately, the use of oil circuit-breakers has practically eliminated all danger due to short-circuits, as these breakers possess the valuable property of opening the circuit when the current wave has its zero value.

187. Resonance.—If, in a circuit consisting of a condenser (of capacitance C farads) in series with an inductance and resistance (of value L henries and R ohms respectively), there is

impressed an alternating voltage of E volts, the current is given by

$$I = \frac{E}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}, \quad . \quad . \quad (209)$$

where $\omega = 2\pi$ frequency. The condensive reactance $\frac{1}{\omega C}$ thus tends to neutralise the inductive reactance ωL. Complete neutralisation is obtained when $\frac{1}{\omega C} = \omega L$, and the condition is known as electrical resonance. The current then becomes equal to $\frac{E}{R}$ and is in phase with the impressed voltage, hence, so far as the current is concerned, the circuit is equivalent to a simple resistance only.

From another point of view, however—as regards the voltage distribution in the circuit—the resonating circuit is by no means equivalent to a simple circuit of resistance R. For while in the latter the potential difference between any two points can never exceed the value of the impressed voltage, in the resonating circuit, potential differences occur which may be large multiples of the impressed voltage.

In commercial transmission circuits the capacitance is usually so small that resonance cannot occur at the fundamental supply frequency, but if the generator e.m.f. wave is distorted, trouble may be experienced due to resonance of one of the higher harmonics. This is owing to the fact that with a constant inductance, the value of capacitance required to produce resonance varies inversely as the square of the frequency.

An approximation to a circuit of massed constants is formed by an underground transmission line connected at one end to generating and transforming apparatus, and open-circuited at the other end. The inductance of the cable is usually negligibly small in comparison with that of the rest of the system, so that the capacitance and inductance are localised to a great extent, the one in the cable, and the other in the terminal apparatus. The phenomena of resonance in such a circuit can be exhibited most clearly by taking a concrete example.

Consider, for instance, a 50-cycle system comprising a 33 000-volt, 0·20 sq. in., three-core cable connected to an alternator and transformer at one end. Let the leakage inductance of the

alternator be 0·025 henries per phase and the resistance 0·5 ohms per phase, the corresponding quantities for the transformer being 0·015 henries per phase and 0·5 ohms per phase, all values being expressed in terms of the high-tension circuit. The total inductance is 0·04 henries, and the cable capacitance necessary to produce resonance at the fundamental frequency of the supply is

$$C = \frac{1}{\omega^2 L}$$
$$= \frac{1}{4\pi^2 \times 50 \times 50 \times 0{\cdot}04}$$
$$= 253 \text{ microfarads},$$

a value which would not be reached on an actual transmission.

For the seventh harmonic, however, the capacitance required for resonance would only be 5·2 microfarads (being one-forty-ninth

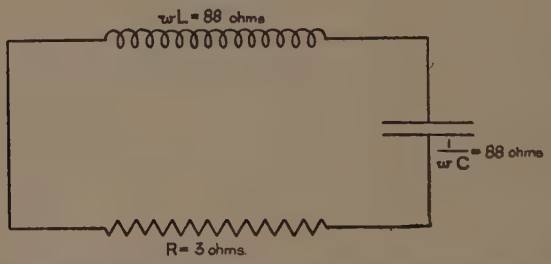

Fig. 122.—Resonating circuit formed by open-circuited cable and terminal apparatus.

of the value required at fundamental frequency), and would be provided by a cable about 18 miles long. Assuming the capacitance of the cable is localised at its mid-point, the additional resistance introduced into the circuit is that of 9 miles of conductor or, say, 2 ohms per phase. The inductance of the cable can be neglected, so that the resonating circuit for the seventh harmonic consists of an inductance of 0·04 henries, a capacitance of 5·2 microfarads, and a resistance of 3 ohms, as shown in Fig. 122. For this harmonic $\omega L = \frac{1}{\omega C} = 88$ ohms.

Supposing now that the generator e.m.f. wave consists of a fundamental having an R.M.S. value E_1 volts to neutral and a seventh harmonic of R.M.S. value E_7 volts to neutral, the

amplitude of E_7 being 10 % that of E_1. Then, the following relationship holds:—

$$\sqrt{1 + (0{\cdot}1)^2}E_1 = \frac{33\,000}{\sqrt{3}},$$

and

$$E_1 = 18\,960 \quad \text{volts},$$

hence

$$E_7 = \frac{E_1}{10} = 1\,896 \quad \text{volts}.$$

The current produced in the circuit due to E_7 is

$$I_7 = \frac{1\,896}{3}$$
$$= 632 \quad \text{amperes},$$

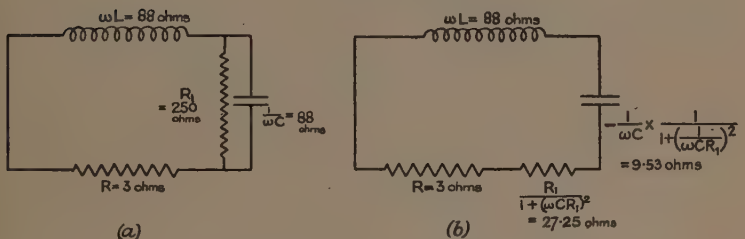

FIG. 123.—Effect of load in preventing resonance.

and the potential difference across the terminals of the capacitance is

$$e_7 = \frac{I}{\omega C}$$
$$= \frac{632}{2\pi \times 50 \times 7 \times 5{\cdot}2 \times 10^{-6}}$$
$$= 55\,300 \quad \text{volts (R.M.S.)}.$$

This is equivalent to a crest value of $\sqrt{2} \times 55\,300 = 78\,200$ volts which would be superimposed, in the worst case, on the crest value of the generator voltage.

The effect of a load at the receiving end of the line is to prevent such dangerous pressure rises due to resonance. For instance, assume that a non-inductive load corresponding to one-quarter of the carrying capacity of the cable is being supplied. This state of affairs may be represented by a resistance R_1 of about 250 ohms connected across the terminals of the

condenser standing for the capacitance of the cable, as shown in Fig. 123 (*a*).

The equivalent series impedance of the circuit formed by the condenser and resistance in parallel is first determined. The joint admittance of this circuit is

$$\sqrt{\frac{1}{R_1^2} + (\omega C)^2}$$
$$= \frac{1}{R_1}\sqrt{1 + (\omega C R_1)^2},$$

and the phase-angle is $\tan^{-1} \omega C R_1$.

The impedance of this parallel circuit is thus

$$\frac{R_1}{\sqrt{1 + (\omega C R_1)^2}},$$

and splitting into its components the resistance is

$$\frac{R_1}{\sqrt{1 + (\omega C R_1)^2}} \times \cos \tan^{-1} \omega C R_1$$
$$= \frac{R_1}{\sqrt{1 + (\omega C R_1)^2}} \times \frac{1}{\sqrt{1 + (\omega C R_1)^2}}$$
$$= \frac{R_1}{1 + (\omega C R_1)^2} \quad . \quad . \quad . \quad . \quad . \quad (210)$$

and the reactance is

$$-\frac{\omega C R_1^2}{1 + (\omega C R_1)^2}$$
$$= -\frac{1}{\omega C} \times \frac{1}{1 + \left(\frac{1}{\omega C R_1}\right)^2} \quad . \quad . \quad . \quad . \quad (211)$$

The effect of a non-inductive load is therefore equivalent to the insertion into the original circuit of an additional resistance of value

$$\frac{R_1}{1 + (\omega C R_1)^2},$$

while the condensive reactance has been decreased by multiplying by

$$\frac{1}{1 + \left(\frac{1}{\omega C R_1}\right)^2}.$$

This equivalent circuit is shown in Fig. 123 (*b*).

Substituting in the above formulæ, the resistance of the load (250 ohms) and capacitance (5·2 microfarads) in parallel for the seventh harmonic is

$$27{\cdot}25 \quad \text{ohms},$$

the reactance is

$$-\ 9{\cdot}53 \quad \text{ohms},$$

and the impedance is

$$28{\cdot}87 \quad \text{ohms}.$$

Hence the resistance of the complete circuit is

$$27{\cdot}25 + 3 = 30{\cdot}25 \quad \text{ohms},$$

the reactance is

$$88 - 9{\cdot}53 = 78{\cdot}47 \quad \text{ohms},$$

and the impedance is

$$84{\cdot}1 \quad \text{ohms}.$$

The current produced in the circuit by the seventh harmonic in the impressed voltage is

$$I_7 = \frac{1\,896}{84{\cdot}1}$$
$$= 22{\cdot}54 \quad \text{amperes},$$

and the potential difference across the condenser and load is

$$e_7 = 22{\cdot}54 \times 28{\cdot}87$$
$$= 651 \quad \text{volts (R.M.S.)},$$

which is equivalent to a crest value of 920 volts.

Thus if the system on open-circuit is liable to resonance, the addition of load removes the danger: (1) by destroying the equality between the condensive and inductive reactances of the circuit, and (2) by limiting the current by the introduction of more equivalent resistance into the circuit. There is, of course, the possibility that a system which is non-resonant on open-circuit may be brought into resonance by a light load, but the increased resistance of the circuit is usually sufficient to prevent any dangerous pressure rises taking place.

Throughout the above discussion, the assumption has been made that the leakage inductance of the terminal apparatus is constant. In actuality, however, this condition is not fulfilled, owing to the variable permeability of the magnetic circuits of such apparatus and other causes, and it should be realised therefore that the above method is only an approximation of the complete solution.

The extreme case to the circuit of massed constants is provided by the transmission line itself, considered apart from any terminal apparatus. Such a line possesses uniformly distributed inductance and capacitance, and has various resonant frequencies corresponding to a fundamental natural period of oscillation and its overtones.

Suppose, for example, a long overhead transmission line connected at one end to the high-tension busbars of a large generating station containing many generating and transforming units working in parallel. The reactance of the terminal apparatus may then be neglected for a first approximation, and the line treated as short-circuited at this end. If the remote end of the line is open-circuited, resonance will occur at a fundamental frequency of $\frac{1}{4l\sqrt{L_0C_0}}$ and its odd harmonics. Since the quantity L_0C_0 is practically constant for all overhead lines, the resonant frequencies are thus determined solely by the length of the line.

Similarly if the line is open-circuited at each end, resonance will occur at a fundamental frequency of $\frac{1}{2l\sqrt{L_0C_0}}$ and its overtones.

Steinmetz [7] has pointed out, however, that while a transmission line has quite definite resonant frequencies corresponding to the fundamental and its harmonics, it will oscillate with any frequency whatever, providing that this is very high. This phenomenon is due to the variation of the line constants at high frequencies. With these frequencies neither the inductance nor the capacitance of a line is perfectly constant: the inductance varies with the frequency by the increased screening effect or unequal current distribution over the cross-section of the conductor; the capacitance increases by brush discharge over the insulator surface, and by the increase of the effective conductor diameter due to corona effect. Hence the frequencies of the very high harmonics are to some extent variable, and since they are close to each other they overlap, so that at very high frequencies the transmission line has no definite frequency of resonance, but responds equally well with any frequency.

In the general case, a transmission circuit is complex, consisting of a line having uniformly distributed constants, connected to

generating, transforming and receiving apparatus where the constants may be considered as massed. When resonance occurs in such a complex circuit the entire circuit oscillates, usually as a full-wave oscillation and its harmonics, and the fundamental frequency of resonance may be very much lower than that corresponding to the transmission line itself. The determination of the resonant frequencies of such a complex circuit is not a simple matter, but methods of attacking the problem have been given by Evans and Sels.[8]

188. Arcing Grounds.—Arcs and sparks in electric circuits produce oscillations in a similar manner to the production of such oscillations in a wireless transmitting circuit. Such a transmitting circuit consists of a spark gap shunted by a condenser and inductance in series. When an alternating e.m.f. is impressed on the circuit the condenser charges up each half-wave, and if the voltage is high enough to break-down the spark gap one or more oscillating discharges occur from the condenser through the gap each half-period. When these oscillations are set up in transmission systems they are known as 'arcing grounds.'

For instance, consider an unearthed transmission system where the earth is brought within striking distance of one of the line conductors, as by the puncture of an insulator on an overhead line, or by the appearance of a weak spot in the insulation of an underground cable. A spark discharge then occurs to earth, and the arc following discharges the conductor by a transient oscillation. That is, the conductor is brought down to earth potential, and the other two conductors of the three-phase system correspondingly rise in voltage from the star to the delta voltage. As soon as the conductor is discharged, the spark gap to earth opens, and the conductor then charges again from the power supply of the system, its voltage to earth rising until sufficient to jump to earth again and start a second transient oscillation. One or more series of oscillations may occur in each half-cycle of the voltage wave, thus forming a 'recurrent' oscillation. Occasionally, however, each transient oscillation persists up to the beginning of the next transient of the same half-wave of voltage, and the recurrent oscillations thus tend to run into each other and approach a 'continuous' oscillation. These phenomena have been shown clearly by oscillograms taken on artificial transmission lines.[9]

In an arcing ground, oscillations are set up on both sides of the faulty point, the frequency of these oscillations being the natural frequency of the section of line in which they occur. In themselves, such oscillations are not specially dangerous, but there is every probability of a pressure rise causing failure of the insulation at some other point of the system. If this happens on one of the other phases, a double earthing is formed, which is equivalent to a short-circuit of the system. Break-down of transformers is also liable to occur, either due to resonance effects, or to localised pressure rises taking place across the end turns of the windings. This latter danger will now be considered in more detail.

189. Pressure Rises in Transformers.—In high-voltage transformers the high-tension winding contains a large number of coils and a great length of conductor, so that its capacitance to earth is of considerable value. Each separate coil has resistance, inductance, and earth capacitance, so that the whole winding is a circuit with distributed electrical constants somewhat analogous to a transmission line. In addition to the earth or shunt capacitance, there is also a series capacitance between adjacent coils. Hence, neglecting resistance, the winding is electrically equivalent to the arrangement shown in Fig. 124 in which

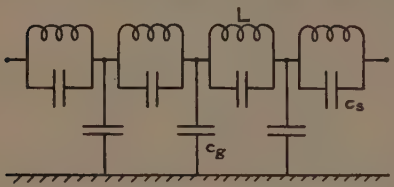

FIG. 124.—Transformer with distributed electrical properties.

L = inductance per unit length.

c_s = capacitance between adjacent coils per unit length.

c_g = capacitance to earth per unit length.

The resultant impedance of L and c_s in parallel is

$$\frac{\omega L}{1 - \omega^2 c_s L},$$

and it acts merely as an inductance at frequencies lower than that given by $1 - \omega^2 c_s L = 0$, and as a condenser at higher frequencies. In the former case the equivalent inductance is

$$L_e = \frac{L}{1 - \omega^2 c_s L}, \quad . \quad . \quad . \quad (212)$$

and in the latter case the equivalent capacitance is

$$c_e = \frac{\omega^2 c_s L - 1}{\omega^2 L}. \quad . \quad . \quad . \quad (213)$$

The electrical behaviour of the transformer therefore when subjected to an oscillating voltage, depends on whether the frequency of the oscillation is greater or less than f_k, the critical frequency which satisfies the equation $1 - \omega^2 c_s L = 0$.

At frequencies lower than f_k the transformer winding acts as L_e and c_g in Fig. 125 (*a*), that is, in the same way as a transmission line whose inductance and capacitance to earth are L_e and c_g respectively. For frequencies higher than f_k the winding can be considered to be a combination of c_e and c_g as in Fig. 125 (*b*), the whole arrangement being similar to the arrangement of the various capacitances of a suspension-type insulator.

If the transformer is connected to the end of a line in which

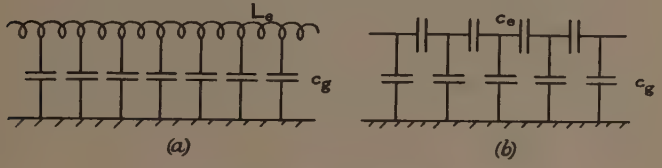

FIG. 125.— Equivalent transformer circuits.

oscillations of frequency $f < f_k$ are being set up, there is danger of resonance occurring in the windings with consequent failure of the insulation. With high-voltage power transformers trouble of this character is most likely to be produced by frequencies ranging between 10 000 and 100 000 cycles per second, and it should be noted that the oscillations set up by arcing grounds are liable to fall within this range as their frequency is determined by the distance between the fault and the line termini. Hence, arcing grounds may produce resonance in the transformer winding resulting in break-down of those coils in the interior of the winding where the maximum stress is experienced.

If the incoming oscillations have a higher frequency than f_k, the transformer winding behaves as a series and shunt arrangement of capacitances, and the potential distribution across the winding is non-uniform, the highest stresses occurring on the end coils. A method of calculating the potential distribution in such

cases has been developed previously in connection with suspension-type insulators (Art. 81) Berg [10] has also carried out an experimental investigation, and shown that arcing grounds of high frequency may set up pressure rises across the end coils ($2\frac{1}{2}$ % of the winding) which amount to 15 times the normal pressure.

The same phenomenon is also produced by a steep-fronted travelling wave or impulse. The two-fold effect of such an impulse in concentrating the pressure caused thereby across the end coils of the transformer winding, and also raising the potential of the winding with respect to earth, is made quite clear by the following analogy due to Lincoln.[11]

Consider the hydraulic transmission of a wave along a flexible piping system. The elasticity of the walls of the pipe represents the capacitance of the electrical system, and the inertia of the contained liquid the inductance of the system. Now either transformer or generator windings have two important differences from an equal length of transmission line in that both the inductance and capacitance per unit length is greatly increased. In the hydraulic analogy, the increase in capacitance per unit length may be represented by imagining a much thinner-walled pipe for the transformer or generator. The increase in inductance per unit length may be represented in the analogy by an increase in the specific gravity of the liquid, say, by substituting mercury for water.

In Fig. 126, therefore, b_1 is the transmission line with relatively heavy walls, although still flexible, and a light liquid contained therein, say, water; b_2 is the transformer or generator which has relatively thin walls, containing a heavy liquid such as mercury When a wave or surge that has been started out on the transmission line reaches the point P, which is the terminus of the line, it is obvious that the following will occur:—

1. A part of the energy of the incoming wave will be reflected and will therefore travel back through the line from the point P.

2. The remainder will begin to travel through part b_2, but the speed of its propagation will be much reduced because the liquid being set in motion is very much heavier, and also because the forces acting upon it through the elastic retaining walls are much smaller.

3. The steepness of the wave front during its propagation

through b_2 will be much increased over that obtaining in b_1 by the above process.

At P_1 there is a tendency to burst the pipe, that is, to break down the insulation to earth, but there is also a heavy stress tending to break through from point P_1 to P_2. Suppose that the portion b_2 of the pipe, instead of being straight as in the figure, were to be coiled upon itself so that a complete coil took place in the distance P_1P_2; there would then be a tendency for the liquid to break through the walls of the pipe into the neigh-

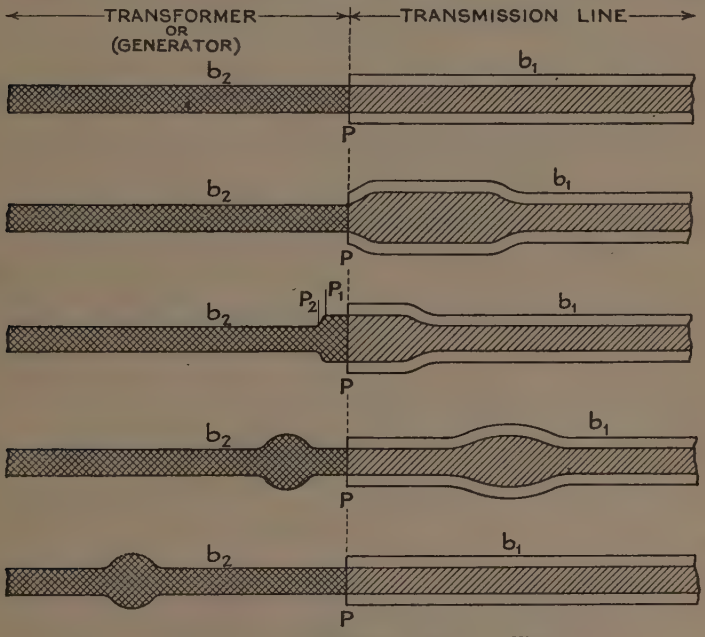

FIG. 126.—Hydraulic analogy of electric travelling waves.

bouring coil. This is exactly what occurs in the electrical system. An incoming wave in penetrating the turns of electrical apparatus causes an excessive voltage stress between turns. The momentary break-down or snapping across of the surge from one turn to the next will do no particular damage unless such break-down is followed by the power current and an arc is thereby established.

There is, due to the normal operation of the apparatus, an insulation stress from conductor to earth, and also one between adjacent turns. The stress to earth may be increased up to twice

its normal value, the stress between turns may be increased up to fifty times.

190. Attenuation of Travelling Waves.—High-frequency travelling waves and impulses of steep wave front caused by switching operations on complex circuits, or due to atmospheric disturbances, are of common occurrence in transmission systems. Fortunately, such impulses or waves are rapidly dissipated by energy losses in the line, so that their effect is more or less local. Furthermore, as the steepness of the wave front of such transients increases, so does the rate of attenuation or decay of the transient, so that a wave shape containing many harmonics is smoothed out and gradually becomes sinusoidal in the course of its travel along the line.

This increase in the rate of decay at high frequencies is due to the variable character of the electrical constants of the line. The influence of skin effect on the resistance and reactance of the conductors has already been dealt with. A more important factor, according to Steinmetz, is the electromagnetic and electrostatic radiation, which though negligible at low and medium frequencies becomes of dominating importance at frequencies between 10^5 and 10^7 cycles. This radiation has the effect of rapidly increasing the effective resistance of the conductor, and also introduces a further source of loss dependent on the voltage and represented by a shunted conductance, so that the energy of the higher frequencies of a complex wave is quickly dissipated.

For example, consider a travelling wave produced in an overhead transmission line by a suddenly liberated electrostatic charge. Such a wave, in order to be sinusoidal in shape, would require a sinusoidal distribution of charge on the line at the instant of discharge, which is not probable, and the wave will usually be complex, consisting of a fundamental and many harmonics. The question of the change in shape of the front of the wave as it travels along the line is thus one of the attenuation or decay of its component frequencies. Steinmetz[12] has calculated the attenuation of a rectangular travelling wave of fundamental frequency 60 kilocycles during its passage over a typical overhead line. At the start, the equivalent frequency of the wave front is infinite, after travelling 0·3 kilometres the equivalent frequency is 1 110 kilocycles, after 3 kilometres 325 kilocycles, after 30 kilometres 103 kilocycles, and after 108 kilometres only 63 kilo-

cycles. It thus appears that the danger zone of steep wave fronts is limited to relatively short distances.

If the amplitude of the travelling wave is such that corona is formed on the conductors, the attenuation of all the component frequencies of the wave will be increased, and the general effect will be to smooth out a steep wave front much more rapidly than would occur on a corona-free line. Thus, taking the 60-kilocycle rectangular wave adopted by Steinmetz as an example, Whitehead[13] shows that for a moderate amount of corona the wave will be approximately sinusoidal after travelling only 0·3 kilometres.

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CHAPTER XIII.

APPARATUS FOR PROTECTION AGAINST DANGEROUS PRESSURE RISES.

191. Phenomena against which Protection is Required.—In considering the installation of apparatus to protect the transmission line and terminal apparatus against the effect of abnormal pressure rises, a distinction should be made between over-voltages that are dangerous and those that are not. Many of those met with in the normal operation of the system are not dangerous, being provided for by the voltage margin represented by the factor of safety on the line insulators. Phenomena which experience has proved to be of a dangerous character are shown in Table 26.

TABLE 26.—*Dangerous Pressure Rises and Chief Types of Protective Apparatus.*

Phenomena.	Protective Apparatus.
1. Direct stroke of lightning.	Ground wire.
2. High-voltage impulses of steep wave front.	Lightning arrestors.
3. Low-voltage high-frequency oscillations.	Surge absorbers.
4. Arcing grounds.	Arcing ground suppressor. Petersen coil.
5. Static overpotential.	Water jet earthing resistances. Earthing choking coils.

Of course, the simplest method of eliminating trouble due to (4) and (5) is to earth the neutral point of the system. The question of earthing the neutral, however, has to be considered from other points of view, and the advantages and disadvantages entailed thereby will be discussed later on in this chapter.

192. Ground Wire.—Overhead ground wires have been in extensive use since the construction of the earliest transmission lines. One wire or stranded conductor is usually employed, run

above the transmission line and earthed at many points, preferably at every pole or tower. It is generally recognised that this method of protecting against direct strokes is very efficient, and this is confirmed by experiments on model lines.

The objections to the ground wire are the additional cost, and the possibility of the wire breaking and falling across the line conductors, thus causing a direct short-circuit. Failure due to the latter cause, however, is rare, as substantial galvanised stranded steel conductors are usually adopted. Indeed, such a steel conductor joining the tops of the towers generally adds greatly to the mechanical strength and stability of the line, particularly if the line is of the flexible type.

In British practice it is usual to run the ground wire *below* the line conductors, thus removing any liability of danger due to

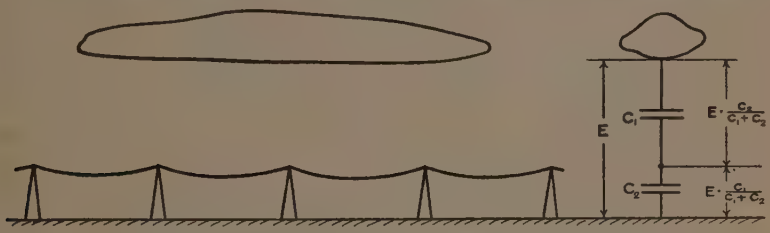

Fig. 127.—Reduction of induced voltage due to ground wire.

breakage, and also serving to carry pilot or telephone cables if required.

Besides taking the brunt of a direct stroke, the ground wire reduces the voltage electrostatically or electromagnetically induced in the conductors by the discharge of a neighbouring thundercloud. For example, referring to Fig. 127, the voltage induced on the line due to the cloud depends on the ratio of the capacitances cloud-transmission line / transmission line-earth. The effect of a ground wire run above the conductors is to bring the equipotential surface corresponding to zero potential from the surface of the earth to the immediate vicinity of the conductors, thus increasing the value of the capacitance between line and earth. The pressure rise may be 30 to 50 % less than in a line not protected by an overhead ground wire. The function of the ground wire is thus preventive in character, whilst that of the lightning arrestor is merely curative.

If a sufficient number of parallel grounded wires were provided

to surround the conductors, all pressure rises due to external causes could be entirely removed. On account of the heavy capital expense involved such a solution would of course be impracticable. Hence one ground wire is generally used in practice, which allows the intensity of over-voltages to be considerably reduced, and lessens the work demanded from the lightning arrestors connected to the system. In exposed situations, however, such as the summit of a range of hills, it would probably be good practice to install one or two additional wires.

Peek[1] has made a laboratory study of the protective effect of ground wires and finds that a single ground wire reduces the induced lightning voltage to one-half of that without ground wire, for two ground wires the reduction is to one-third, while for three ground wires it is to one-fourth. These results were obtained under the favourable condition of good earths and low impedance for the earth connections. In a dry country with poor earths the protective value of a ground wire would be low. Its value would also be low even in a damp country if earths were made very infrequently, since then the impedance of the wire between successive earth connections would be considerable.

Another protective action of the ground wire is its damping effect on any wave or other disturbance travelling along the line by its action as a short-circuited secondary. The ground wire must consume a considerable part of the energy of a travelling wave and thus greatly increase the energy dissipation of the wave, especially when it is considered that the wire is generally made of steel, and thus possesses a large high-frequency resistance.

193. Horn-gap Arrestors.—The term 'lightning arrestor' is commonly, but inaccurately, used to cover all devices which are employed to protect a transmission system from the effects of over-voltages, no matter whether these are due to external or internal causes. The horn-gap arrestor was one of the first types to be developed, and still enjoys considerable merit because of its great simplicity. It consists of two horn-shaped pieces of metal separated by a small air-gap and connected in shunt between each conductor and earth. The distance between the two electrodes is such that the normal potential difference between line and earth is insufficient to jump the gap, but abnormally high voltages will break down the gap and so find a path to earth. The arc following the initial discharge constitutes a flexible conductor and rises

owing to the upward tendency of the heated air. This forms a bend in the current path, and following the principle that an electric circuit always tends to enlarge the area it encloses, the arc is drawn up by electromagnetic action towards the top of the horns. Here it is extinguished owing to the line pressure being unable to maintain an arc of such length. The time taken for the complete operation is usually from 3 to 5 seconds.

As regards the gap setting, the ideal arrangement would be to by-pass all voltage waves exceeding normal amplitude, but this is impracticable. On the other hand, the setting must be for a voltage less than the test voltage of the line insulation or transformers, so as to ensure that over-voltages will discharge to earth through the appointed device, and not elsewhere. Since all equipment is designed for a test voltage of at least twice normal it is generally deemed sufficient if the arrestor comes into operation somewhat below this figure.

The simple connection of the arrestor between line and earth would be insufficient to relieve the system from the effects of high-frequency waves of moderate amplitude. It is very probable that such waves would pass straight on to the transformers or other terminal apparatus, and there pile up the voltage in such a manner as to puncture the insulation. The end turns of transformer windings are specially heavily insulated to withstand this trouble, and sometimes the impedance of the end turns alone is relied upon to reflect the voltage wave back to the lightning arrestor, where it is discharged to earth. More usually a choking coil, consisting of several turns of bare copper wire, is connected in the line between the arrestor and the apparatus to be protected. The reactance of this coil is sufficiently high to ensure that all high-frequency waves are reflected back to the horns, while at the supply frequency it is negligible. In practice, twenty or thirty turns on a diameter of about 12 inches are found to be quite effective for the purpose.

Fig. 128 (*a*) illustrates the method of installing a simple horn-gap arrestor in conjunction with choking coils. For the utmost protection to the terminal apparatus the arrestor should be located as close to the end of the transmission line as possible. Also the connections from choking coil to arrestor, and arrestor to earth, should be as short and direct as possible.

The commonest form of choking coil is wound in a simple

helix, the turns of which are usually braced between clamps to prevent movement when heavy short-circuit currents traverse the coil. For outdoor work a suspension-type coil is used, the coil being wound round a wooden tension insulator which can be inserted in the run of the conductor as it is sufficiently robust to take the dead-end pull of the line if need be. Continental practice favours a pancake coil, the turns being wound with flat strip. This design has the advantage of ease in winding and supporting, but the insulation between turns is apt to be weak. Naturally very great electric and mechanical stresses are liable to come into existence between the turns of a protective choking coil, and

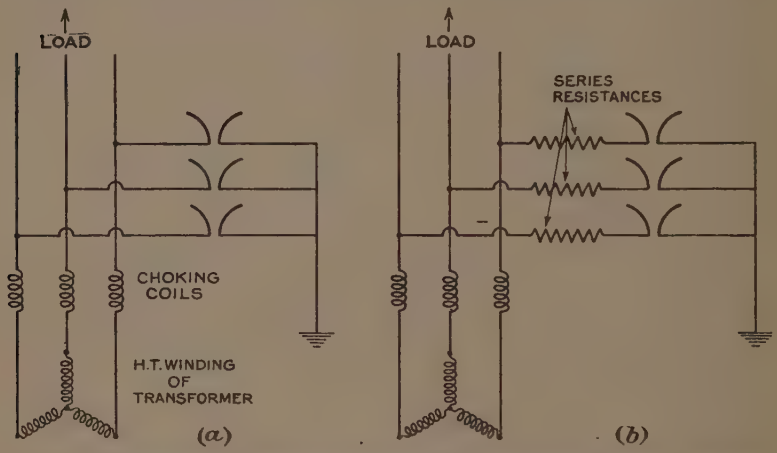

FIG. 128.—Installation of horn-gap arrestor and choking coils on a three-phase line.

it is desirable to have each turn separated by a good air space. The solenoid type of coil lends itself better to this construction.

Regarded simply from the point of view of relieving the line of excess voltages, a simple horn gap without series resistance is the best. In this case, however, a discharge across one gap in the case of a system with earthed neutral, or on two gaps simultaneously on an insulated-neutral system, constitutes a dead short-circuit for the dynamic current of the system. This introduces operating difficulties, as a horn gap is only capable of rupturing quite small currents, certainly not in excess of 10 amperes. It therefore becomes necessary to provide a resistance in series with the gap to limit the flow of dynamic current to a

value which can be safely interrupted by the arrestor. On the other hand, such a resistance very materially reduces the efficiency of the arrestor.

The selection of a value for the resistance is therefore a matter for compromise, the value generally adopted being such as will limit the dynamic current to between 1 and 5 amperes with normal voltage impressed. The resistance should be connected on the line side of the horns as in Fig. 128 (*b*) and not between horns and earth, since should the arcs from adjacent horns be blown together a short-circuit would occur in the latter case.

The resistances for use with horn gaps may be of the liquid metallic, or carborundum type. Resistances of the liquid type are usually placed in U-shaped earthenware vessels. The water is mixed with about its own weight of glycerine to prevent freezing, and it is advisable also to cover each surface of liquid with a layer of oil so as to avoid losses through evaporation. The advantages of the liquid resistance are its cheapness and simplicity, but it requires more attention than the other types. Carborundum rods or built-up resistances of a similar nature are cheap and in fairly common use, but are rather apt to disintegrate under the passage of heavy currents. The aim in selecting any resistance is to obtain a high thermal capacity, since this will enable the arrestor to discharge through a longer period of time. Wire-wound resistances, oil immersed, are good from this point of view, and furthermore their value does not vary with temperature like that of a liquid resistance. If a resistance of this type is used, however, it should be wound as non-inductively as possible.

In some installations an effort to secure protection has been made by employing two or more sets of horn gaps. Thus there might be horns without resistances set to arc over at 200 °/₀ of normal voltage, and horns with series resistances set at 160 °/₀ normal volts. The function of the horns without resistances is to give a direct path to earth for the severest over-voltage waves. When they operate, it is recognised that power current will follow and cause the protective circuit-breakers to operate. However, as the over-voltage conditions would constitute a serious menace to the system, this disadvantage is accepted as unavoidable. The function of the horns with series resistances is to operate on the less serious, but still undesirable, over-voltage waves falling between 160 °/₀ and 200 °/₀ of normal voltage. Incidentally they

also come into action when the other horns operate, but owing to the former being connected direct to earth they will take the major portion of the discharge. Since these over-voltages are less dangerous, more time can be allowed for their passage from the system if, by so doing, a power arc can be prevented, and a system shut-down avoided.

It is difficult to believe that a horn gap having a sufficient resistance in series to make it self-extinguishing has any protective value. An analogous case is met with in hydraulic pipe-lines, which are fitted with a pressure-compensation pipe as a relief valve against water-hammer. This pipe can only protect the line against water-hammer when it has a section equal to that of the main pipe-line; if the section is much smaller, water will overflow (like sparking-over of a horn-gap arrestor), and the presence of water-hammer is indicated, but no appreciable relief is given. A similar state of affairs exists with electrical phenomena: the surge impedance of an overhead line usually falls between 400 and 600 ohms, that is, a travelling voltage wave has associated with it a current wave of value roughly 2 amperes for every kilovolt potential of the wave. Thus a travelling wave of 100 kilovolts would have an accompanying current of 200 amperes. It is fairly evident that an arrestor which can only by-pass 10 amperes from the 200 amperes as the wave rushes along into the transformer winding will have a correspondingly small influence in reducing the amplitude of the voltage rise across the apparatus.

Theoretical formulæ for calculating the break-down voltage of a cylindrical horn gap have been worked out by Grutter[2] and others, but are not found to be of much use in practice. For one reason, the formulæ give the 50-cycle break-down voltage of the gap, but the voltage at which they will break-down when subjected to high-frequency lightning voltage may be very much different, depending on the impulse ratio of the gap. Also the state of the atmosphere and roughness of the horn surface make a difference, and the horns should preferably be set experimentally, being set as close together as circumstances will permit. Table 27, given by Coates,[3] will enable a first approximation to be made. The figures are average settings for horns at sea-level, corresponding to a spill-over voltage of 150 % normal. At higher altitudes a longer gap is necessary, the arc-over voltage between fixed electrodes being very closely proportional to the air density.

TABLE 27.—*Horn-gap Settings at Sea-level.*

Normal Volts.	Gap.	Normal Volts.	Gap.
10 000	$\frac{1}{2}$ in.	30 000	$2\frac{1}{4}$ in.
15 000	1 „	40 000	$3\frac{1}{8}$ „
20 000	$1\frac{5}{8}$ „	50 000	4 „
25 000	$1\frac{3}{4}$ „	60 000	$4\frac{3}{4}$ „

It is seen that on low-voltage installations the gap has to be set very closely, with consequent danger of accidental discharge by insects or other objects bridging the gap. Alteration of calibration also is liable to occur, due to corrosion or formation of small globules of melted metal. To permit of a longer gap being used the arrangement shown in Fig. 129 can be employed. The main gap is set at a good distance, say, 1 inch for 5 000 volts. The auxiliary gap, which is provided with an electrode having an adjustable platinum point is, however, set to discharge, say, at 25 % above the normal voltage. If a foreign body gets into the auxiliary gap the resulting discharge quickly dies out owing to the high resistance in series therewith. On the other hand, an arc formed by a pressure rise persists, and ionises the air in the main gap, causing the same to arc across and relieve the line.

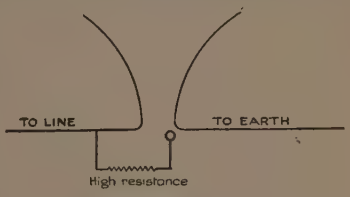

FIG. 129.—Diagram of horn gap with auxiliary electrode.

Another modification of the simple horn gap is the Burke arrestor which embodies as a characteristic feature a triangular choking coil wound from copper strip, one side of the coil being used as one of the horns. The arrangement of the device is as shown in Fig. 130. It will be seen that the line current passes round the triangular reactance coil and from the centre of the latter goes away to the load. The main gap has a series resistance, and there is also an auxiliary gap which is connected direct to earth. Very severe over-voltages will jump the double gap and go direct to earth in preference to passing through the resistance, while discharges of less severity pass to earth *via* the main gap and the resistance. The resistance itself consists of one or more rods of a material similar in appearance to carborundum. Its ohmic value is such as to limit the dynamic current with normal

voltage to from 4 amperes at 3 300 volts, to 2 amperes at 66 000 volts.

It is claimed that the choke coil in this arrestor has a magnetic blow-out effect, causing the arc to rise rapidly up the horns. Furthermore, a wave travelling along the line meets its first impedance at the first sharp upward bend of the triangle opposite to which the earthed horn is mounted. This piles up the voltage and allows a considerably wider gap to be used than with other forms of horn arrestor, without, however, diminishing the sensitiveness of the device.

There are several other forms of arrestor which are modifications in some degree of the horn and resistance device. The graded-resistance horn arrestor, for example, has a tapered or

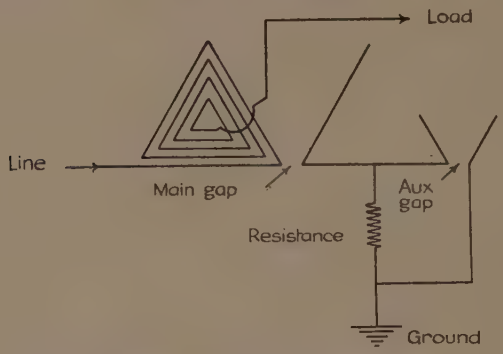

FIG. 130.—Diagram of Burke horn-gap arrestor.

stepped gap. A moderate discharge takes place across the narrowest portion of the gap which has a high resistance in series therewith. If the current flow is not sufficient to relieve the line the arc jumps across to the next wider gap, and so on, the successive gaps having lower and lower resistances in series. Under the severest conditions the discharge will be taken at the last gap where no resistance at all is in series.

Space does not permit of a description of these various modifications in detail, as they only enjoy a limited popularity. More efficient types of protective apparatus can now be obtained at a reasonable cost, and the greatest value of the horn gap at the present time is as an adjunct to some of these modern arrestors.

194. Multiple-gap Arrestor.—This type of arrestor, first developed by Wurts,[4] has, like the horn gap, been in use for

a considerable time. It consists of a series of small metal cylinders insulated from one another, and separated by airgaps of about $\frac{1}{32}$ of an inch wide. The first of the series is connected to the line, and the last to earth, and the number of gaps necessary depends upon the line voltage. To assist in the suppression of the arc and thereby prevent dynamic current from the line flowing through the arrestor, the cylinders are made of an alloy of zinc called a non-arcing metal. The vapour formed by an arc between these cylinders has a rectifying effect similar to mercury vapour, thereby suppressing the arc every other half-cycle. It is common practice to knurl the surface of the cylinders so as to present as large a cooling surface as possible, this also helping to extinguish the dynamic current.

The theory of the arrestor is briefly as follows. There is a certain capacitance between consecutive cylinders, and also between each of these cylinders and earth, and this leads to a non-uniform distribution of the potential between the various gaps. The phenomenon has been previously discussed in connection with the analogous case of the suspension insulator (Art. 81). It is sufficient here to state that the potential per gap is greatest at the line end of the arrestor, and gradually diminishes towards the gaps at the earthed end. The result is, that when the voltage across the arrestor reaches a certain critical value, break-down occurs between the first and second cylinders. The second cylinder is then connected to the first by an arc so that its potential rises accordingly until break-down occurs between the second and third cylinders; and so on. The dynamic current then follows the discharge and in so doing produces a sensibly uniform fall of potential along the line of cylinders, with the result that the maximum potential difference between cylinders is considerably less than that required for the initial break-down. The dynamic current continues to flow until the line voltage passes through zero to the next half-cycle, when the arc-extinguishing quality of the metal electrodes comes into action. Some of the zinc is vaporised by the heat of the discharge, and when the current in the gap passes through the zero point of the wave the zinc vapour prevents its re-establishment in the opposite direction. Before the voltage again reverses, the arc vapour in the gap has cooled to a non-conducting state, cutting off further flow of current. As in the case of the horn arrestor there is a

current limit, although a higher one, beyond which the arc will be maintained despite the quenching action, so that limiting resistances must be employed in most instances. These resistances are usually of graphite mixture or metallic, and are connected in series with the group of gaps as shown in Fig. 131. Their value is such as to limit the dynamic current to less than 20 amperes.

The multiple-gap arrestor is not satisfactory for use on systems with line pressures exceeding, say, 33 000 volts, due to the fact that the necessary increase in the number of gaps to prevent arcing over by the normal voltage is out of all proportion to the increase in voltage. There is also much uncertainty as to the number of gaps required, this depending on the position of the arrestor relative to surrounding earthed objects. With the earth potential brought very near to the arrestor, the potential gradient at the end near the line frequently becomes high enough to ionise the air between the cylinders, thus carrying the line potential

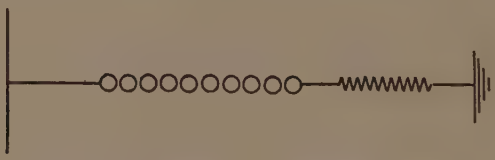

FIG. 131.—Diagram of multiple-gap arrestor.

to lower cylinders until the remaining gaps are so few that a discharge occurs.

In order to obtain a more equal distribution of potential over the arrestor, and so allow of a reduction in the number of gaps necessary, a metal earth-shield or antenna is sometimes placed near the gaps at the high-potential end of the arrestor and connected to the line wire.[5] Other modifications also have been proposed, but are not in general use, as the cost of the arrestor for the high voltages at which such special designs would be necessary is so great as to make other types of arrestor a better commercial proposition.

As with other types of lightning arrestor, the multiple-gap arrestor loses its efficiency as a discharge path for intense disturbances when it is provided with a series resistance. Thomas[6] has shown, however, that the difficulty can be largely surmounted by shunting some of the gaps with a resistance and using a smaller series resistance than with the unshunted arrangement. The ar-

restor with shunted gaps is known as the 'low-equivalent arrestor,' and the principle of its construction is shown in Fig. 132.

It will be seen that the point *B* is at earth potential when conditions are normal, so that a discharge will take place when the pressure is sufficient to break down the series gaps between *A* and *B*. The impulsive rush of current following the break-down, being of high frequency, will choose the straight through path to earth *via* the shunted gaps between *B* and *C*, instead of the alternative path through the shunt resistance, finally passing to earth *via* the small series resistance. As soon as the impulsive rush of current is over, the arcs in the shunted gaps die out, with the result that any dynamic current attempting to follow will have to flow through both resistances, which are now in series. This current will be so reduced by the high resistance in its path that it will not be sufficient to maintain the arcs in the series gaps, which therefore go out as well.

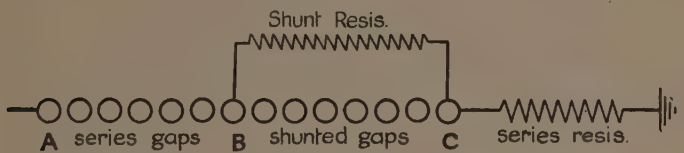

FIG. 132.—Diagram of 'low-equivalent' multiple-gap arrestor.

The effect of shunting some of the gaps is therefore to give the arrestor a certain amount of selective action, since it interposes a low resistance in the path of high-frequency disturbances, and a high resistance in the path of low-frequency discharges. If wire resistances are used this selective action will be improved since they cannot be wound absolutely non-inductively, and so their impedance will increase to some extent with the frequency. Thus the higher the frequency of the discharge, the greater the impedance of the shunted resistance, and therefore the greater the tendency for such a discharge to bridge all the gaps and to by-pass the shunted resistance. In one form of construction the series resistance is dispensed with altogether, the arrestor consisting simply of a large number of gaps some of which are shunted and some not.

As compared with the horn arrestor, the multiple-gap type has the advantage of greater sensitivity to minor pressure rises. Furthermore, it has the property of extinguishing any dynamic

current following the discharge at the zero value, instead of in the neighbourhood of the maximum value as with a horn gap. There is therefore no danger of the arrestor becoming the seat of high-frequency oscillations after it has operated. On the other hand, the multiple-gap type is more costly, especially for the higher line voltages.

195. Lightning Arrestor Spark Gaps.—During recent years there have been evolved types of lightning arrestor such as the electrolytic, oxide film and autovalve arrestors which can be designed to give satisfactory service no matter what the voltage of the system on which they are installed. These modern arrestors have a very high discharge capacity owing to the fact of their possessing a valve characteristic. In other words, they act like a counter e.m.f. equal to the line voltage, and the energy to be dissipated on discharge is limited to the energy of the over-voltage.

In general, these arrestors consist of two parts, first, a spark gap for discharging an abnormal voltage, and second, a means for preventing the normal line voltage from maintaining a power arc across the gap. Much research work has been carried out to determine the best form of spark gap for this purpose, and the simple horn gap formerly used has now been practically superseded.

In order to understand the *raison d'être* of these later types of spark gap the fundamental phenomena relating to spark-over must be briefly reviewed. Consider, first of all, the case of a needle gap. When a 50-cycle voltage is applied to such a gap and gradually increased, spark-over occurs at some definite value of the voltage. This is the minimum voltage required for the gap to discharge, and the phenomenon requires a finite period of time. When a steep wave front or high-frequency voltage is applied to the same gap, however, spark-over does not occur at the instant the minimum or 50-cycle voltage is reached but at a higher point on the rapidly rising voltage wave. The 'slower' the gap, the higher the voltage rises before break-down. This dielectric spark lag is due to the fact that in the non-uniform field of the needle gap balls of corona first form around the electrodes, and grow in size until the stress between them is sensibly uniform and great enough to break down the remaining air. Hence both time and energy are required before spark-over can take place. In Fig. 133 are shown

a series of typical spark-over curves relating to needle gaps in air, from which it will be noted that changes in frequency cause a considerable change in spark-over voltage.

A sphere gap, on the other hand, has the characteristic of possessing a spark-over voltage independent of the frequency, *i.e.* voltages of all frequencies and wave fronts discharge across the gap at the same value. In this case the field between the electrodes is practically uniform. The only energy required prior to break-down is the extremely small amount necessary to charge the spheres, and consequently the time lag is practically zero. The curves of Fig. 134, which exhibit the results of tests

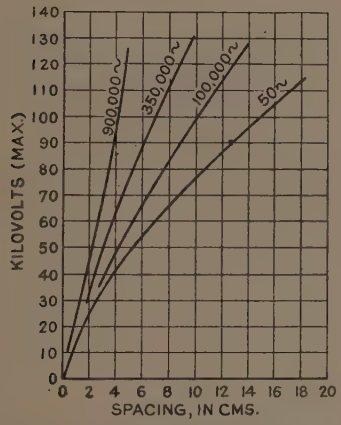

FIG. 133.—Spark-over voltage for needle gaps.

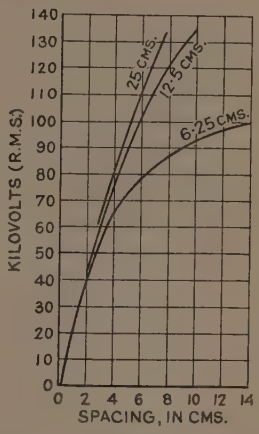

FIG. 134.—Spark-over voltage for sphere gaps.

on sphere gaps, show no variation of spark-over voltage with frequency at spacings less than, or comparable with, the sphere diameter.

The following points therefore should be noted :—

1. Two gaps with equal 50-cycle spark-over voltages may have extremely different lightning or impulse spark-over voltages because of time lag.

2. This time lag is the greater in a non-uniform field, or for electrodes where corona precedes spark-over, and is a minimum for a uniform field.

3. The spark-over voltage increases, and the time lag decreases, with increasing steepness of wave front.

With any particular set of electrodes the ratio between the voltage at impulse frequency, say, 500 kilocycles per second, required to cause spark-over, and that necessary at line frequency, is termed the impulse ratio of the gap. The impulse ratio between needle points is of the order of 1·5 to 2·3, depending on the spacing to some extent. A sphere gap of course has an impulse ratio of unity. With regard to horn gaps, these may have either sphere or needle-gap characteristics, depending upon the ratio of the diameter of the electrodes to their distance apart. If the setting is very small, as with low voltage, the gap will have sphere-gap properties and therefore a unity impulse ratio. If the gap is wide, as with high voltages, then its properties will be more nearly those of a needle gap.

It is important to utilise these principles in designing transmission systems. That is, lightning arrestor spark gaps should have an impulse ratio of unity or as low as possible, while insulators, and insulation generally, should have a high impulse ratio. In this connection it should be noted that the impulse ratio for the usual types of suspension insulator varies between 1·2 and 1·6, while for the pin-type insulators the figures are about 1·3 to 2·3. With solid dielectrics the impulse ratio is generally higher than with air. Thus varnished cloth or oiled presspahn between flat discs and immersed in oil shows an impulse ratio of from 2·2 to 2·5.

In comparing the protective values of lightning arrestor spark gaps it is necessary to take into account, not only the impulse ratio, but also the gap setting imposed by operating conditions. Thus, if the spark gap is placed out of doors without any protection from the weather, it must be set so that it will withstand normal line voltage with a factor of safety under the most unfavourable conditions. The 50-cycle spark-over voltage of a gap is very much decreased if the surfaces of the electrodes are covered with moisture due to rain or fog. The decrease in spark-over voltage due to moisture differs greatly with the shape of the electrodes, and, speaking generally, the more uniform the gap the greater the reduction. Thus the influence is a minimum for points and a maximum for plane surfaces. For spheres the wet spark-over voltage is 40 to 50 °/₀ of the dry spark-over voltage, and for horns the corresponding ratio is about 60 to 70 °/₀, varying with the spacing to some extent.

On the other hand, the impulse spark-over voltage of any form of gap is practically unaffected by moisture on the electrodes. Denoting by α the impulse ratio of a spark gap, and by β the setting factor or ratio of dry spark-over voltage to wet spark-over voltage, the relative protective values of two gaps for high-frequency voltages vary inversely as $\frac{\alpha_1\beta_1}{\alpha_2\beta_2}$.

To illustrate this point consider the following types of gap:—

1. Horn gap installed out of doors,
2. Sphere gap out of doors, and
3. Sphere gap covered.

The figures in Table 28 give representative values of α and β for each case, and the superiority of the covered sphere gap is clearly demonstrated. Calling the protective value of the latter 100 %, the protective value of the outdoor sphere gap is 50 %, and that of the simple horn gap only about 36 %.

TABLE 28.—*Relative Protective Values of Spark Gaps.*

	α.	β.	$\alpha\beta$.	Relative Protective Values.
Outdoor horn gap . .	1·5	1·5	2·25	36
Outdoor sphere gap . .	1·0	2·0	2·0	50
Covered sphere gap . .	1·0	1·0	1·0	100

It is therefore seen that in order to fully realise the high protective value of a sphere gap it must be covered, or the surfaces of the electrodes kept fairly dry. Of course a simple sphere gap is not possible for use with all types of arrestor since a horn is often necessary to assist in breaking the dynamic arc. Wide horn gaps are often provided with spherical or hemispherical electrodes at their lower parts so as to increase the speed of the gap.

A more recent development in high-speed gaps is the 'impulse protective gap' due to Allcutt.[7] This device has a selective action so that it operates at a lower voltage with impulse frequency than it does at the supply frequency. In other words, the apparent impulse ratio of the gap is less than unity. In view of the comments previously made on the covered sphere gap such added facility may at first glance appear unnecessary. It should be

borne in mind, however, that even with spheres, the arrestor will not discharge at any frequency until a voltage in excess of the gap setting is reached. Connected apparatus will therefore be subjected to all high-frequency waves up to perhaps 150 % of normal voltage, and is exposed to trouble due to localised pressure rises or resonance caused by these waves.

In principle the impulse protective gap is very simple, as shown in Fig. 135. The gap proper consists of two sphere-horn electrodes S_1 and S_2 which are connected respectively to the line and to the arrestor, the latter being usually of the electrolytic type. In addition to these two main electrodes an auxiliary electrode A is provided, which is connected to one of the horns through a condenser K_2, and to the other horn through a condenser K_1 and an unbalancing resistance UR. At the normal line frequency the impedance of UR is negligible compared to that of the condensers, and if the two latter are equal in value the potential of the auxiliary electrode will be about half-way between the potential of the two main electrodes. If, then, the auxiliary electrode is mounted half-way between the main electrodes, it will have practically no influence on the distribution of the electrostatic field between them, nor on the 50-cycle discharge voltage of the gap.

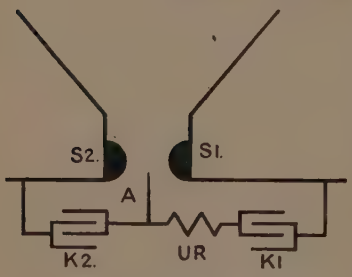

FIG. 135.—Diagram of impulse protective gap.

Suppose now that a high-frequency voltage is impressed on the gap. The distribution of potential is then disturbed, due to the difference in the impedance of the circuits (UR alone offers appreciable impedance in this case), with the result that practically the whole of the voltage is concentrated across the gap between A and S_1. This gap at once breaks down, the rest of the gap between A and S_2 immediately following. In effect, the length of the gap is halved the instant a high-frequency wave occurs. On the other hand, a slight time lag is introduced due to the fact that, in the initial break-down, one of the electrodes is pointed. However, an apparent impulse ratio of 0·6 to 0·7 can be obtained with the commercial forms of the gap.

196. Electrolytic Arrestor.—This arrestor has been undoubtedly one of the most effective lightning protective devices brought out, and is in very extensive use on the high-voltage systems of America. The action of the arrestor depends on the characteristics of a film of aluminium hydroxide, which is deposited on aluminium plates immersed in electrolyte. Such a film presents a very high resistance to the passage of current if the impressed voltage is comparatively low. Above a definite limit, however, known as the critical voltage, the film punctures and permits current to pass freely. The magnitude of this break-down voltage varies between 350 and 450 volts, depending on the composition of the electrolyte employed.

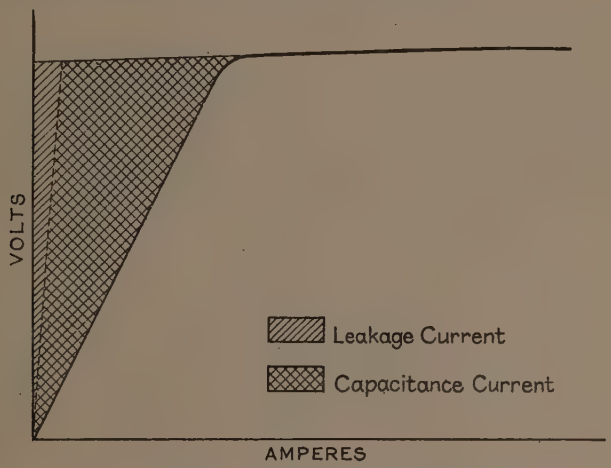

Fig. 136.—Volt-ampere characteristic of electrolytic cell.

The volt-ampere characteristic of an electrolytic cell having aluminium electrodes is shown in Fig. 136. Suppose that such a cell is connected to a circuit whose voltage can be varied. Then, if a low voltage is applied, a film of thickness corresponding to the voltage will form. If now the voltage is increased, there will be a rush of current for a few seconds until the thickness of the film has increased to an amount corresponding to this higher voltage. If this is repeated, the same phenomena will be observed until the critical voltage of the cell has been reached, when the film will not be able to increase its thickness any further, with the result that there will be a free flow of current.

It will be readily seen that this property of the aluminium

cell can be made use of in order to relieve the line of excess pressures. Stacks of cells can be connected between each line and earth, the number of cells being such that the break-down voltage is greater by any desired amount than the voltage set up across the stack by the ordinary line pressure. It is not necessary that the number of cells should be such that, on charging, the film is built up to the voltage corresponding to the critical voltage. For, supposing the film is built up with a voltage below critical, then any high-frequency discharge over this voltage will break it down, since the time element required for this is much less than the time required at commercial frequency to build the film further. As long as conditions are normal no current will flow through the arrestor (except capacitance current which will be considered later), but if the pressure rises the films break down, thus allowing a free path to earth for the discharge. The films re-form immediately this discharge has passed because the normal line pressure is not sufficient to enable dynamic current to flow. Any desired degree of protection can be obtained by having a sufficient number of cells per stack, and the arrestor has an extremely high discharge capacity as no series resistance is necessary.

The construction adopted for the arrestor is as shown in Fig. 137. Spun aluminium trays of double-conical section are nested one within the other, electrical contact between them being prevented by small porcelain spacers. Electrolyte is poured into each tray so as to make contact from one to the next, and the whole stack of trays is immersed in transformer oil. The oil serves the double purpose of increasing the thermal capacity of the arrestor, and of preventing evaporation of the electrolyte. Under extremely severe lightning discharges the electrolyte may splutter a little, and it is therefore usual to interpose a cylindrical micarta lining between the trays and the steel containing tank.

Each pair of trays in the electrolytic arrestor are normally separated by an insulating film, and thus the whole apparatus can be looked upon as a series arrangement of condensers. For this reason it is impossible to have the stacks permanently in circuit, otherwise continual capacitance and leakage currents would flow to earth, raising the temperature of the electrolyte and lowering the discharge capacity. It is therefore necessary to

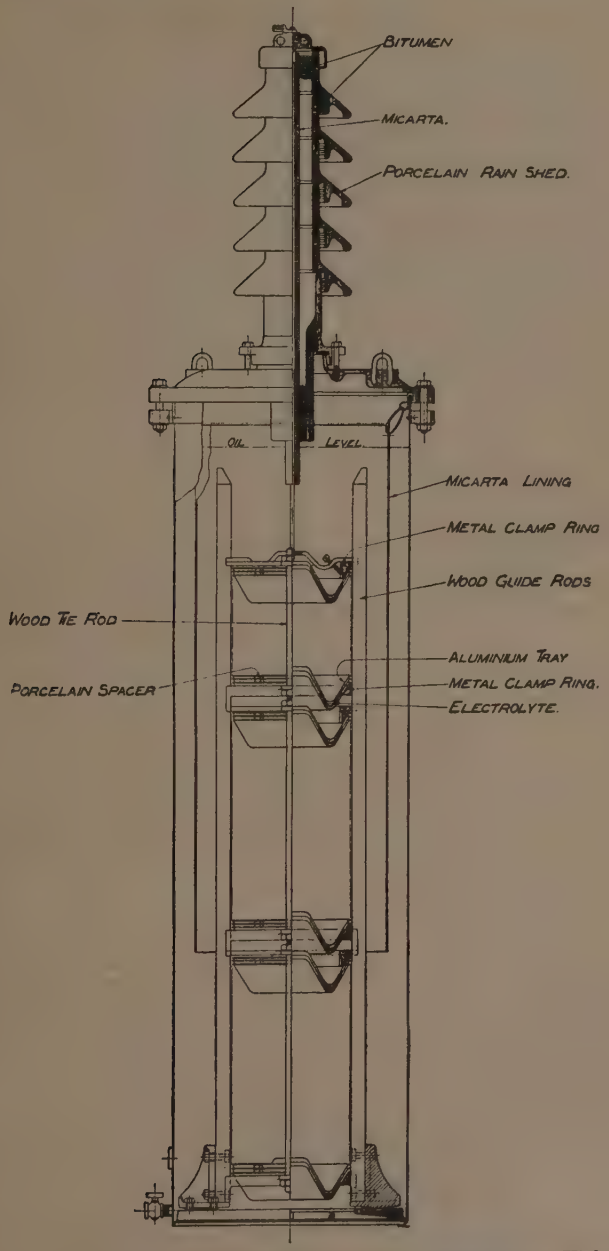

FIG. 137.—Construction of electrolytic arrestor. (Metropolitan-Vickers Electrical Co.)

interpose a spark gap in the connection between the line and each stack.

For this purpose the Westinghouse Co. employ the impulse protective gap, and Fig. 138 shows the application of this gap to an electrolytic arrestor. In the actual construction of the gap ordinary insulators are used for the condensers K_1 and K_2, and the unbalancing resistance UR is mounted on the upper insulator. One property of the arrestor which is of great importance is that when standing the film slowly dissolves in the electrolyte, and requires re-forming by the momentary passage of current. This charging process, as it is termed, is generally carried out once each day, or in very hot weather when electrical storms are frequent, twice daily. This is done by bridging the two horns of the gap for a few seconds, thus connecting the arrestor direct to the line. When this is done there is an initial rush of current which re-forms the film, and the operator can tell whether this reforming is complete by the appearance of the arc on breaking the contact at the horns. If, by any chance, the film has been allowed to dissolve entirely, there is a very heavy current rush on charging. To reduce this rush of current, and to eliminate the danger of pressure rises which might be caused by oscillations set up thereby, it is usual to charge through a limiting resistance. It will be seen in Fig. 138 that the earth-side horn is split, so that when the horns are bridged above this split the charging current has to flow through the resistance CR. When a discharge from the line occurs, however, it takes place across the spheres, so that only the resistance of the electrolyte is in circuit, thus giving maximum freedom of discharge. The resistance of commercial arrestors varies from about 60 ohms for a 22-kilovolt installation, to 320 ohms for a 110-kilovolt installation, so that the discharge capacity is high.

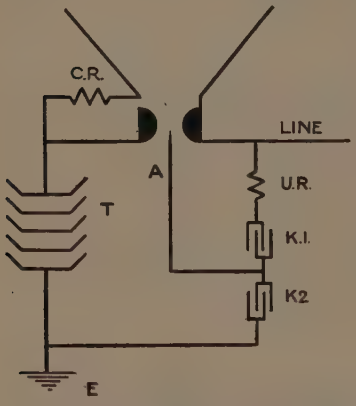

FIG. 138.—Arrangement of electrolytic arrestor and impulse protective gap.

The chief disadvantages of the electrolytic arrestor are the

freezing of the electrolyte at low temperatures (which makes the arrestor practically inoperative), and the necessity of daily charging. This latter consideration considerably increases the maintenance expense, particularly in the case of arrestors located on the more remote parts of the line.

197. Oxide Film Arrestor.—This type of arrestor has not been in use for many years but appears, in some respects, to be superior to the electrolytic arrestor. Its action depends upon the fact that certain dry chemicals change very rapidly from good conductors to almost perfect non-conductors when heat is applied. Thus, lead peroxide which has a specific resistance of about 1 ohm per inch cube at ordinary temperatures becomes converted to red lead at about 150° C., its specific resistance being then of the order of 24 million ohms per inch cube. A further increase in temperature reduces the red lead to litharge, which is practically an insulator.

The actual arrestor working on this principle consists of a number of cells held together under slight pressure in sections or stacks. Each cell is made up of sheet-metal electrodes about $7\frac{1}{2}$ inches in diameter dipped in an insulating varnish, which covers them with a film, and separated by a porcelain ring about 0·5 inches thick. The space between the electrodes inside the ring is filled with lead peroxide powder. The number of cells used in an arrestor is such that the voltage per cell is approximately 300 volts.

Fig. 139 shows the volt-ampere characteristic of a cell compared with that of an aluminium arrestor cell. It will be seen that right up to the critical point the leakage current is very small, and then the cell breaks down with absolute suddenness. When the insulating film on the electrodes breaks down it is punctured in very fine pinholes, and therefore any dynamic current following the discharge has to flow through these pinholes. Thus, there is intense local concentration of current, which produces local heating to such an extent that the neighbouring lead peroxide becomes converted to the insulating litharge. Hence the pinholes are almost immediately re-sealed, thus preventing any flow of dynamic current. According to Field,[8] the time taken for this re-sealing is certainly less than one four-thousandth of a second. The ohmic resistance of an arrestor during discharge varies between 0·01 and 0·1 ohms per cell,

depending on the path followed by the discharge, so that the impedance offered to the flow of a lightning discharge is very small. In the course of time all the lead peroxide must be used up, but the quantity employed in the cells appears to suffice for several years normal service.

Although the oxide film arrestor does not take such a large capacitance current as the electrolytic type, it is necessary to place a spark gap in series therewith because of the deteriorating effect of the alternating current on the sheet-metal plates. The General Electric Co. employ for this purpose covered hemisphere gaps, set at about 170 % of the normal voltage of the system.

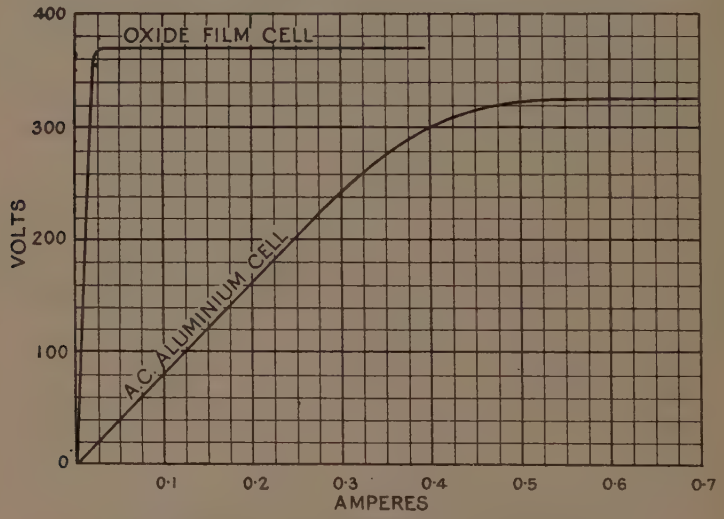

Fig. 139.—Volt-ampere characteristic of oxide film cell.

The great advantage possessed by the oxide film arrestor is that it does not require daily charging, and it may thus be installed at points on transmission systems where daily attendance is difficult or expensive to provide. On the other hand, varnish has an impulse ratio of above unity, so that as the action of the arrestor first depends on the puncture of a varnish film, it appears as though the device as a whole would be less sensitive to steep-fronted waves than to low-frequency disturbances.

198. Autovalve Arrestor.—The autovalve arrestor, originated by Slepian,[9] is one of the most modern developments for the protection of transmission lines from the effects of abnormal pressure

rises. In principle it is essentially a multiple spark-gap type of arrestor, but makes use of a glow discharge instead of an arc discharge.

The lower curve in Fig. 140 shows the characteristic of an arc discharge in air. If, however, the electrodes of a gap are made of material of considerable resistivity, any discharge is distributed over the faces of the electrodes, and local heating with formation of an arc is not possible. In this case, therefore, the discharge is maintained as a glow with volt-ampere characteristics, as shown in the upper curve of Fig. 140. Studies have shown that when flat

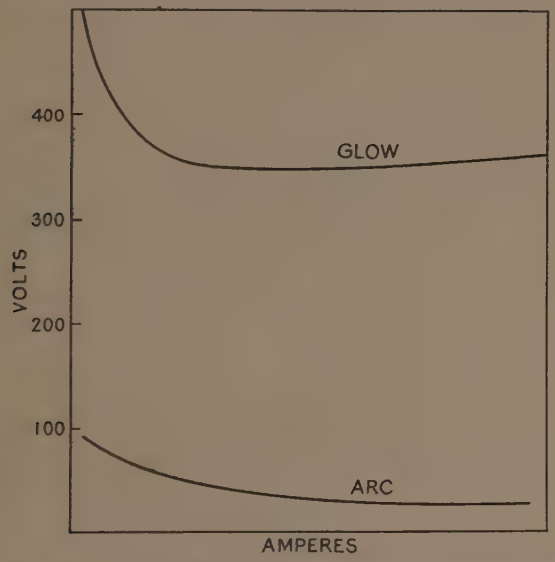

FIG. 140.—Volt-ampere characteristics of arc and glow discharge.

electrodes are separated by insulating spacers of the thickness to give gap lengths of 3 to 5 mils the glow-discharge voltage is practically constant at 350 volts (instantaneous value) over a wide range of current values. The break-down voltage is also of the same magnitude. Thus a structure composed of a column of flat electrodes suitably spaced will operate as a valve, the critical voltage being approximately 350 volts per gap. The characteristics of the arrestor over this critical voltage will depend on the thickness, area, and resistivity of the electrodes.

In the commercial form, the arrestor is built up of a series of

disc electrodes made of a composition material of suitable resistivity, each pair of discs being separated by mica spacers. Each column of gaps is connected between line and earth through a series gap. The number of gaps is chosen so as to give one gap for each 200 volts (R.M.S.) between line and earth, thus providing a voltage margin of approximately 25 % before break-down occurs. The disc area is selected to give a resistance rather lower than the electrolytic. For example, arrestors for a 22-kilovolt installation have a resistance of about 45 ohms, and those for use on a line voltage of 110 kilovolts have an approximate resistance of 225 ohms.

The autovalve arrestor has the advantage over the electrolytic type in that the initial cost and maintenance charges are lower,

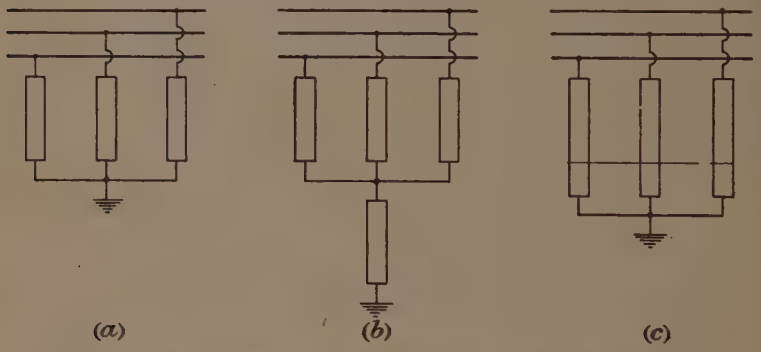

FIG. 141.—Arrangement of lightning arrestors on a three-phase circuit.

and there is no necessity for daily attendance. Hence it has a wider field of application than the electrolytic, as it can be used to protect isolated transformer stations, automatic substations, or similar plant where the installation of electrolytic arrestors would not be justified from an economic point of view. The autovalve type is sufficiently flexible in design to meet the wide range of requirements of the various applications of arrestors on an economic basis, and appears to have a great future before it.

199. Arrangement of Lightning Arrestors.—The arrangement of electrolytic and other valve-type arrestors on a three-phase circuit depends on whether or not the neutral point of the circuit is earthed. In Fig. 141 (*a*) the three arrestor stacks or columns are star connected and the neutral point connected direct to

earth. This is suitable for use on a system with earthed neutral, because if one of the lines becomes earthed the other lines will still be at the star voltage above earth, and will not therefore cause the arrestor to discharge.

If the neutral point of the system is not earthed, then four stacks are necessary as shown in Fig. 141 (*b*). The reason for this is, that in the event of an earth on one line, the other two lines will be raised to the full line voltage above earth instead of the normal star voltage. As a result, if the arrangement of Fig. 141 (*a*) were used with an unearthed neutral, an earth on one line would immediately cause the arrestors on the other two lines to operate, unless they were set to operate at a pressure between line and earth greater than the line pressure of the system, *i.e.* more than 173 % of the normal voltage across them. The protection afforded in such a case would obviously be too coarse.

With the neutral point of the system connected to earth through a resistance, the precise voltage impressed across each stack of the arrestor depends on the earth current. To guard against mishap, however, it is customary to use the four-stack or multiplex arrangement in all cases except where the neutral point is earthed solid.

An arrangement of stacks recommended for use with the autovalve arrestor is shown in Fig. 141 (*c*). In this method of connection three columns or stacks are used, each designed for line or delta voltage, and an interconnection between stacks is provided at the star-voltage point in each stack. The arrangement is based on the idea that the requirements from the standpoint of discharge-current capacity are fixed by the most severe conditions, which are those imposed by disturbances of external or atmospheric origin. Discharges due to these external causes pass through the three line elements in parallel and then through the earth element. Thus, in the multiplex connection, the *IR* drop in the arrestor resistance of the line and earth elements in series is four times that across a line element. With the method of connection shown in Fig. 141 (*c*) the resistance is reduced to that of an arrestor for line voltage only. In other words, the addition of 29 % in the material used increases the discharge capacity by about 132 %.

200. Condensers.—In order to relieve the line of those high-frequency disturbances which are not of sufficient amplitude to

discharge over an arrestor, electrostatic condensers are often used. These have the useful property of possessing an impedance $\frac{1}{\omega C}$ which is inversely proportional to the frequency of the discharge. For example, supposing that a 50-cycle transmission system had a battery of condensers of value 0·05 microfarads installed between each line and ground. Assuming that the wiring connections to the condensers were of negligible impedance, the apparent resistance to the passage of the 50-cycle current would be 63 500 ohms. If, however, an oscillation having a frequency of 500 kilocycles per second occurred in the line, the apparent resistance of the condenser battery thereto would only be about 6·35 ohms.

It is almost the same, as far as these high-frequency discharges are concerned, as though the line were connected direct to earth at the point where the condensers are installed. It is this property of the condenser of presenting a very small obstacle in the path of high-frequency currents, while allowing practically no current to pass at line frequency, which makes it so valuable as a protective device against those high-frequency oscillations always accompanying lightning phenomena.

The first satisfactory design of condenser for this purpose was due to Moscicki,[10] and consists of a long thin glass tube, provided inside and out with a coating of silver deposited chemically and then electroplated. This method of construction obviates any danger of air bubbles being enclosed between the coating and the glass, which would be a source of danger. The thickness of the tube must be kept as low as possible in order to get a high value of capacitance, and special precautions must also be taken to prevent 'edge' trouble due to concentration of the electrostatic stress at the edges of the metallic coatings. Each condenser is mounted inside a long metal tube, the space between being filled with a mixture of water and glycerine to give the apparatus a large thermal capacity, and thus prevent a dangerous rise of temperature on passing a heavy discharge. The glycerine also prevents freezing in cold weather. The tubes are finally assembled in metallic frames in the form of batteries, a sufficient number being connected in parallel to give the requisite capacitance.

The Moscicki condenser met with extensive application, but proved rather difficult to develop for the high voltages now in more general use, and its fragile character due to the use of glass

also hindered its commercial exploitation. Other materials such as impregnated paper and mica have been tried out for high-tension condensers, but the latest type uses as dielectric strips of acetyl cellulose known under the name of 'Cellon.' This material is remarkably suitable for the purpose, as it has good electrical properties, and can be obtained in large sheets or strips of very uniform quality. Most important of all, it is free from fissures and air pockets which have been the cause of much trouble with condensers using other dielectric materials. In the manufacture of cellon condensers, strips of the material are silver plated on each side so that the plating forms the metallic conductors of the condenser. This plating is carried out by chemical means by passing the band of dielectric through appropriate chemical baths. A high-resistance film is then applied to the edges of the metal with the object of producing a gradual tailing-off of the potential at these points, and avoiding trouble due to local stress concentration. After treatment, the strips are wound up into rolls which are fitted into short insulating tubes provided with metal end-caps making cartridge elements which can be connected in series. Several of these cartridges are connected together and mounted inside a porcelain tube container, which is fitted with metal end-caps to provide terminal connections for the condenser. Any number of these porcelain-covered condenser units can be connected in series by screwing the metal caps together, and Fig. 142 illustrates such an arrangement with a disconnecting fuse-switch mounted on the top. This fuse-switch is of a special design and introduces a high resistance into circuit just prior to disconnection,

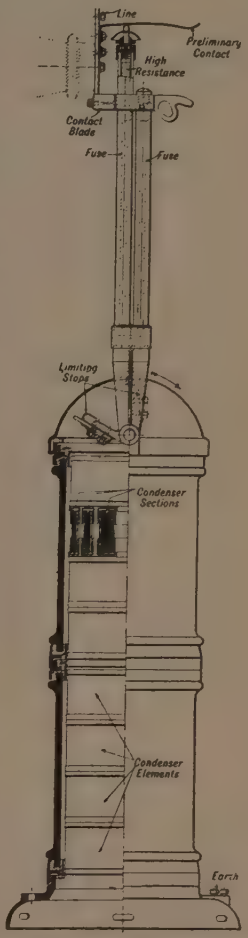

FIG. 142. — High-tension condenser with disconnecting fuse-switch. (Isenthal & Co.)

so that the arcing is minimised, and there is no trouble set up by pressure rises due to this cause. The condenser can be fitted with rain shields and is then very suitable for outdoor installation, or it may be arranged for suspension mounting.

Although condensers are permanently in circuit, the power taken by them is exceedingly small because the current at supply frequency is small, and furthermore it leads the applied voltage by nearly 90 degrees. The departure of the current from true quadrature depends upon the magnitude of the dielectric losses and these are quite low. When installing condensers, choking coils are connected between them and the apparatus to be protected. These choking coils act as an impassable barrier to the high-frequency waves, which have then to find their way to earth through the condenser.

201. Surge Absorbers.—It must not be understood from the above remarks that condensers are able to dissipate the energy of a discharge as though the discharge had taken place through a resistance. As a matter of fact a condenser can only absorb or dissipate that small percentage of the energy represented by the losses in the dielectric. The action of the condenser is really to shunt the high-frequency discharge away from the apparatus to be protected, and the dissipation of the energy is due to the resistance of the line conductors and the earth connections. It thus appears desirable to place in series with the condensers a non-inductive resistance, so as to rapidly absorb the energy of a disturbance. Such a combination of resistance and capacitance is known as a surge absorber.

Another type of surge absorber is provided by an arrangement of a non-inductive resistance in parallel with an inductance coil. Both of these devices were first proposed by Campos,[11] and undoubtedly have a high protective value. The best conception of their action is obtained by considering them as resistances to which a condenser or inductance is connected in series or shunt to act as a switch. This imaginary switch automatically connects resistance in circuit when a high-frequency discharge of energy occurs, and automatically disconnects it when conditions are normal and resistance is no longer required.

It is now standard practice on some transmission systems to lead the current from an overhead line into terminal stations through 200 or 300 feet of underground cable. Such a cable

forms a type of surge absorber capable of dissipating considerable quantities of high-frequency energy, owing to the combined effect of dielectric and sheath losses.

202. Arcing Ground Suppressors.—On fully-insulated transmission systems, arcing grounds are always liable to give trouble. Furthermore, should a spill-over occur on a line insulator, there is the possibility of the arc continuing during an appreciable length of time sufficient to do serious damage to the insulator, even if it should not totally destroy it. In order to protect against trouble due to these causes, the arc suppressor due to Creighton [12] has

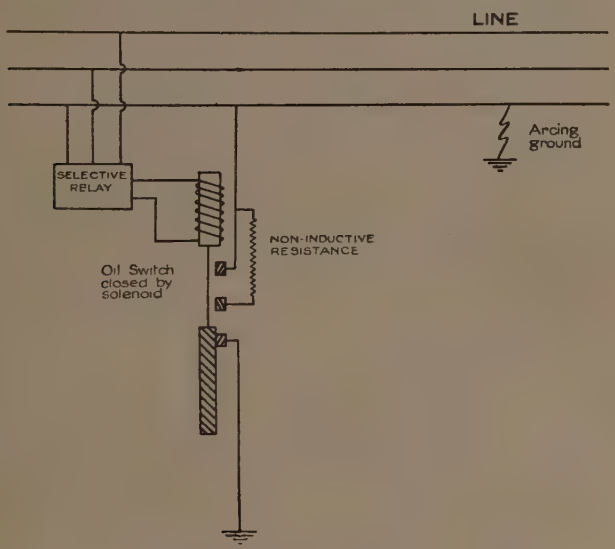

FIG. 143.—Diagram of arcing ground suppressor.

been occasionally used. This is an automatic device for momentarily short-circuiting the arc through a switch. By providing a metallic connection between the conductor and earth the arc is suppressed, and it will usually not re-form when the switch is again opened because the air in the path of the arc has had time to cool, and the line pressure, which was sufficiently high to maintain the arc once started, is not able to break down the insulation of the new layers of cooler air.

The diagram of Fig. 143 illustrates the principle of action of the apparatus. The chief feature is the selective relay which energises the solenoid operating the switch on the faulty line.

The relay operates on the principle of a leakage detector. For high-pressure systems it is of the electrostatic type, but for low voltages it works on the electromagnetic principle. When the relay operates, it picks out that phase which is brought near to earth potential by a fault, and through the intermediary of the trip coil and oil switch connects the faulty phase to earth for a fraction of a second, after which it is automatically disconnected again. Of course three earthing oil switches are employed, one in each phase, but only one is shown in the figure. Owing to the increasing practice of earthing the neutral point of three-phase systems the need for the arc suppressor has almost disappeared, and at the present time it is chiefly of historical interest.

203. Petersen Coil.—The neutral earthing reactor proposed by Petersen [13] is another device sometimes employed to protect against

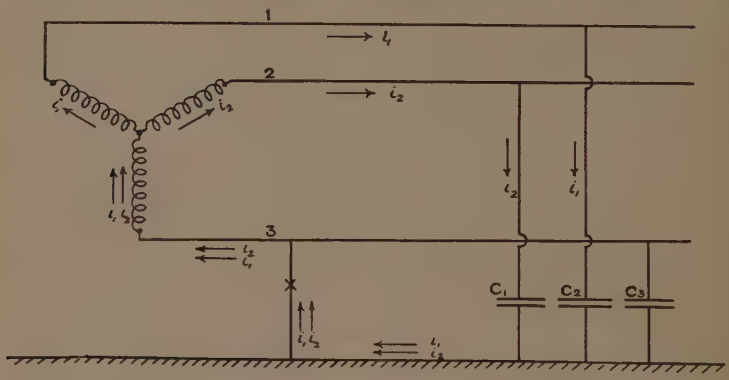

FIG. 144.—Flow of currents in three-phase system with arc from one phase to earth.

the effect of arcing grounds on those transmission systems for which earthing is not desirable for one reason or another. For example, the terminal apparatus may not be able to safely withstand short-circuit, or a single transmission circuit only being in use frequent interruptions to the supply cannot be tolerated. The device itself consists of an air-core reactor connected between the neutral of the circuit and earth. The reactance is of such a value as to neutralise the capacitance of the circuits when an accidental earthing with arc formation occurs. Under this accidental condition the reactor is electrically in parallel with the active capacitances, and the residual current is of very small magnitude.

The theory of operation can be understood by referring first to

Fig. 144, which represents the high-voltage side of the transformer bank, and the transmission circuit on which there is an arc from phase 3 to earth. The charging currents will flow through the capacitances c_1 and c_2 from phases 1 and 2 to earth, through the arc back on conductors 3 to the transformer, and thence to conductors 1 and 2 as shown. Capacitance c_3 is short-circuited by the arc. Now if a reactor is inserted between the neutral and earth as in Fig. 145, a current i_l will flow through the reactor and the arc as shown. i_l is opposite in phase to $i_1 + i_2$, and if a suitable value of reactor is chosen the current through the arc will be neutralised, and the final flow of current will be as shown. In actual practice, there is an active component of current due to the losses in the reactor and lines, and the current cannot be balanced out but exists

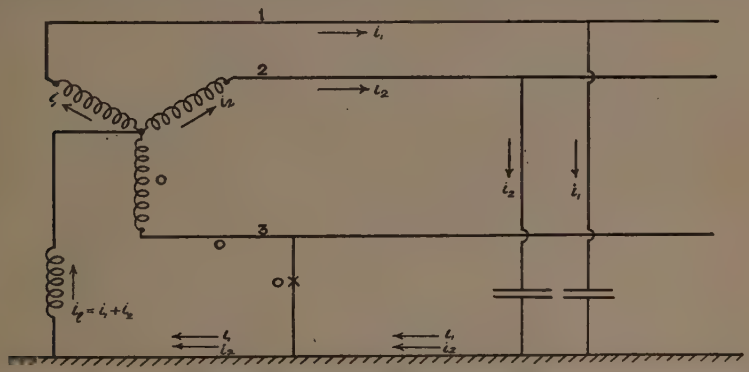

Fig. 145.—Flow of currents in three-phase system with neutral earthing reactor, and arc from one phase to earth.

as a small residual current through the arc. This, however, is soon extinguished, as it is insufficient to support a permanent arc.

It will be obvious that the conditions are entirely changed if the fault is a definite leak, which has not the re-sealing features of an earth arc on extinction, and there is nothing in the conditions outlined which allows of discriminating protection. It is convenient therefore to install a time-lag relay, which on a sustained discharge through the reactor will operate to close a switch, shunting the reactor and thereby solidly earthing the neutral, and throwing the business of clearing the fault on to the other protective gear installed.

Although there are a number of these coils in use on the

Continent, the device does not appear to be of general application. In particular, it is not quite satisfactory for installing on transmission systems operating at the highest voltages, or even on moderate-voltage systems if the length of transmission is very great. For in this case the charging current is large, and the reactance required for the Petersen Coil is correspondingly low. The reactance of the transmission line, on the other hand, is high, so that it is difficult to maintain a balance for arcs at different points on the line, and there may be enough residual current to maintain the arc. There are other technical difficulties associated with the operation of the neutral earthing reactor, and there is no doubt that the device has only a limited field of application.

204. Water-jet Earthing Resistances.—These devices have been much employed on the Continent to relieve overhead lines

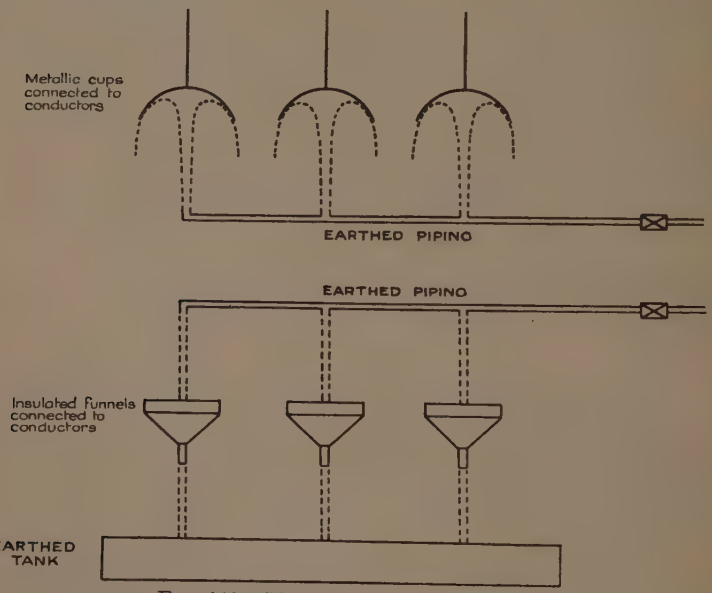

FIG. 146.—Water jet earthing resistances.

of dangerous electrostatic charges. They consist essentially of permanent leaks to earth through continuous jets of water. The diagrams in Fig. 146 show two common types. In the first, the jets of water issue from earthed nozzles and impinge against inverted metallic cups connected by means of isolating switches to the line. In the second type, the jets are at the top and the

water flows into insulated troughs or funnels each connected to one pole of the system, and thence into an earthed tank. In this case, each pole has two leakage paths to earth in parallel, so that it is often used in places where the water is very pure and therefore of high resistance.

Since these water jets constitute a permanent leak to earth they are very efficient in relieving the line of gradual accumulation of static charges, but the resistance is naturally very high in order to prevent the loss of too great an amount of energy. For example, the Allgemeine Elektricitäts Gesellschaft recommend that the leakage current for a 77 000-volt system should have a value of about 0·08 amperes, corresponding to a resistance for each jet of 555 000 ohms.[14] It is obvious that a resistance of this value is utterly incapable of giving any protection against steep-fronted travelling waves due to lightning effects. Furthermore, the safeguard against static over-potential is very dearly bought, especially if the water jets are permanently in circuit. Taking the 77 000-volt case again, and assuming that the cost per unit of energy at the station is 0·5 pence, the value of the wasted energy would be about £200 per annum.

To reduce the loss of energy it is common practice to cut the water jets out of circuit unless thunderstorms are expected, but this reliance on the human element renders the apparatus less valuable than an equally effective device which is always ready to act.

From the above it will be seen that water-jet earthing resistances are able to deal only with static charges on the line, and only then if the accumulation of charge is slow. Since there are other types of arrestor which will safeguard the line against this effect in addition to travelling waves, the installation of water-jet earthing resistances can hardly be justified.

205. Earthing Choking Coils.—For large systems, where it is necessary to provide means to dissipate static charges, the water jet has often too high a resistance and earthing choking coils have to be resorted to. These have the advantage of dissipating very little energy because the current through them is practically wattless. Also there is no waste of water which, in any but a hydro-electric station, might be a serious item.

In order to have a sufficiently large reactance and prevent too much current flowing to earth these coils have a laminated iron

core, so that there are small iron losses in addition to small copper losses. But, since the inductance of the coil is the main factor in limiting the permanent earth current, the resistance can be made so small that even rapidly accumulating static charges can be successfully dealt with.

These choking coils are of no value in relieving the line of travelling wave effects because, their reactance being proportional to the frequency of the current flowing through them, they will offer an enormous impedance to high-frequency waves. Their function, like that of water-jet earthing resistances, is to relieve the line of static over-potentials only.

Earthing choking coils are connected between each line and earth, so that their insulation will normally have to withstand the star voltage of the system. They have to be insulated, however, for the full line voltage, since that is the voltage the insulation will have to withstand if one of the lines should become earthed.

Of course, if the neutral point of the transmission system is earthed solidly, or through a resistance, a path to earth is provided for any static charges on the conductors, thereby rendering unnecessary any special apparatus for this purpose.

206. Modern Tendencies.—The more important types of apparatus for the protection of transmission lines and terminal stations from the effects of pressure rises have now been described. There are many devices to choose from, each of which has its own particular field of usefulness, and it is now possible to provide reasonable protection against discontinuity of service during atmospheric disturbances.

For the highest voltages, *e.g.* over 110 kilovolts, there is a tendency to do away altogether with lightning-arrestor equipment, and to rely solely on the insulation. The pressure of the system is already so exceedingly high that if the insulation of the line and the transformers is designed with a reasonable factor of safety it should stand up to almost anything in the nature of an over-voltage, with the possible exception of a direct lightning stroke of great severity.

It is also probable that the near future will see some developments with the idea of making the line more or less self-protective. For example, it has been suggested that the properties of the corona might be utilised as a protection against lightning and

other similar types of disturbance. The spacing and sizes of conductor of transmission lines could often be arranged so that the normal operating voltage was only just below the corona-forming voltage. If an over-voltage then occurred, the energy of the disturbance would be quickly dissipated in the corona itself.

For low- or medium-voltage lines the spacing necessary for corona formation would be so small as to constitute a serious danger of the wires swinging together, or of flash-over. In these cases it would be possible to equip lines of standard spacing with some form of studs or points, which would produce a corona-discharge curve equal to or better than the normal corona power-loss curve. Experiments have been made by Nagel[15] and Whitehead,[16] using barbed wires, and also wires having mounted on them corona-forming studs, with satisfactory results. Apparently sufficient protection for the station apparatus could be obtained by arranging for this artificial corona formation on the two terminal miles at each end of the transmission line.

207. The Potentials of a Three-phase Transmission System.—Before going on to discuss the subject of earthing, the potential relations to earth will be considered of a three-phase system on which no deliberate earthing has taken place.

On a three-phase system with solidly-earthed neutral the potential difference between any conductor and earth is equal to the phase-to-phase voltage divided by $\sqrt{3}$. On the other hand, if the neutral is not earthed, the position of the earth-potential point is unstable, and varies with the relative condition of the insulation on the three phases. Its position also depends on the magnitude of the capacitance current of the system. A simple means of determining the potential of such a system has been given by Field.[17]

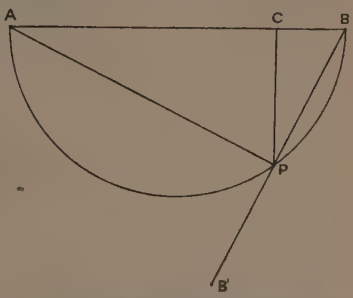

FIG. 147.—Single-phase system with leakage and capacitance.

First of all consider the case of a non-earthed single-phase system whose voltage is represented by the rotation of the vector in Fig. 147. If the pole A be connected to earth the

diagram may be considered as rotating round the point A as centre. Similarly, if the pole B is earthed instead of A, the vector would rotate round B as centre. If, however, the sole connections of A and B to earth are by the leakage conductances of the insulation, the point at earth potential will be in some position between A and B, depending on the relative magnitude of these leakage conductances.

Assume now, that there is a susceptance to earth of B farads in connection with pole A, and a path to earth of conductance G mhos in connection with pole B. The current to earth through the susceptance at A will be 90 degrees ahead of the pressure producing it, whereas the current through the conductance at B will be in phase with the pressure. The position of the zero-potential point must be such that these two currents are equal and opposite at every instant. It will be found that the locus of the zero-potential point is a semicircle described upon the line AB as diameter, as shown in Fig. 147.

The diagram will then rotate round a point P as centre, where PA represents in phase and magnitude the pressure acting on the susceptance between pole A and earth. This will produce a current 90 degrees in advance of the pressure in the direction PB' and of amount $B \times \overline{PA}$ amperes. On the other hand, the vector PB represents in phase and magnitude the pressure acting upon the conductance G and producing in it the current $G \times \overline{PB}$ amperes in the direction PB.

The condition that P is at earth potential is that these currents are equal and opposite, that is

$$B \times \overline{PA} = G \times \overline{PB}$$

or

$$\frac{\overline{PA}}{\overline{PB}} = \frac{G}{B}.$$

If a perpendicular PC is dropped from the point P the following geometrical relations are obtained

$$\frac{\overline{CA}}{\overline{CB}} = \frac{\overline{PA}^2}{\overline{PB}^2} = \left(\frac{G}{B}\right)^2.$$

The combined effect of capacitance and leakage in the case of a three-phase system may now be considered. Taking first the effect of leakage, a point P' can be found on the diagram of Fig. 148

which will represent the position of earth potential due to this cause. It can be readily shown that the position of P' is the same as the centre of gravity of three masses placed at A, B, and C, and proportional to the conductances of the leakage paths to earth from these respective phases.

For example, assume that the system comprises a transformer and overhead line, the leakage conductances of each phase of the complete system being

$$\begin{aligned} \text{Phase } A &= 2 \times 10^{-4} \text{ mhos,} \\ B &= 4 \times 10^{-4} \text{ mhos,} \\ C &= 6 \times 10^{-4} \text{ mhos,} \end{aligned}$$

and the sum of the conductances G' thus being 12×10^{-4} mhos.

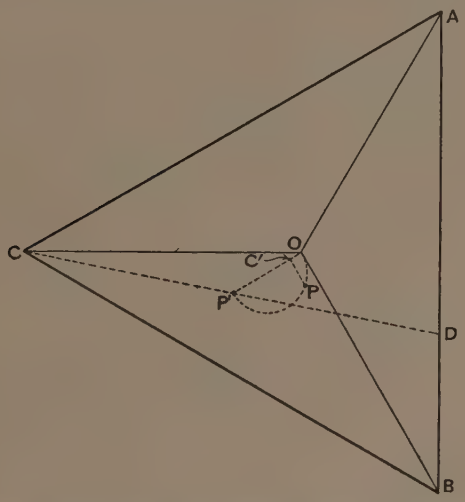

FIG. 148.—Effect of leakage and capacitance currents on a three-phase system.

Neglecting capacitance currents, the currents from A and B alone would be equal if the line AB revolved round the point D, the lengths BD and AD being such that $4\overline{BD} = 2\overline{AD}$. The total leakage path from A and B may now be considered as concentrated at D, and as in this particular instance the leakage conductance from C has an equal value, the point P' midway along the line CD will represent the point at earth potential.

The effect of capacitance currents must now be taken into account. The same value of susceptance usually applies to each conductor owing to the practice of transposing any unsymmetrical circuits.

Hence capacitance effects can be taken into consideration by assuming a susceptance equal to three times the conductor susceptance connected at O, the central point of the diagram. The total conductance of the leakage paths may similarly be considered as concentrated at the point P'.

Upon OP' describe a semicircle and divide OP' at a point C' such that

$$\frac{\overline{OC'}}{\overline{C'P'}} = \left(\frac{G'}{3B}\right)^2.$$

From the point C' draw a perpendicular to meet the semicircle at the point P. The point P will then be the zero-potential point in the diagram.

It is important to note the considerable stability given to the neutral point of a three-phase transmission system by the capacitance currents flowing in the conductors.

208. Earthed *versus* Isolated Neutrals.—The conditions arising out of operation with earthed or isolated neutrals have to be borne in mind when installing all protective devices, whether for the prevention of dangerous currents, or potentials. There has been for many years a divergence of opinion on the subject, but modern practice strongly inclines towards earthing for the following reasons:—

1. On a solidly-earthed system, the voltage to earth at all points is definitely limited to the star voltage of the system. This often permits of considerable economies in the matter of insulation. For example, transformers for the highest voltages can be built with the neutral connected directly to the core, and only one high-tension bushing. Such a transformer would be insulated for 2·73 times the star voltage, instead of 3·46 times, which is standard for transformers with isolated neutral.

2. All trouble due to arcing grounds or static pressure rises is eliminated.

3. Many of the most important relay protective devices can only operate to full advantage if the neutral is earthed.

The chief argument in favour of the isolated neutral is the possibility of maintaining continuity of service in case one phase becomes earthed until it is convenient to disconnect and repair the faulty line. This has been done in the early days of power transmission, when mileage was small and voltage low. With the more

extensive systems of the present day this method of operation is impracticable, owing to the fact that most earths are not of a solid nature but are accompanied with severe arcing. Thus, not only are the unearthed conductors subject to a pressure to earth equal to $\sqrt{3}$ times the normal value, but high-frequency disturbances are set up which place the whole system under severe strain, and often result in break-down of apparatus at different points of the system.

In the case of an earthed-neutral system, there will occur practically as many primary line failures, such as break-down of insulators or other apparatus, but as soon as one phase becomes earthed a short-circuit is produced on one leg of the earthed-neutral transformer. The current in this short-circuit flows over definite and known paths which renders possible the use of automatic protective devices to disconnect the faulty line. The selective action of relay apparatus is thus made more positive, and secondary break-downs due to pressure rises are almost unknown. Furthermore, as most systems feed their important stations by more than one circuit, the loss of a line due to trouble does not necessarily involve any stoppage of the supply.

The trend of opinion is clearly evidenced by the result of an inquiry held by a Sub-committee of the American Institute of Electrical Engineers on the conditions prevailing in that country.[18] Out of eighty systems operating at between 15 000 and 150 000 volts, and representing a total mileage of 21 140, from which replies were received, it was found that a mileage of 15 660 or 74 % was earthed either solidly or through a resistance.

As regards methods of earthing, this is usually done at the sending station, a neutral point on the high-voltage circuit being obtained by using delta-star raising transformers. Where solid earthing is resorted to, transformer neutrals at the receiving stations or substations are often earthed as well. The only difficulty introduced by having several earths on one network is in the calculation of the short-circuit current. Such calculations are comparatively simple with only one neutral earthed, but where several are involved each contributes to the earth current, the amount and direction depending upon the location of these earth points. It is thus sometimes very difficult to predetermine the earth current with sufficient definiteness as to allow of accurate selection and adjustment of protective relays.

The question of the desirability of limiting the earth current

in transmission systems is one on which opinion varies. There is little doubt that the greatest protection from high-voltage strain comes from the use of a solidly-earthed neutral, and this is also the best arrangement from the point of view of relay protection. Providing, then, that the terminal apparatus is designed to withstand the magnetic and thermal effects of absolute short-circuit currents for the time necessary for the relays to clear the circuit, no disadvantage is entailed by earthing solidly. This is also the more popular method, as is shown by the fact that of the 15 660 miles of line mentioned above as operating with neutral earthed, 13 040 miles or 62 % of the total mileage was dead-earthed.

In cases where from one reason or another a current-limiting resistance between neutral and earth is requisite, this should be of comparatively low value. The maximum voltage which such a resistance would be called upon to bear is the voltage to neutral of the system, and the maximum current to be passed is that required to operate the circuit-breaker on the heaviest line. It is obvious that this current will be much heavier if the lines are only protected with overload devices and not leakage or balanced protective gear. Even if the latter be used, however, overload protection will in most cases be used as well, so that the earthing resistance should in all cases, if possible, be of capacity enough to carry for the requisite time the current necessary to trip the overload gear on the heaviest line.

Earthing resistances are usually built up of cast-iron grids, and must be insulated for the voltage to neutral excepting only at the end where they are permanently connected to earth. Their time rating at maximum current usually varies between 30 to 60 seconds, during which interval their temperature rise may be about 400 degrees C. When used on high-voltage systems they are very expensive, and take up a large amount of room. They are also liable to be burnt out in service.

Another type of earthing resistance is the Brazil carbon-powder, or in its later form, the carbon-slab resistance. Cast-iron grids increase in resistance with temperature, but the carbon type possesses a negative temperature coefficient, so that when red-hot the resistance is only a small fraction of its cold value. It is thus possible to arrange the initial resistance to be much higher than with other types of apparatus. If the fault current which first passes is not sufficient to open the circuit-breakers it continues to

flow until, the resistance having gone down, the current has increased by the required amount. The shock to the system due to a fault is thus considerably minimised.

Earth connections used in transmission work fall into two categories: those used solely for lightning arrestors, and those used in conjunction with relay protective apparatus. In the case of the former, the ohmic value of the earth connection itself is not particularly important. Fifteen or twenty ohms is probably good enough, since it is usually in series with protective resistances of relatively much greater value, and when called into action there will be an extremely high potential impressed across the combination. On the other hand, the earth contact resistance for relay protection must be as low and as constant as possible, since the series resistance is relatively small or non-existent, and the close setting of relays depends on an accurate knowledge of the circuit conditions as a whole.

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[18] 'Present Day Practices in Grounding of Transmission Systems.' *First Report of Sub-committee on Grounding of Protective Devices Committee, Trans. A.I.E.E.*, Vol. 42, p. 446 (1923).

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PRINTED IN GREAT BRITAIN BY THE ABERDEEN UNIVERSITY PRESS, ABERDEEN